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Summer 2020

Exercise: Quantum Computing

Problem set 12 (to be discussed in week of July 20st, 2020)

Problem 1 Higher-order Trotter formula

Show that

$$e^{t(A+B)} = \left(e^{tA/N/2}e^{tB/N}e^{tA/N/2}\right)^N + O(t^2/N^2)$$
(1)

for $t \in \mathbb{R}$, $N \in \mathbb{Z}^+$, and complex matrices A and B.

Problem 2 Analytic solution of two-site Ising model

Consider the one-dimensional transverse-field Ising model with two sites and open boundary conditions, i.e.,

$$H = -\frac{1}{2}Z_1Z_2 + \frac{g}{2}(X_1 + X_2) \tag{2}$$

acting on a Hilbert space of two qubits.

- a) Compute the eigenvectors and eigenvalues of H
- b) Use these results to construct the transfer matrix $T = e^{iHt}$ with $t \in \mathbb{R}$
- c) Without re-doing the entire calculation, what is the transfer matrix for the case of periodic boundary conditions (discussed in the Tuesday lecture)?
- d) Compute the state

$$|\psi(t)\rangle = e^{itH} \left|00\right\rangle \tag{3}$$

and

$$\langle Z(t) \rangle \equiv \frac{1}{2} \langle \psi(t) | (Z_1 + Z_2) | \psi(t) \rangle .$$
(4)