Quantum theory of condensed matter II

Mesoscopic physics (Quantum transport)

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	Fri	12:00 - 14:00	9.2.01
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Sheet 7

1. Equation of motion for the reduced density matrix

Consider a generic system-bath model represented by the Hamiltonian:

$$\hat{H} = \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{B}} + \hat{H}_{\mathrm{S-B}}$$

where $\hat{H}_{\mathrm{S}}$ describes a microscopic system coupled via $\hat{H}_{\mathrm{S-B}}$ to the macroscopic reservoir described by $\hat{H}_{\mathrm{B}}$.

1. Prove that the equation of motion for the density matrix of the total system in the interaction picture reads:

$$\dot{\hat{\rho}}_{\mathrm{I}}(t) = -\frac{i}{\hbar}[\hat{H}_{\mathrm{S-B,\,I}}(t), \hat{\rho}_{\mathrm{I}}(t)] \tag{1}$$

   where $\hat{H}_{\mathrm{S-B,\,I}}$ is the system-bath Hamiltonian in interaction picture at time t. **(2 Points)**

2. Prove that the equation (1) is equivalent to the integro-differential equation:

$$\dot{\hat{\rho}}_{\mathrm{I}}(t) = -\frac{i}{\hbar}\left[\hat{H}_{\mathrm{S-B,\,I}}(t), \hat{\rho}_{\mathrm{I}}(0)\right] - \frac{1}{\hbar^2}\int_0^t \mathrm{d}t' \left[\hat{H}_{\mathrm{S-B,\,I}}(t), \left[\hat{H}_{\mathrm{S-B,\,I}}(t'), \hat{\rho}_{\mathrm{I}}(t')\right]\right]. \tag{2}$$

   (2 Points)

3. Consider now the reduced density operator $\hat{\rho}_{\mathrm{red,I}}(t) := \mathrm{Tr_B}\left\{\hat{\rho}_{\mathrm{I}}(t)\right\}$ where $\mathrm{Tr_B}\{\bullet\}$ is the partial trace over the reservoir degrees of freedom and assume the separation of the initial density operator $\hat{\rho}_{\mathrm{I}}(0) = \hat{\rho}_{\mathrm{S}}(0) \otimes \hat{\rho}_{\mathrm{B}}(0)$. Prove that, up to the second perturbative order in the interaction $\hat{H}_{\mathrm{S-B}}$ one can write:

$$\begin{aligned}\dot{\hat{\rho}}_{\mathrm{red,I}}(t) \;=\; & -\frac{i}{\hbar}\mathrm{Tr_B}\left\{\left[\hat{H}_{\mathrm{S-B,\,I}}(t), \hat{\rho}_{\mathrm{S}}(0)\otimes\hat{\rho}_{\mathrm{B}}(0)\right]\right\} + \\ & -\frac{1}{\hbar^2}\int_0^t \mathrm{d}t' \mathrm{Tr_B}\left\{\left[\hat{H}_{\mathrm{S-B,\,I}}(t), \left[\hat{H}_{\mathrm{S-B,\,I}}(t'), \hat{\rho}_{\mathrm{red,I}}(t')\otimes\hat{\rho}_{\mathrm{B}}(0)\right]\right]\right\}.\end{aligned} \tag{3}$$

   Hint: It is useful to start by proving the following relation:

$$\hat{\rho}_{\mathrm{I}}(t) = \hat{\rho}_{\mathrm{S}}(0)\otimes\hat{\rho}_{\mathrm{B}}(0) + O(\hat{H}_{\mathrm{S-B}})$$

Frohes Schaffen!