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Exercise: Quantum Computing
Problem set 12 (to be discussed in week of July 22st, 2019)

Problem 1 Higher-order Trotter formula

Show that

$$e^{t(A+B)} = \left(e^{tA/N/2} e^{tB/N} e^{tA/N/2}\right)^N + O(t^2/N^2) \tag{1}$$

for $t \in \mathbb{R}$, $N \in \mathbb{Z}^+$, and complex matrices A and B.

Problem 2 Analytic solution of two-site Ising model

Consider the one-dimensional transverse-field Ising model with two sites and open boundary conditions, i.e.,

$$H = -\frac{1}{2} Z_1 Z_2 + \frac{g}{2}(X_1 + X_2) \tag{2}$$

acting on a Hilbert space of two qubits.

a) Compute the eigenvectors and eigenvalues of H

b) Use these results to construct the transfer matrix

c) Without re-doing the entire calculation, what is the transfer matrix for the case of periodic boundary conditions discussed in the lecture?

d) Compute the state

$$|\psi(t)\rangle = e^{itH} |00\rangle \tag{3}$$

and

$$\langle Z(t)\rangle \equiv \frac{1}{2} \langle\psi(t)| (Z_1 + Z_2) |\psi(t)\rangle \ . \tag{4}$$