12.1 In order to show existence of the maps R_T^S, d, F_n on $\mathbb{W}\,\Omega$ one has to show that applying the maps, which exist already on $\check{\Omega}$, to the kernel of the projection η yield zero in the quotient.

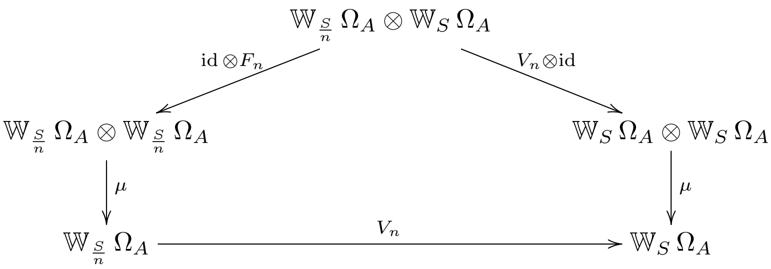
Let for $n \in \mathbb{N}$

$$\omega = \sum_\alpha V_n(X_\alpha) dy_{1,\alpha} \cdots dy_{q,\alpha} \in \check{\Omega}^q_{\mathbb{W}_S(A)}$$

such that $0 = \eta_{\frac{S}{n}} \left(\sum_\alpha x_\alpha F_n dy_{1,\alpha} \ldots F_n dy_{q,\alpha}\right) \in \mathbb{W}_{\frac{S}{n}}\,\Omega^q_A$.

1. Show that $\eta_T(R^S_T(N^q_S)) = 0$. THis has been done in class.
2. Show that $\eta_S(dd\omega) = 0$. This is a rather straight forward calculation.
3. Show that $\eta_{\frac{S}{m}} F_m(\omega) = 0$ and $\eta_{\frac{S}{m}} F_m(d\omega) = 0$. [Hint : argue, that it is sufficient to assume that $m = p$ is prime. Consider the two cases $p|n$ and $p \nmid n$.

12.2 Show that the pentagon diagram commutes.



Recall again that every element of $\mathbb{W}_{\frac{S}{n}}$ can be written as a sum of elements of the form $\eta_{\frac{S}{n}}(xF_n dy_1 \cdots dy_q)$ with $x \in \mathbb{W}_{\frac{S}{n}}(A)$ and $y_i \in \mathbb{W}_S(A)$. Then the diagram commutes by the following calculation.

$$\begin{aligned}
V_n \eta_{\frac{S}{n}}(xF_n dy_1 \cdots dy_q) \cdot \eta_S(zdw_1 \cdots dw_r) &= \eta_S(V_n(x) dy_1 \cdots dy_q) \cdot \eta_S(zdw_1 \cdots dw_r) \quad \text{by definition of } V_n \\
&= \eta_S(V_n(x) dy_1 \cdots dy_q \cdot zdw_1 \cdots dw_r) \quad \text{by multiplicativity of } \eta_S \\
&= \eta_S(V_n(xF_n(z)) dy_1 \cdots dw_r) \quad \text{by definition of } F_n \text{ and } V_n \text{ on } \mathbb{W}(A) \\
&= V_n \eta_{\frac{S}{n}}(xF_n(z) F_n dy_1 \cdots F_n dw_r) \quad \text{by definition of } V_n \\
&= V_n(\eta_{\frac{S}{n}}(xF_n dy_1 \cdots F_n dy_q) \cdot \eta_{\frac{S}{n}} F_n(zdw_1 \cdots dw_r)) \quad \text{by multiplicativity of } \eta_{\frac{S}{n}} \\
&= V_n(\eta_{\frac{S}{n}}(xF_n dy_1 \cdots F_n dy_q) \cdot F_n \eta_S(zdw_1 \cdots dw_r)) \quad \text{by definition of } F_n
\end{aligned}$$

12.3 Show that on $\mathbb{W}\,\Omega_A$ the identity $F_m V_n = V_n F_m$ for Frobenius and Verschiebung hold if $(M, N) = 1$.