

Exercise: Quantum Computing

Problem set 12 (to be discussed in week of January 30th, 2023)

Problem 1 Higher-order Trotter formula

Show that

$$e^{t(A+B)} = \left(e^{tA/N/2} e^{tB/N} e^{tA/N/2} \right)^N + O(t^2/N^2) \quad (1)$$

for $t \in \mathbb{R}$, $N \in \mathbb{Z}^+$, and complex matrices A and B .

Problem 2 Analytic solution of two-site Ising model

Consider the one-dimensional transverse-field Ising model with two sites and open boundary conditions, i.e.,

$$H = -\frac{1}{2}Z_1Z_2 + \frac{g}{2}(X_1 + X_2) \quad (2)$$

acting on a Hilbert space of two qubits.

- Compute the eigenvectors and eigenvalues of H
- Use these results to construct the transfer matrix $T = e^{iHt}$ with $t \in \mathbb{R}$
- Without re-doing the entire calculation, what is the transfer matrix for the case of periodic boundary conditions (discussed in the Tuesday lecture)?
- Compute the state

$$|\psi(t)\rangle = e^{itH} |00\rangle \quad (3)$$

and

$$\langle Z(t) \rangle \equiv \frac{1}{2} \langle \psi(t) | (Z_1 + Z_2) | \psi(t) \rangle . \quad (4)$$