University of Regensburg Winter Term 18/19

Quantum theory of condensed matter II

Mesoscopic physics (Quantum transport)

Prof. Milena Grifoni	Tue	8:00 - 10:00	9.2.01
	Fri	12:00 - 14:00	9.2.01
PD Dr. Andrea Donarini	Tue	10:00 - 12:00	5.0.21

Sheet 10

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1. Mesoscopic beam splitter

Consider the device with three terminals sketched below, restrict yourself to a single mode per terminal and assume that time reversal symmetry holds.

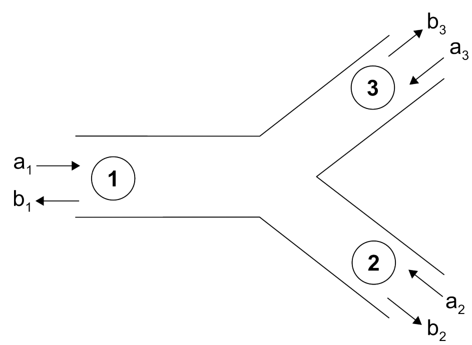

Figure 1: A mesoscopic beam splitter.

1. Assume furthermore that the device is invariant under the exchange of terminal 2 and terminal 3 and show that under these conditions the scattering matrix connecting the incoming amplitudes a_α to the outgoing ones b_α (with $\alpha = 1, 2, 3$) can be parametrized as

$$S = \begin{pmatrix} r & t & t \\ t & r' & t' \\ t & t' & r' \end{pmatrix}, \tag{1}$$

where r, r', t and $t' \in \mathbb{C}$. **(2 Points)**

2. The 4 parameters of the scattering matrix S obtained in the previous point are not independent. Why? Assume all the parameters of S to be real and show that, for nonzero t, either

$$t^2 = \frac{1-r^2}{2}, \qquad r' = -\frac{1+r}{2}, \qquad t' = \frac{1-r}{2}$$

or

$$t^2 = \frac{1-r^2}{2}, \qquad r' = \frac{1-r}{2}, \qquad t' = -\frac{1+r}{2}$$

has to hold. What is the maximum value allowed for t^2? Which scattering matrices can fulfil instead the condition $t = 0$? Give a physical interpretation of the results.

(3 Points)

3. Consider now a fully symmetric system, invariant under any permutation of the three terminals. What changes in the scattering matrix S? Can r' be zero in this case?

1. Wick's theorem

1. Show that, for a system of non-interacting fermions described by the Hamiltonian in the energy basis

$$\hat{H} = \sum_{\alpha} \epsilon_{\alpha} \hat{c}^{\dagger}_{\alpha} \hat{c}_{\alpha} \left(= \sum_{i=1}^{N} \hat{h}_i \right)$$

   the following relation for the many-body grandcanonical expectation value holds:

$$\langle \hat{c}^{\dagger}_{\alpha_1} \hat{c}^{\dagger}_{\alpha_2} \hat{c}_{\alpha_3} \hat{c}_{\alpha_4} \rangle = \langle \hat{c}^{\dagger}_{\alpha_1} \hat{c}_{\alpha_4} \rangle \langle \hat{c}^{\dagger}_{\alpha_2} \hat{c}_{\alpha_3} \rangle \delta_{\alpha_1 \alpha_4} \, \delta_{\alpha_2 \alpha_3} - \langle \hat{c}^{\dagger}_{\alpha_1} \hat{c}_{\alpha_3} \rangle \langle \hat{c}^{\dagger}_{\alpha_2} \hat{c}_{\alpha_4} \rangle \delta_{\alpha_1 \alpha_3} \, \delta_{\alpha_2 \alpha_4},$$

   where

$$\langle \hat{c}^{\dagger}_{\alpha_1} \hat{c}^{\dagger}_{\alpha_2} \hat{c}_{\alpha_3} \hat{c}_{\alpha_4} \rangle \equiv \frac{1}{Z} \, \mathrm{Tr} \left\{ \hat{c}^{\dagger}_{\alpha_1} \hat{c}^{\dagger}_{\alpha_2} \hat{c}_{\alpha_3} \hat{c}_{\alpha_4} \; \exp\left[-\beta(H - \mu N)\right] \right\}$$

   and Z is the grandcanonical partition function. The trace is taken over the full Fock space. Hint: Consider the use of the eigenbasis of $\hat{h}$. **(2 Points)**

2. Derive from 2.1 that, for noninteracting fermions, in every other given single particle basis $\{|n\rangle\}$ the following relation holds:

$$\langle \hat{c}^{\dagger}_{n_1} \hat{c}^{\dagger}_{n_2} \hat{c}_{n_3} \hat{c}_{n_4} \rangle = \langle \hat{c}^{\dagger}_{n_1} \hat{c}_{n_4} \rangle \langle \hat{c}^{\dagger}_{n_2} \hat{c}_{n_3} \rangle - \langle \hat{c}^{\dagger}_{n_1} \hat{c}_{n_3} \rangle \langle \hat{c}^{\dagger}_{n_2} \hat{c}_{n_4} \rangle.$$

   Note that this is valid even if in this basis the Hamiltonian

$$\hat{H} = \sum_{n,m} h_{nm} \hat{c}^{\dagger}_{n} \hat{c}_{m}$$

   would contain non-diagonal terms, h_{nm} for $n \neq m$. Hint: Diagonalize H first, using a unitary transformation $\hat{c}_n = \sum_{\alpha} u_{n\alpha} \hat{c}_{\alpha}$. Apply the equation proven in 2.1. Finally perform the canonical transformation in the reverse direction.

Frohes Schaffen!