Magnetic susceptibility of the QCD vacuum [arXiv:1209.6015]

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A riddle
A riddle (solution: Andre Geim)
Introduction

- external magnetic field as a probe of the QCD vacuum
  - affects chiral symmetry breaking [Gusynin et al '96]
  - drastic changes in hadron spectrum
  - influences deconfinement transition [Agashian '08]
  - phase diagram structure [Bali et al '11]
  - broken Lorentz symmetry $\rightarrow$ new order parameter(s) [Smilga et al '84]
  - para- or diamagnetism?

- phenomenological relevance for systems with strongly interacting matter & magnetic fields
  - dense neutron stars, magnetars
  - non-central heavy ion collisions
  - early universe cosmology
Introduction

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Magnetism in statistical physics

- response to magnetic field \( B = F_{xy} \) is encoded in free energy density

\[
f(eB) = -\frac{T}{V} \log Z(eB)
\]

- giving a magnetization

\[
\mathcal{M} = -\frac{\partial f}{\partial (eB)}
\]

- leading coefficient is the susceptibility

\[
\xi = -\frac{\partial^2 f}{\partial(eB)^2}\bigg|_{eB=0}
\]

- \( \xi > 0 \): paramagnetism, \( \xi < 0 \): diamagnetism

- spin- & orbital angular momentum contributions
  - free electrons: 3 (S para) : -1 (L dia) [Landau ’30]
Magnetism in QCD

- from the partition function

\[ Z = \int \mathcal{D}U e^{-\beta S_g} \prod_f \det(\Phi_f + m_f), \]

the magnetization is given by

\[ \frac{\partial \log Z}{\partial (eB)} = \sum_f \left\langle \text{tr} \frac{1}{\Phi_f + m_f} \frac{\partial \Phi_f}{\partial (eB)} \right\rangle = -\frac{1}{2} \sum_f \frac{1}{m_f} \left\langle \text{tr} \frac{1}{\Phi_f + m_f} \frac{\partial \Phi_f^2}{\partial (eB)} \right\rangle \]

- now use

\[ \frac{\partial \Phi_f^2}{\partial (eB)} = -\frac{q_f}{e} \cdot \sigma_{xy} + \frac{\partial D_f^2}{\partial (eB)}, \]

where \( \sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2i \)

- this implies

\[ \frac{T}{V} \frac{\partial \log Z}{\partial (eB)} = \frac{1}{2} \sum_f \frac{q_f}{m_f} \left( \left\langle \bar{\psi}_f \sigma_{xy} \psi_f \right\rangle + \left\langle \bar{\psi}_f L_{xy} \psi_f \right\rangle \right), \]
Magnetism in QCD (cont’d)

- so the magnetization is

\[
\frac{T \partial \log Z}{V \partial (eB)} = \frac{1}{2} \sum_f \frac{q_f/e}{m_f} \left( \langle \bar{\psi}_f \sigma_{xy} \psi_f \rangle + \langle \bar{\psi}_f L_{xy} \psi_f \rangle \right),
\]

where we defined

\[
L_{xy} \equiv -\frac{\partial D^2_f}{\partial (q_f B)} = -i(x \partial_y - y \partial_x) + \ldots
\]

- altogether this gives

\[
\xi = \sum_f \frac{q_f/e}{2m_f} \left( \frac{\partial \langle \bar{\psi}_f \sigma_{xy} \psi_f \rangle}{\partial (eB)} + \frac{\partial \langle \bar{\psi}_f L_{xy} \psi_f \rangle}{\partial (eB)} \right) \bigg|_{eB=0}
\]

\[
= \sum_f \left( \xi^S_f + \xi^L_f \right)
\]

spin- and angular momentum-related terms
Magnetic susceptibility of condensate

- at $F_{\mu\nu} = 0$: $\langle \bar{\psi} \Gamma \psi \rangle = 0$ for $\Gamma \neq 1$
- at $F_{\mu\nu} \neq 0$ Lorentz invariance implies [Smilga et al '84]

$$\langle \bar{\psi}_f \sigma_{\mu\nu} \psi_f \rangle = q_f F_{\mu\nu} \cdot \langle \bar{\psi}_f \psi_f \rangle \cdot \chi_f + \ldots$$

