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Exercises on Quantum Chromodynamics problem sheet 2

Worksheet 2: Annihilation of an electron-positron pair in a pair of muons.

The goal of this exercise is to calculate the total cross section of the reaction

$$e^-(p) + e^+(p') \to \mu^-(k) + \mu^+(k'). \tag{1}$$

Since electron mass is very much smaller than that of a muon, it can be neglected.

Problem 1

Calculate the invariant matrix element of the reaction (1) averaged over the electron polarizations and summed over the muon polarizations, in the approximation of massless electrons, and show that it equals

$$\begin{aligned}|M|^2 =& \frac{1}{4}\frac{e^4}{s^2}\,\mathrm{Tr}\left[\gamma^\rho(\not{k}' + m)\gamma^\sigma(-\not{k} + m)\right]\mathrm{Tr}\left[\gamma_\sigma\,\not{p}'\,\gamma_\rho\,(-\not{p})\right]\\ =& \frac{8e^4}{s^2}\left[(k,p)^2 + (k,p')^2 + m^2(p,p')\right]\end{aligned} \tag{2}$$

where $s = (p + p')^2$ and m is the muon mass.

Problem 2

Re-express $|M|^2$ in terms of the center-of-mass scattering angle θ and the invariants s and m.

Problem 3

Since the initial state and the final state particles have different mass, we cannot simply use the expression $d\sigma/d\Omega = 1/(64\pi^2 s)|M|^2$ as in the lectures, but have to start with the master formula that relates cross section to $S-$matrix elements:

$$d\sigma = \frac{1}{\mathcal{J}}|M|^2 d\Phi\,, \tag{3}$$

where $\mathcal{J}$ is the initial-state flux factor and $d\Phi$ is the elementary final-state phase space. Show that for the present case $\mathcal{J} = 2s$ and $d\Phi = \sqrt{1 - 4m^2/s}\,d\Omega/(32\pi^2)$.

Problem 4

Show that the annihilation cross section is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{e^+e^-\to\mu^+\mu^-} = \frac{\alpha^2}{4s}\sqrt{1-\frac{4m^2}{s}}\left[1+\frac{4m^2}{s}+\left(1-\frac{4m^2}{s}\right)\cos^2\theta\right]. \tag{4}$$

