7.1 Let A be a commutative unital ring. how that for any big Witt complex over A the formula $V_n d = n d V_n$ holds.

7.2 Determin the square root of unity $[-1]_S$ in $\mathbb{W}_S(A)$.

1. Argue that it is sufficient to do this for $A = \mathbb{Z}$.
2. Show that for $m \in \mathbb{Z}$ there are unique integers r_e for $e \in S$ such that

$$[m]_S = \sum_{e \in S} r_e V_e([1]_{\frac{S}{e}})$$

3. Evaluate the n^{th} ghost component of this equation to find

$$m^n = \sum_{e|n} e r_e.$$

4. Show that from this by Möbius inversion the formula

$$[m]_S = \sum_{n \in S} \frac{1}{n} \left(\sum_{e|n} \mu(e) m^{\frac{n}{e}} \right) V_n([1]_{\frac{S}{n}})$$

follows.

5. Let

$$g(n) := \begin{cases} -1 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ 0 & \text{otherwise} \end{cases}$$

One gets $f(n) := \sum_{e|n} g(e) = (-1)^n$. Use this to determine

$$[-1]_S = -[1]_S + V_2([1]_{\frac{S}{2}}).$$

7.3 Determine $d \log \eta_S([-1]_S)$ for every big Witt complex.

1. Use the result from the previous exercise to show that

$$d \log \eta([-1]_S) = dV_2 \eta([1]_{\frac{S}{2}}) + V_2(d \log \eta([-1]_{\frac{S}{2}})).$$

2. By induction follow that

$$d \log \eta_S([-1]_S) = \sum_{r \in \mathbb{N}} 2^{r-1} dV_{2^r} \eta_{\frac{S}{2^r}}([1]_{\frac{S}{2^r}}).$$

3. Show moreover that

$$d \log \eta([-1]_S) \cdot d \log \eta([-1]_S) = 0.$$

4. And also that

$$dd \log \eta([-1]_S) = 0.$$

7.4 Use the previous result to show $F_n(d \log \eta_S([-1]_S)) = d \log \eta_{\frac{S}{n}}([-1]_{\frac{S}{n}})$.