- significant for experiments:
  - $\mu$ anomalous magnetic moment [Czarnecki et al '03]
  - radiative meson transitions [Colangelo et al '05]
  - chiral-odd $\gamma$ distribution amplitudes [Braun et al '02]
- calculated using
  - sum rules, vector dom. [Belyaev et al '84, Ball et al '03]
  - in quark-meson model, NJL model [Ruggieri et al '11]
  - SU(2) and SU(3) quenched [Buividovich et al '10]
- gives spin contribution:

$$\xi^S_f = \frac{(q_f/e)^2}{2m_f} \tau_f$$
Objectives

- so the observables are

\[ \xi = \sum_f (\xi_f^S + \xi_f^L), \quad \xi_f^S = \frac{(q_f/e)^2}{2m_f} \tau_f = \frac{(q_f/e)^2}{2m_f} \langle \bar{\psi}_f \psi_f \rangle \cdot \chi_f \]

- measure \( \chi_f \sim \tau_f \) on the lattice with dynamical staggered fermions
  - use physical quark masses
  - perform proper renormalization
  - extrapolate to the continuum limit
  - is the spin contribution para- or diamagnetic?
  - determine dependence on temperature
  - up, down and strange flavors
τ in the free case

- using chiral symmetry and \([\sigma_{xy}, \gamma_5] = 0\)

\[
\text{tr} \frac{1}{\hat{D} + m} \sigma_{xy} = \text{tr} \frac{\gamma_5}{\hat{D} + m} \gamma_5 \sigma_{xy} = \text{tr} \frac{\gamma_5}{\hat{D} + m} \sigma_{xy} = m \text{tr} \frac{\sigma_{xy}}{-\hat{D}^2 + m^2}
\]

- rewrite \(\langle \overline{\psi} \sigma_{xy} \psi \rangle\)

\[
N_c \frac{mqB}{\pi} \int \frac{d^2 p}{(2\pi)^2} \sum_n \sum_{s=\pm 1} \frac{s}{p^2 + (2n + 1 + s)qB + m^2}
\]

- only the unpaired LLL with \(s = -1\) remains:

\[-N_c \frac{mqB}{\pi} \int \frac{d^2 p}{(2\pi)^2} \frac{1}{p^2 + m^2}\]

- which is exactly linear! and divergent

\[
N_c \frac{mqB}{4\pi^2} \left[ \gamma - \log(4\pi) + \log \left( \frac{m^2}{\Lambda^2} \right) \right]
\]
Simulation setup

- Symanzik improved gauge action + $N_f = 1 + 1 + 1$ stout smeared staggered quarks
  - physical quark masses: $m_u = m_d = m_s/28.15$
  - physical quark charges: $-q_u/2 = q_d = q_s = e/3$
- temperature/lattice spacing is tuned by
  $$T(\beta) = [N_t a(\beta)]^{-1}$$
- continuum limit: $\beta \to \infty$ ($T = 0$)
  or $N_t \to \infty$ ($T > 0$)
- magnetic field is quantized
  $$a^2 eB = \frac{6\pi N_b}{N_s^2}, \quad N_b \in \mathbb{Z}, \quad 0 \leq N_b \ll N_s$$
  or, equivalently
  $$eB = 6\pi N_b \cdot \frac{N_t^2}{N_s^2} \cdot T^2$$
\begin{equation}
\langle \bar{\psi} \gamma_5 \gamma_\mu \gamma_5 \psi \rangle = \frac{q_u}{e} \cdot eB \cdot \tau_u = \frac{2}{3} eB \cdot \tau_u
\end{equation}
Renormalization

