Exercise: Quantum Computing

Problem set 2 (to be discussed in week of May 11, 2020)

Problem 1 Controlled-U

Show that

\[ A^\dagger \otimes A = H \]

with

\[ A = R_{\pi/2}HR_{\pi/4} \]  \hspace{1cm} (1)

and

\[ R_{\phi/2} \otimes R_{-\phi/2} = R_\phi. \]

Problem 2 C\textsuperscript{n}NOT with work qubits

The recursive definition of C\textsuperscript{n}NOT given in the lecture has exponential cost for large \( n \). If we have \( n - 2 \) additional “work qubits”, we can implement a gate whose cost only grows linearly with \( n \). Show for \( n = 4 \) that

\[
\begin{pmatrix}
|c_0\rangle \\
|c_1\rangle \\
|c_2\rangle \\
|c_3\rangle \\
|t\rangle
\end{pmatrix} = \begin{pmatrix}
|c_0\rangle \\
|c_1\rangle \\
|c_2\rangle \\
|c_3\rangle \\
|0\rangle \\
|0\rangle \\
|t\rangle
\end{pmatrix}
\]

and then generalize this to \( n \) gates.

Problem 3 Generalization of Deutsch-Jozsa algorithm

Let us consider a generalization of the Deutsch-Jozsa algorithm, where the input function \( f \) is not constrained to be either constant or balanced. Consider the scenarios of measuring \( r = 0 \) and \( r \neq 0 \). Show that measuring \( r \neq 0 \) guarantees that the function is not constant and measuring \( r = 0 \) guarantees that the function is not balanced.
Problem 4  Quantum parallelism

Write a circuit for $U_f$ with $N = 2$ and $f(x) = x \mod 2$, i.e., $f(x) = 0$ if $x$ is divisible by 2 and $f(x) = 1$ in all other cases. Generalize the circuit to general $N$.

Problem 5  Deutsch-Jozsa algorithm for $N = 4$ (optional)

Implement the Deutsch-Jozsa algorithm for the function of Problem 4 in the quantum computing simulator (http://github.com/lehner/sqc) for the case of $N=4$. 