Problem 5

Show that the total cross section in the high-energy limit $s \gg m^2$ equals

$$\frac{4\pi\alpha^2}{3s} \tag{5}$$

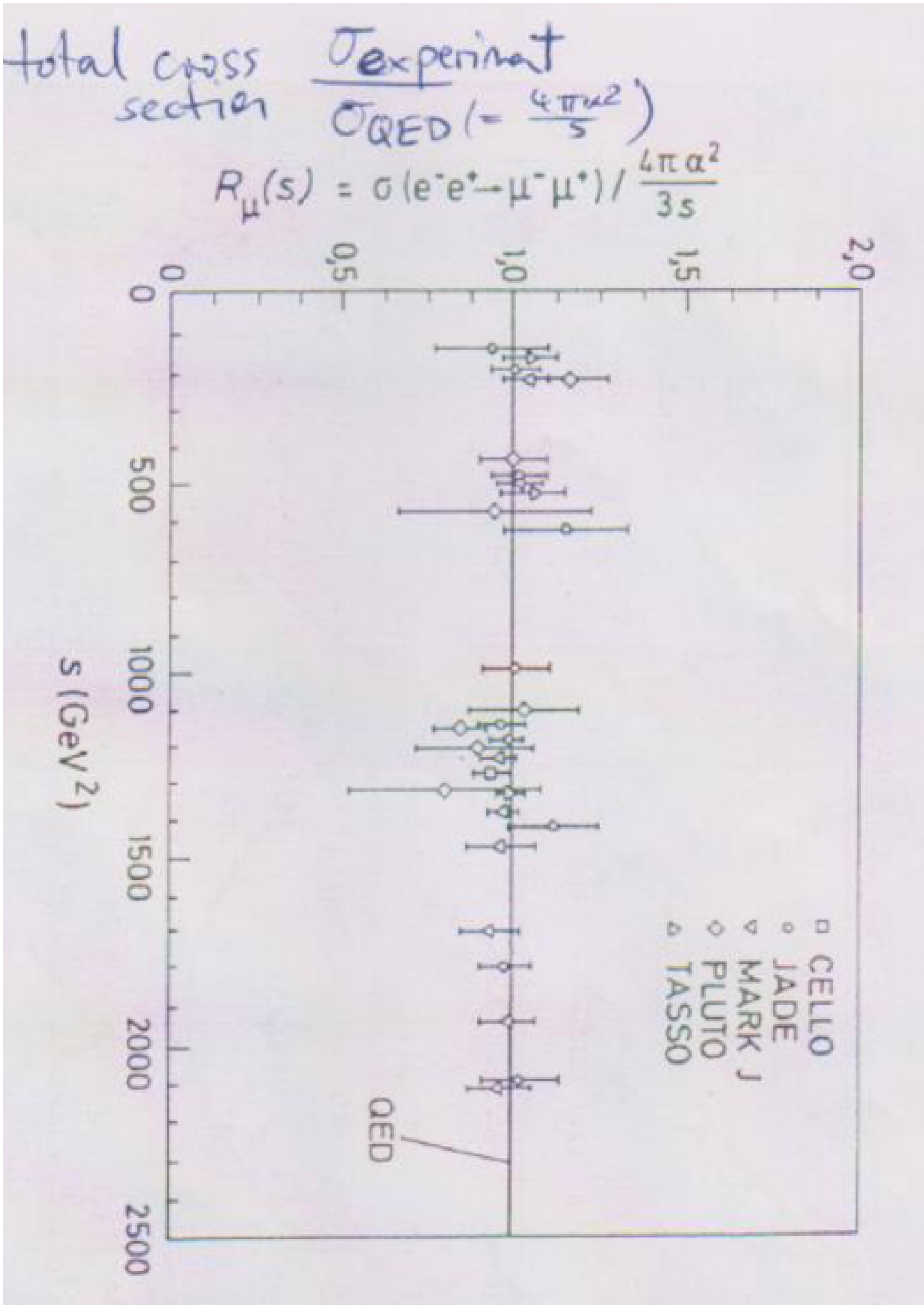

Figure 1: Ratio $R_{\mu\mu}$ of experimental measurement relative to theory prediction, as a function of the centre of mass energy s. From Haidt (1984). See also Z.Phys.C 26 (1985) 507

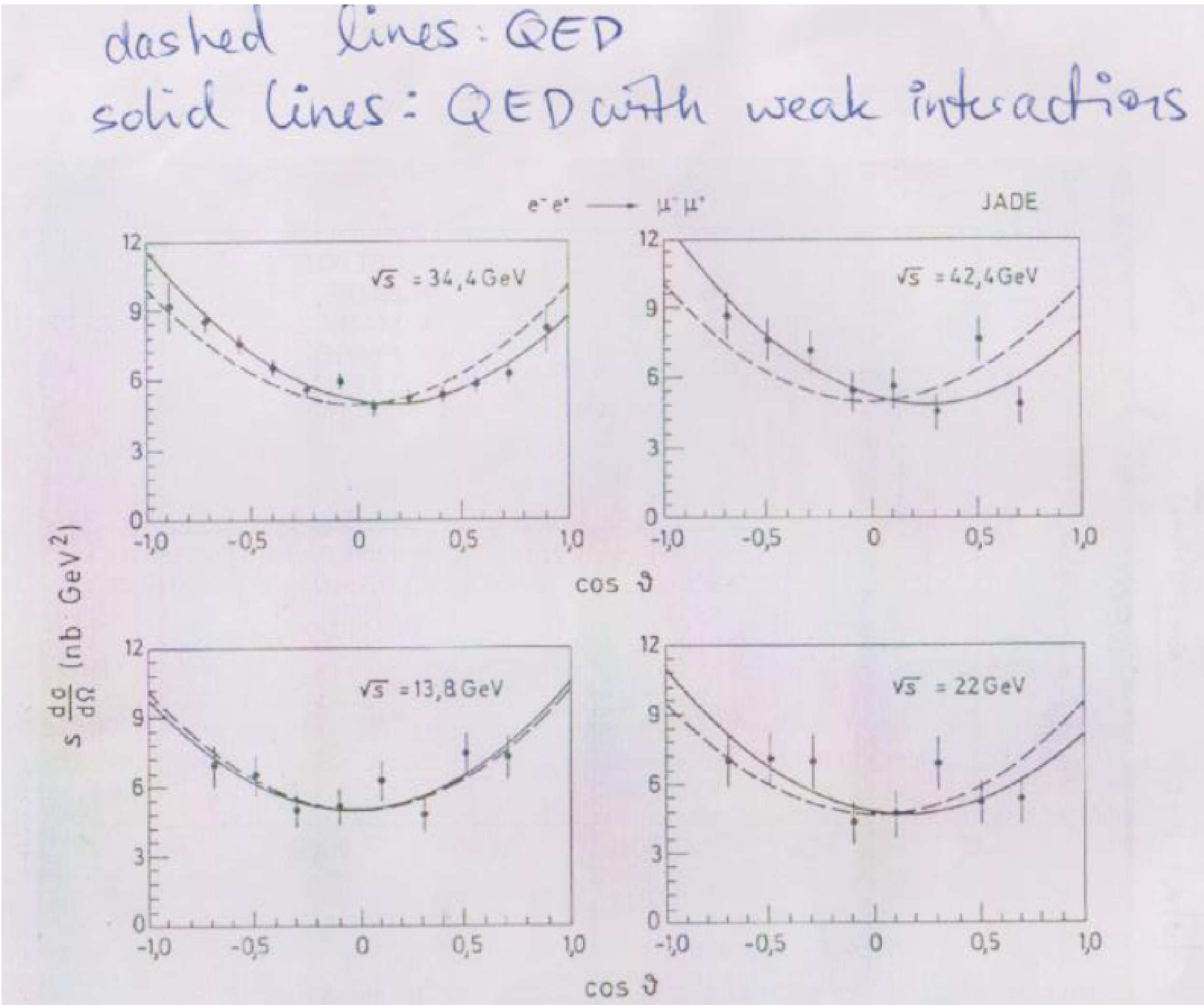

Figure 2: Angular distributions for $e^+e^- \to \mu^+\mu^-$. Data from JADE collaboration at given centre of mass energies $\sqrt{s}$. Dashed lines are predictions of QED, full lines take into account effects of the weak interaction. From Bartel (1985). See also Z.Phys.C 26 (1985) 507