Lattice studies of topologically nontrivial non-Abelian gauge field configurations in an external magnetic field

P. V. Buividovich (Regensburg University)

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Introduction

• **Chiral Magnetic Effect:** charge separation in an external magnetic field due to chirality fluctuations

• **Chirality fluctuations:** reflect the fluctuations of the topology of non-Abelian gauge fields

• In real **QCD**: instanton tunneling (zero temperature) or sphaleron transitions
Describing CME in Euclidean space

CME is a dynamical phenomena, Euclidean QFT (and lattice) rather describe stationary states.

Background fields?

- Induce static chirality imbalance due to chiral chemical potential [Kharzeev, Fukushima, Yamamoto,...]

- Consider topologically nontrivial classical solutions (instantons, calorons, dyons)

- Consider correlators of observables with local chirality $\rho_5 \sim \bar{\phi} \gamma_5 \phi$ [ArXiv:0907.0494]
Describing CME in Euclidean space

Charge separation in stationary states?

Observables?

• Electric currents (allowed by torus topology)

• Spin parts of magnetic and electric dipole moments (but is it charge separation?)

• Global dipole moment prohibited by torus topology

• Local density of electric charge
Charge separation in the instanton background

**Instanton** ~ point in Euclidean space

Tensors from which the Lorentz-invariant current can be constructed:

- Distance to the instanton center $r^\mu$
- Electromagnetic field strength tensor $F_{\mu\nu}$
- Dual field strength tensor $\tilde{F}_{\mu\nu}$

$$j_\mu = \alpha F_{\mu\nu} r_\nu + \beta \tilde{F}_{\mu\nu} r_\nu$$

All such expressions average to zero

No global current
Charge separation in the instanton background
Dirac spectrum for instanton with magnetic field

Motivated by recent work [ArXiv:1112.0532 Basar, Dunne, Kharzeev]

- What is the Dirac eigenspectrum for instanton in magnetic field?
- Are there additional zero modes with different chiralities?
- How does the structure of the eigenmodes change?
Dirac spectrum for instanton with magnetic field

Overlap Dirac operator, $16^4$ lattice, $\rho = 5.0$

$$\frac{1}{\sqrt{B}} \sim \frac{L}{\sqrt{2\pi\Phi}} < \rho, \Phi > 2 - \text{“Large Instanton Limit”}$$
IPR and localization of Dirac eigenmodes

- **IPR**, inverse participation ratio:
  \[
  IPR = \sum_x \rho(x)^2, \quad \sum_x \rho(x) = 1
  \]
  Localization on a single point: \( IPR = 1 \),
  Uniform spread: \( IPR = 1/V \)

- \( \rho(x) \) is the eigenmode density:
  \[
  \rho(x) = \bar{\psi}(x)\psi(x)
  \]

- Geometric extent of the eigenmodes:
  \[
  \sqrt{\langle x_\mu^2 \rangle} = \sqrt{\langle x_\mu^2 \rangle - \langle x_\mu \rangle^2}
  \]
  \[
  \langle f(x) \rangle = \sum_x f(x)\rho(x)
  \]
IPR of low-lying Dirac eigenmodes

![Graph showing IPR of low-lying Dirac eigenmodes]

- Mode 0
- Mode 1
- Mode 2
- Mode 3
- Mode 4
- Mode 5

The graph plots the IPR against $\Phi_B$ for the specified modes.
Geometric extent of the zero mode

\[ \langle \chi_{\mu}^2 \rangle \]

\[ \Phi_B \]
Geometric extent of low-lying modes

\[ \langle x_\mu \rangle \]

\[ \Phi_B \]

\( \mu = 1, \text{ Mode 0} \)
\( \mu = 3, \text{ Mode 0} \)
\( \mu = 1, \text{ Mode 1} \)
\( \mu = 3, \text{ Mode 1} \)
\( \mu = 1, \text{ Mode 5} \)
\( \mu = 3, \text{ Mode 5} \)
Geometric structure of low-lying modes

Magnetic field =>

Mode Number =>
There are no additional zero modes

Zero modes are extended in the direction of the magnetic field

Zero modes become more localized in transverse directions

Overall IPR only weakly depends on the magnetic field

Geometric parameters of higher modes weakly depend on magnetic fields
Charge separation at finite temperature: caloron background [work with F. Bruckmann]

- Caloron: A generalization of the instanton for one compact (time) direction = Finite $T$
  [Harrington, Kraan, Lee]
- Trivial/nontrivial holonomy: stable at high/low temperatures
- Strongly localized solution

Action density

Zero mode density
Charge separation at finite temperature: caloron background

- Pair of BPS monopole/anti-monopole, separated by some distance.
- Explicit breaking of parity invariance
- Only axial symmetry
- $\langle j_\mu \rangle \neq 0$ not prohibited by symmetries
- Can calorons be relevant for the description of the CME/Charge separation?

