Quantum theory of condensed matter II

Mesoscopic physics (Quantum transport)

Prof. Milena Grifoni	Tue	8:00 - 10:00	9.2.01
	Fri	12:00 - 14:00	9.2.01
PD Dr. Andrea Donarini	Tue	10:00 - 12:00	5.0.21

Sheet 12

1. Interference effects in the transport through a double quantum dot

Consider the double quantum dot coupled to leads described by the following (for simplicity spinless) Hamiltonian:

$$H = H_{\rm S} + H_{\rm T} + H_{\rm res} \tag{1}$$

where

$$\begin{aligned} H_{\rm S} &= \sum_{\alpha=L,R} \varepsilon_\alpha d_\alpha^\dagger d_\alpha + b(d_L^\dagger d_R + d_R^\dagger d_L) \\ H_{\rm T} &= \sum_{\alpha k} t_\alpha\, c_{\alpha k}^\dagger d_\alpha + H.c. \\ H_{\rm res} &= \sum_{\alpha k} \varepsilon_k c_{\alpha k}^\dagger c_{\alpha k} \end{aligned} \tag{2}$$

1. Calculate, using the equation of motion technique, the retarded Green's function of the system in presence of the reservoirs

$$G^r_{\alpha\beta}(t) = -\frac{i}{\hbar}\theta(t)\langle\{d_\alpha(t), d_\beta^\dagger(0)\}\rangle. \tag{3}$$

   In particular, prove that its Fourier transform is a matrix which satisfies the equation:

$$(\omega - \mathbf{h} - \mathbf{\Sigma}_L - \mathbf{\Sigma}_R)\,\tilde{\mathbf{G}}^r(\omega) = \mathbf{I}_2 \tag{4}$$

   where

$$\mathbf{h} = \begin{pmatrix} \varepsilon_L & b \\ b & \varepsilon_R \end{pmatrix}, \qquad \mathbf{\Sigma}_L = -i\frac{\hbar\Gamma_L}{2}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \mathbf{\Sigma}_R = -i\frac{\hbar\Gamma_R}{2}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

   and $\Gamma_\alpha = \frac{2\pi}{\hbar}|t_\alpha|^2\mathcal{D}_\alpha$ with $\mathcal{D}_\alpha$ the density of states of lead α.

   (4 Points)

2. Calculate the conductance through the system in the limit $T \to 0$ by means of the formula:

$$G = \frac{e^2}{h}\int_{-\infty}^{+\infty} {\rm d}\xi \left(-\frac{\partial f}{\partial \xi}\right) {\rm Tr}\left\{\hbar^2 \mathbf{\Gamma}_L \mathbf{G}^r \mathbf{\Gamma}_R \mathbf{G}^a\right\}, \tag{5}$$

   where $\mathbf{\Gamma}_\alpha = -\frac{2}{\hbar}{\rm Im}\mathbf{\Sigma}_\alpha$.

   Prove the result:

$$G = \frac{e^2}{h}\frac{b^2\hbar^2\Gamma_L\Gamma_R}{|(\mu_0 - \varepsilon_L + i\frac{\hbar\Gamma_L}{2})(\mu_0 - \varepsilon_R + i\frac{\hbar\Gamma_R}{2}) - b^2|^2}$$

   (2 Points)

3. Consider now the limit $\varepsilon_L = \varepsilon_R = \varepsilon$ and $\Gamma_L = \Gamma_R = \Gamma$. Prove that the conductance formula assumes in this case the form:

$$G = \frac{e^2}{h}\frac{\hbar^2\Gamma^2}{4}\left|\frac{1}{\mu_0 - \varepsilon - b + i\frac{\hbar\Gamma}{2}} - \frac{1}{\mu_0 - \varepsilon + b + i\frac{\hbar\Gamma}{2}}\right|^2$$

Plot the result as a function of μ_0 for different values of b, taking from $b > \Gamma$ and $b < \Gamma$. Comment the results. What happens in the limit $b = 0$? **(2 Points)**

4. Assume now a different coupling to the leads which produces the tunnelling self-energies:

$$\mathbf{\Sigma}_L = \mathbf{\Sigma}_R = -i\frac{\hbar\Gamma}{4}\mathbf{I}_2$$

Prove that the zero temperature conductance through the junction reads now:

$$\frac{e^2}{h}\frac{\hbar^2\Gamma^2}{4}\left(\left|\frac{1}{\mu_0 - \varepsilon - b + i\frac{\hbar\Gamma}{2}}\right|^2 + \left|\frac{1}{\mu_0 - \varepsilon + b + i\frac{\hbar\Gamma}{2}}\right|^2\right)$$

Compare the expression just obtained and compare it with the one obtained in the previous point. Which is giving the highest conductance? Why? What happens this time to the conductance in the limit $b = 0$?

Hint: Use again Eq. (4) and Eq. (5) to calculate the retarded Green's function and the conductance, respectively.

Frohes Schaffen!