The QCD transition in external magnetic fields

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Trento, Nov. 2012  


JHEP 1202 (2012) 044, 1111.4956  
PoS LATTICE2011 (2011) 192, 1111.5155  
PRD 86 (2012) 071502, 1206.4205
Motivation(?!)

- magn. fields in QCD
- magn. field on the lattice: not too hard

here QCD lattice simulations:
  - up + down + strange, physical masses & continuum limit
  - Eucl. space & constant external magn. field
  - $\sqrt{eB} \approx 0.1 \ldots 1$ GeV (quantized and bounded)
  - condensate at $T = 0$: magn. catalysis
  - transition and observables at finite $T$

\[ \sqrt{eB} \approx 0.1 \ldots 1 \text{ GeV} \]
Main result: Phase diagram

pseudo-critical temperature as a function of magn. field:

\[ T_c \text{ decreases by O}(10) \text{ MeV} \]

renorm. condensate of light quarks (red), strange number susc. (blue)
similar for chiral susceptibility

\[ T_c \text{ decreases by O}(10) \text{ MeV} \]
Nature of the transition

- vanishing $B$: crossover
- nonzero $B$:

volume dependence of $u$-condensate and susceptibility

$\Rightarrow$ no effect
the relative change of the light susceptibility with temperature around the critical one: not becoming divergent for stronger magn. fields

⇒ transition remains a crossover up to $\sqrt{eB} \sim 1$ GeV
Condensate at $T = 0$

- grows with $B = \text{‘magn. catalysis’}$:

$$\Delta (\Sigma_u + \Sigma_d)/2 \Rightarrow \text{magn. catalysis confirmed, linear dependence for large } B$$

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Condensate at $T=0$: comparison to $\chi$PT & NJL model

$\Rightarrow$ well approximated

unless $eB > \frac{0.1}{0.3}$ GeV$^2$ (should we trust these approaches there?!)

Cohen, McGady, Werbos 07, Andersen 12
Gatto, Ruggieri 10
Inverse magnetic catalysis

- again condensate, for finite $T$:

$$\Delta(\Sigma_u + \Sigma_d) / 2$$

$\Rightarrow$ non-monotonic behavior:

magn. catalysis turns into inverse magn. catalysis around $T_c$

captured by models? understanding?  

[cf. talk by T. Kovács!](#)
Consequence of inverse magnetic catalysis

- again condensate, now as function of $T$ at fixed $B$:

![Graph showing the effect of magnetic fields on temperature](image)

magn. catalysis and inverse magnetic catalysis (vertically) ✓
⇒ crit. temperature decreases
Condensate at finite $T$: comparison to $\chi$PT & PNJL model

$\Rightarrow \chi$PT ok unless $T > 100$ MeV (should we trust this approach there?!)

PNJL? this one uses Polyakov loop potential from $N_f = 2$ lattice data
Isospin condensate

- difference $\bar{u}u - \bar{d}d$: only through el. charge (vanishes at $B = 0$)

\[ \Sigma_u - \Sigma_d \equiv \frac{2m_u = d}{M_N^2 F_\pi^2} \left[ \bar{u}u(B) - \bar{d}d(B) \right] \]

$\Rightarrow$ order parameter
Polyakov loop

- near $T_c$ for different $B$'s:

$\Rightarrow$ monotonically growing with $B$ ($\Leftrightarrow$ condensate)

$\Rightarrow$ crit. $T$ decreases again
Mass sensitivity

- what if we put \((m_{\text{light}}, m_{\text{light}}, m_{\text{strange}}) \to (m_{\text{strange}}, m_{\text{strange}}, m_{\text{strange}})\)?

\[ T \text{-dep. of } u \text{-susceptibility (top) and change of } u \text{-condensate (bottom)} \]

\( \Rightarrow \) effects of decreasing \( T_c \) & inverse magn. catalysis disappear

light quark masses are important
Comparison to other lattice simulations

$1 + 1 + 1$ (staggered, smeared) vs at phys. masses

$1 + 1$ (staggered) at higher-than-phys. masses

D’Elia et al. 10

similar in $SU(2)$

Ilgenfritz et al. 12

$T_c(B)$ different

monotonicity different

rationale: masses as before, cont. limit (diff. lattice actions)
Remarks on renormalization

- empirically: charged pion mass and Sommer scale as function of $B$

\[ \Rightarrow B \text{ does not change the lattice spacing} \]
Remarks on renormalization

- $B$ always enters as $(eB)$ and $(eB)^{bare} = (eB)^r$

Local $U(1)$ gauge invariance still present:

\[
\psi^r = \sqrt{Z_2} \psi^b
\]

\[
\downarrow \text{gauge trafo} \downarrow
\]

\[
e^{i\alpha^r} \psi^r = \sqrt{Z_2} \psi^b e^{i\alpha^b} \Rightarrow \alpha^r = \alpha^b
\]

\[
e^r A^r_\mu = Z e \sqrt{Z_3} e^b A^b_\mu
\]

\[
\downarrow \text{gauge trafo} \downarrow
\]

\[
\partial_\mu \alpha^r + e^r A^r_\mu = Z e \sqrt{Z_3} \left( e^b A^b_\mu + \partial_\mu \alpha^b \right) \Rightarrow Z e \sqrt{Z_3} = 1
\]

\[1 \Rightarrow e^r B^r = 1 \cdot e^b B^b\]
Remarks on renormalization

divergence of partition function $Z$/free energy density $f$: Leutwyler, Smilga 92

$$\log \frac{Z}{V_4} = - f' + O\left(\frac{1}{a^4}\right) + O\left(\frac{m^2}{a^2}\right) + O\left(m^4 \log a\right)$$

no new divergences by temperature or chem. pot.

$$\bar{\psi}\psi = \partial_m \log \frac{Z}{V_4} = - \partial_m f' + 0 + O\left(\frac{m}{a^2}\right) + O\left(m^3 \log a\right)$$

add. divergence removed by e.g. $\bar{\psi}\psi(\bar{\psi}\psi)$ ($T=0$)
mult. divergence removed by mult. back by mass

new divergences with $B$-field? free energy of free quarks (one flavor):

$$f - f(B=0) = N_c \frac{(qB)^2}{24\pi^2} \left(\text{finite} - \log(m^2 a^2)\right)$$

related to charge renorm. (background field method)

$$\bar{\psi}\psi - \bar{\psi}\psi(B=0) = N_c \frac{(qB)^2}{24\pi^2} \left(\partial_m \text{finite} - \text{finite}\right)$$

$\Rightarrow$ divergence only in free energy

cf. talk by G. Endrődi
magn. catalysis turns into inverse magn. catalysis around $T_c$

$T_c(B) \downarrow$

transition sensitive to light quark mass ← understanding?

- $\chi$PT and (P)NJL models fine in some range of validity
- isospin condensate
- renormalization almost unchanged by magn. field

- more observables from $B$

cf. talk by G. Endrődi

open: more realistic $B$-fields on the lattice (still Euclidean)