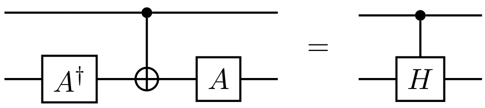
Universität Regensburg, Institut für Theoretische Physik
Prof. Dr. Christoph Lehner, Andreas Hackl Winter 2022/23

Exercise: Quantum Computing
Problem set 2 (to be discussed in week of November 7, 2022)

Problem 1 Controlled-U

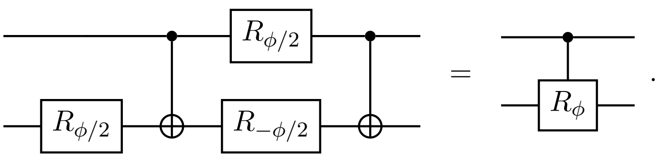
Show that



with

$$A = R_{\pi/2} H R_{\pi/4} \tag{1}$$

and



Solution: We first show that

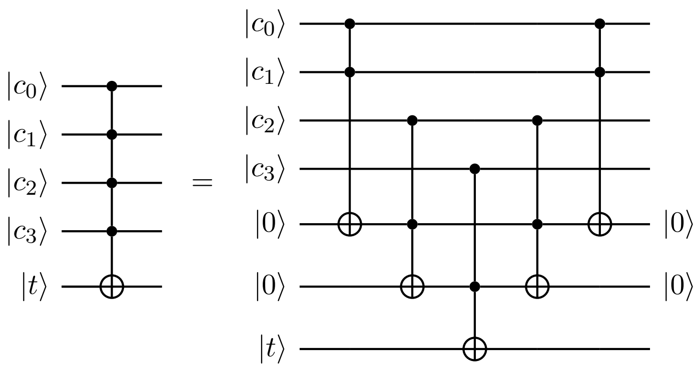
$$AXA^\dagger = H \tag{2}$$

such that

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1+i}{2} \\ \frac{i}{\sqrt{2}} & \frac{1-i}{2} \end{pmatrix} \tag{3}$$

Problem 2 C^{n}NOT with work qubits

The recursive definition of C^{n}NOT given in the lecture has exponential cost for large n. If we have $n-2$ additional "work qubits", we can implement a gate whose cost only grows linearly with n. Show for $n = 4$ that



and then generalize this to n gates.

Problem 3 Generalization of Deutsch-Jozsa algorithm

Let us consider a generalization of the Deutsch-Jozsa algorithm, where the input function f is not constrained to be either constant or balanced. Consider the scenarios of measuring $r = 0$ and $r \neq 0$. Show that measuring $r \neq 0$ guarantees that the function is not constant and measuring $r = 0$ guarantees that the function is not balanced.

Problem 4 Quantum parallelism

Write a circuit for U_f with $N = 2$ and $f(x) = x \bmod 2$, i.e., $f(x) = 0$ if x is divisible by 2 and $f(x) = 1$ in all other cases. Generalize the circuit to general N.

Problem 5 Deutsch-Jozsa algorithm for $N = 4$ (optional)

Implement the Deutsch-Jozsa algorithm for the function of Problem 4 in the quantum computing simulator (http://github.com/lehner/sqc) for the case of N=4.