Numerical study of current densities in the caloron background
Electric current definition for chiral fermions

Fermion propagating in a fixed gauge field configuration

\[ j_\mu(x) = \langle \bar{\psi}(x) \gamma_\mu \psi(x) \rangle = Tr[\gamma_\mu D^{-1}(x, x)] \]

Overlap lattice Dirac operator [ArXiv:hep-lat/9707022, Neuberger]

\[ D = \frac{1 + \gamma_5 \text{sign}(\gamma_5 D_w)}{2} \]

\( D_w \) is a local Wilson-Dirac operator, \text{sign}() is nonlocal

\[ j_\mu(x) = \frac{\partial}{\partial A_\mu(x)} \text{Det}[D] + O(\alpha^2) \]

Current is not exactly conserved
Net electric current along the magnetic field

- $16^3 \times 4$ lattice
- monopole/anti-monopole distance = 8
Current density profile along the caloron axis

$B = 0.12 \parallel \parallel$ caloron axis
Current density profile along the caloron axis

\[
\langle j_3(0, L/2, L/2, x_3) \rangle \cdot 10^5
\]

- \(16^3 \times 4, \Phi_B = 10\)
- \(20^3 \times 4, \Phi_B = 16\)
- \(24^3 \times 4, \Phi_B = 23\)
- \(24^3 \times 6, \Phi_B = 10\)

\(B = 0.24 \parallel \parallel \) caloron axis
Transverse profile of the current density

\[ \langle j_3(0, x_1, L/2, (x_3^m + x_3^a)/2) \rangle \cdot 10^5 \]

- $16^3 \times 4, \Phi_B=5$
- $20^3 \times 4, \Phi_B=8$
- $24^3 \times 4, \Phi_B=11$
- $24^3 \times 6, \Phi_B=5$

\[ B = 0.24 \parallel \parallel \text{caloron axis} \]
Net electric charge

- $16^3 \times 4$ lattice
- monopole/anti-monopole distance = 8
Net electric charge vs. magnetic field
Charge density profile along the caloron axis

\[ \langle j_0(0, L/2, x_3) \rangle \cdot 10^3 \]

- \( 16^3 \times 4, \Phi_B = 5 \)
- \( 20^3 \times 4, \Phi_B = 8 \)
- \( 24^3 \times 4, \Phi_B = 11 \)
- \( 24^3 \times 6, \Phi_B = 5 \)

\[ B = 0.12 \parallel \parallel \text{caloron axis} \]
Charge density profile along the caloron axis

\[ B = 0.24 \parallel \parallel \text{caloron axis} \]
Transverse charge density profile at monopole

\[ \langle j_0(0, x_1, L/2, x_3^{(m)}) \rangle \cdot 10^3 \]

- \(16^3 \times 4, \Phi_B = 5\)
- \(20^3 \times 4, \Phi_B = 8\)
- \(24^3 \times 4, \Phi_B = 11\)
- \(24^3 \times 6, \Phi_B = 5\)

\(B = 0.12 \parallel \parallel \text{caloron axis}\)
Transverse charge density profile at midpoint

\[ \langle j_0(0, x_1, L/2, (x_3^{(m)} + x_3^{(a)})/2) \rangle \cdot 10^3 \]

- \(16^3 \times 4, \Phi_B = 5\)
- \(20^3 \times 4, \Phi_B = 8\)
- \(24^3 \times 4, \Phi_B = 11\)
- \(24^3 \times 6, \Phi_B = 5\)

\[ x_1 \]

\[ B = 0.12 \ || \ | \] calorson axis
Transverse charge density profile @ anti-monopole

\[ \langle i_0(0, x_1, L/2, x_3^a) \rangle \cdot 10^3 \]

- \( 16^3 \times 4, \Phi_B = 5 \)
- \( 20^3 \times 4, \Phi_B = 8 \)
- \( 24^3 \times 4, \Phi_B = 11 \)
- \( 24^3 \times 6, \Phi_B = 5 \)

\[ B = 0.12 \parallel \parallel \text{caloron axis} \]
Some physical estimates

- Fix lattice spacing from \( d = \pi \rho^2 T \)
- \( \rho = 0.33 \text{ fm} \) - characteristic calorion size, from [ArXiv:hep-ph/0607315, Gerhold, Ilgenfritz, Mueller-Preussker] \[ a \approx 0.10 \text{ fm} \ (16^3 \times 4) \]

\[ J_0(eB) \approx (2 \text{ GeV}^{-1})eB \]

- Consider a dilute gas of calorions/anticalorions
- Concentration \( n \sim 1 \text{ fm}^{-4} \) [ArXiv:hep-ph/0607315]
- Charge fluctuations

\[ \langle \langle Q^2 \rangle \rangle = \frac{1}{4} J_0^2(eB)nV \]
Some physical estimates

- **4D volume:**
  
  \[ V \approx V_{3d} \times (c\tau) \]

  - Fireball volume
    
    \[ V_{3d} \approx (5 \text{ fm})^3 \]

  - Collision duration
    
    \[ c\tau \approx 0.2 \text{ fm} \]

  - \( eB \approx m_\pi^2 \approx 0.02 \text{ GeV}^2 \)

- **Au-Au,** \( b = 4 \text{ fm}, \sqrt{s} = 200 \text{ GeV} \)

  - [ArXiv:0907.1396, Skokov, Illarionov, Toneev]

  - Compare with \( \langle \langle Q^2 \rangle \rangle \approx 0.01 \)
    
    - in [ArXiv:0711.0950, Kharzeev, McLerran, Warringa]
Conclusions

**Instanton**
- Charge separation not allowed by symmetries
- Magnetic-field-induced fluctuations of electric dipole moment
- No additional zero modes in the magnetic field

**Caloron**
- Charge generation if \( B \parallel \text{caloron axis} \)
- Charge and current distributions are strongly localized
- Reasonable estimates for charge fluctuations