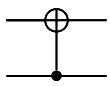
Universität Regensburg, Institut für Theoretische Physik
Prof. Dr. Christoph Lehner, Andreas Hackl Winter 2022/23

Exercise: Quantum Computing
Problem set 1 (to be discussed in week of October 31st, 2022)

Problem 1 Matrix representation of CNOT

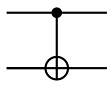
In the lecture we showed that the matrix representation of



is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} . \tag{1}$$

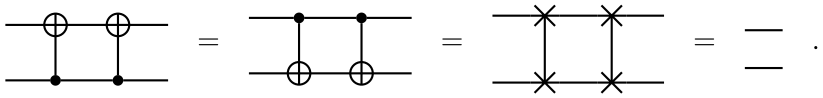
Remember that we draw the least-significant qubit at the top and use the standard basis $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$. Derive the matrix representation for



in the same basis.

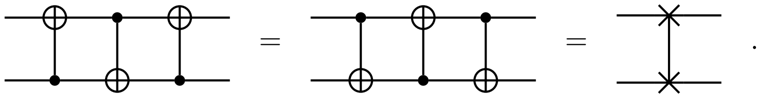
Problem 2 Inverse of CNOT

Show that



Problem 3 Relation of SWAP and CNOT

Show that



Problem 4 Universal one qubit gates

Show that the H and R_ϕ gates can generate an arbitrary two-dimensional unitary matrix. You may first show

$$X = HR_\pi H \tag{2}$$

and then restrict yourself to infinitesimal unitary matrices of the form

$$U = \mathbb{1} + i\varepsilon M + O(\varepsilon^2) \tag{3}$$

with Hermitian matrix $M^\dagger = M$ and $\varepsilon \in \mathbb{R}$. Show that

$$U_1 = H R_{\varepsilon 2x} H\,, \qquad U_2 = R_{\pi/2} H R_{\varepsilon 2y} H R_{-\pi/2}\,, \qquad U_3 = R_{\varepsilon z}\,, \qquad U_4 = X R_{\varepsilon t} X \tag{4}$$

with $x, y, z, t \in \mathbb{R}$ are all of this form and their product maps out a general M.

Problem 5 Universal gates

Show that adding a CNOT gate to a universal one-qubit gate is sufficient to generate an arbitrary unitary matrix in S^N. Use the gate $s^{(ij)}$ defined in the lecture to extend the proof of Problem 4 to the general case of M in 2^N dimensions.