Quantum theory of condensed matter II

Mesoscopic physics (Quantum transport)

Prof. Milena Grifoni	Tue	8:00 - 10:00	9.2.01
	Fri	12:00 - 14:00	9.2.01
PD Dr. Andrea Donarini	Tue	10:00 - 12:00	5.0.21

Sheet 9

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1. An electronic Fabry-Pérot interferometer

Consider a one dimensional wire with two identical scatterers located at $x = -d/2$ and $x = +d/2$, respectively. Further assume that the scattering potential is approximated by

$$U(x) = U_0[\delta(x + d/2) + \delta(x - d/2)].$$

1. Calculate the scattering matrix for the single scatterer. Prove that the associated transmission probability reads:

   $$T_\delta = \frac{\hbar^2 v^2}{\hbar^2 v^2 + U_0^2}$$

   with the electronic velocity $v = \sqrt{2E/m}$ (E is the kinetic energy of the scattered electrons).
   Hint: Recall from QM I the special matching conditions for the wavefunctions at δ potentials.

   (2 Points)

2. Calculate the transmission probability through the double δ-barrier using the combination of scattering matrices. Prove that the result reads:

   $$T(E) = \frac{T_\delta^2}{1 - 2R_\delta \cos\theta + R_\delta^2}$$

   where $\theta = 2[dmv/\hbar + \arctan(\hbar v/U_0)]$ and $R_\delta = 1 - T_\delta$. Plot $T(E)$ for $U_0 = 9\,\mathrm{eV\AA}$, $d = 50\text{Å}$ and $0 \leq E \leq 250\,\mathrm{meV}$.

   Hint: Do not forget to consider the free propagation of the electronic wavefunction between the scattering barriers.

   (2 Points)

3. What is the maximum value for the transmission coefficient calculated in the previous point? In particular calculate the position of the resonances (*i.e.* the maxima of $T(E)$) both in the low and high energy limit, *i.e.* $\hbar v \ll U_0$ and $\hbar v \gg U_0$. Give a physical interpretation of the result.

   (2 Points)

2. A nanotube electron wave guide

Transport through nano-junctions shows very different characteristics depending on the transparency of the contacts to the source and drain electrodes. Read for example the following paper about a carbon nanotube nanojunction:

> Fabry-Pérot interference in a nanotube electron waveguide,
> Wenjie Liang, Marc Bockrath, Dolores Bozovic, Jason H. Hafner, M. Tinkham, and Hongkun Park,
> Nature **411**, 665 (2001).

Consider in particular Fig. 1, showing a conductance trace as a function of the gate voltage:

1. Which is the minimum amount of transport channels able to give conductance of this size? Can you conclude something about the left/right symmetry of the tunnelling barriers?

2. The conductance oscillations measured in this sample are periodic in the gate voltage. Compare this result with the one you obtained in the previous exercise and rationalize it in terms of the low energy dispersion relation of a carbon nanotube.

Frohes Schaffen!