Problem 1  Higher-order Trotter formula

Show that
\[ e^{t(A+B)} = \left( e^{tA/N} e^{tB/N} e^{tA/N} \right)^N + O(t^2/N^2) \] (1)

for \( t \in \mathbb{R}, \ N \in \mathbb{Z}^+, \) and complex matrices \( A \) and \( B. \)

Problem 2  Analytic solution of two-site Ising model

Consider the one-dimensional transverse-field Ising model with two sites and open boundary conditions, i.e.,
\[ H = -\frac{1}{2} Z_1 Z_2 + \frac{g}{2} (X_1 + X_2) \] (2)

acting on a Hilbert space of two qubits.

a) Compute the eigenvectors and eigenvalues of \( H \)
b) Use these results to construct the transfer matrix
c) Without re-doing the entire calculation, what is the transfer matrix for the case of periodic boundary conditions discussed in the lecture?
d) Compute the state
\[ |\psi(t)\rangle = e^{iH} |00\rangle \] (3)

and
\[ \langle Z(t) \rangle = \frac{1}{2} \langle \psi(t) | (Z_1 + Z_2) |\psi(t)\rangle . \] (4)