- for the condensate [Leutwyler '92]

\[
\langle \bar{\psi}_f \psi_f \rangle (B, T) = \frac{1}{Z_S} \langle \bar{\psi}_f \psi_f \rangle^r (B, T) + \zeta_S \cdot m_f/a^2 + \ldots
\]

- renormalization prescription, e.g.:

\[
m_f \left( \langle \bar{\psi}_f \psi_f \rangle (B, T) - \langle \bar{\psi}_f \psi_f \rangle (0, T) \right)
\]

- for the tensor polarization

\[
\langle \bar{\psi}_f \sigma_{xy} \psi_f \rangle (B, T) = \frac{1}{Z_T} \langle \bar{\psi}_f \sigma_{xy} \psi_f \rangle^r (B, T) + \zeta_T \cdot q_f B m_f \log(m_f^2a^2) + \ldots
\]

in the free theory \( \zeta_T = \frac{3}{4\pi^2} \)

- renormalization prescription:

\[
Z_T \left( 1 - m_f \frac{\partial}{\partial m_f} \right) \cdot \langle \bar{\psi}_f \sigma_{xy} \psi_f \rangle (B, T)
\]

- \( Z_T(\mu) \) is calculated perturbatively in \( \overline{\text{MS}} \) scheme
Mass dependence & log-divergence

\[ Z_T \tau_f = c_0 + c_1 m_f + c_2 m_f \log(m_f^2 a^2), \quad c_i = c_i^{(0)} + c_i^{(1)} a^2 \]

\[ \left(1 - m_f \frac{\partial}{\partial m_f}\right) Z_T \tau_f = c_0 - 2c_2 m_f \]
Final results at $T = 0$ and $\mu = 2$ GeV

- in the chiral limit
  $$\tau_{u}^r = -40.3(1.4) \text{ MeV}, \quad \tau_{d}^r = -38.9(1.5) \text{ MeV}$$

- at physical quark masses
  $$\tau_{u}^r = -40.7(1.3) \text{ MeV}, \quad \tau_{d}^r = -39.4(1.4) \text{ MeV}, \quad \tau_{s}^r = -53.0(7.2) \text{ MeV}$$

- which gives for the susceptibility
  $$\chi_f = \frac{\tau_f^r}{\langle \bar{\psi} f \psi_f \rangle} \rightarrow \begin{cases} 
\chi_u = -(2.08 \pm 0.08) \text{ GeV}^{-2} \\
\chi_d = -(2.02 \pm 0.09) \text{ GeV}^{-2} \\
\chi_s = -(3.4 \pm 1.4) \text{ GeV}^{-2}
\end{cases}$$

- compare: $\chi_{ud} = -2.11(23) \text{ GeV}^{-2}$ (QCD sum rules)
  $\chi_{ud} = -1.547(6) \text{ GeV}^{-2}$ (quenched SU(2))
  $\chi_{ud} = -4.24(18) \text{ GeV}^{-2}$ (quenched SU(3))
Finite temperature

- divergent term $q_f B m_f \log(m_f a)$ is $T$-independent
- combined fit for different lattice spacings
- can be used to define $T_c = 163(2)(3)$ MeV
Conclusions

- QCD vacuum in external magnetic fields
- Magnetization separates into spin- and angular momentum contributions, and the former is
  \[ \xi^S = \sum_f \frac{(q_f / e)^2}{2m_f} \tau_f \]
- \( \langle \bar{\psi} \sigma_{\mu\nu} \psi \rangle (B, T) \sim \tau_f(T) \cdot B \) determined
  - discretization errors and finite volume effects under control
  - renormalization of log-divergence
  - first results in dynamical QCD on both \( \tau \) and \( \chi \)
- \( \tau_f < 0 \Rightarrow \) the spin contribution is diamagnetic
- Details in [arXiv:1209.6015]