10.1 Describe the left adjoint of the forgeful functor, which sends an (A, λ_A)-module to its underlying set

The forgetful functor is given by

$$(M, \lambda_M) \mapsto M.$$

It has a left adjoint F sendiung a set S to a free (A, λ_A)module $(F(S), \lambda_{F(S)})$, where $F(S)$ is the free A-module generated by the symbols

$$\lambda_{F(S),n}(s) \quad \text{for all } n \in \mathbb{N}, s \in S$$

and the lambda-operation $\lambda_{F(S)} : F(S) \to$ is componentwise defined by

$$\lambda_{F(S),m}\left(\sum_{\substack{s\in S\\ n\in\mathbb{N}}} a_{s,n}\lambda_{F(S),n}(s)\right) = \sum_{\substack{s\in S\\ n\in\mathbb{N}}} \psi_{A,m}(a_{s,n})\lambda_{F(S),mn}(s)$$

It is clear, that the $\lambda_{F(S),m}$ are $\psi_{A,m}$-linear, and satisfy by definition $\lambda_{F(A),1} = \mathrm{id}_{F(S)}$ and $\lambda_{F(S),m} \circ \lambda_{F(S),n} = \lambda_{F(S),mn}$. THus by what we said above, $(F(S), \lambda_{F(S)})$ is an (A, λ_A)-module. The unit of the adjunction is given by sending $s \in S$ to $\lambda_{F(S),1}(s) \in F(S)$ and the counit is given by seniding $\sum a_{x,n}\lambda_{F(M),n}(x) \in F(M)$ to $\sum a_{x,n}\lambda_{M,n}(x) \in M$. The triangle idetities hold.

10.2 Let (A, λ_A) be a λ-ring. Describe $F(A)$ for the set A.

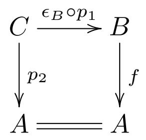
10.3 Show that (A, ψ_A) is an (A, λ_A)-module.

10.4 Show that the (A, λ_A)-modules from two previous exercises are in general not the same

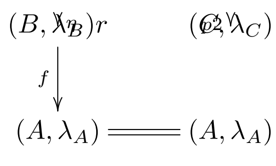
10.5 Let $U : \mathscr{A}_\lambda \to \mathscr{A}$ be the forgetful functor from the category of λ-rings to the category of rings. Show that U creates limits in the category of λ-rings.

10.6 Describe the unit and the counit of the adjoint functors U_{A,λ_A} and R_{A,λ_A}.

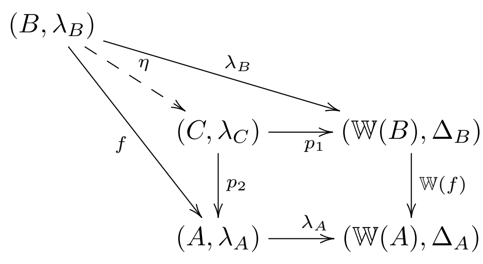
The counit of this adjunction is given by $\epsilon_B \circ p$ via :



The unit is given by the unique map η in



defined by the universal property of the pull back



10.7 Describe the unit and counit of the adjunction (U', R').