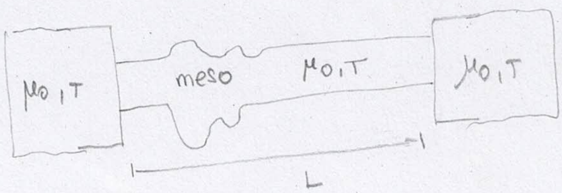
CH. 4. "CONDUCTANCE OF BALLISTIC MESOSCOPIC CONDUCTORS

We have seen that in linear response, the conductance reduces to the evaluation of equilibrium correlators of the mescoscopic conductor + reservoirs.



$$\hat{\rho} = \frac{e^{-\beta(\hat{H} - \mu\hat{N})}}{Z} \quad , \quad G = \lim_{\omega \to 0} \frac{-\Im m\, \tilde{\chi}_{II}(\omega)}{\omega}$$

In general, $\hat{H}$ refers to the total system, including the reservoirs, cf. Eq. (2.19) $\Rightarrow$ $\hat{H} = \hat{H}_L + \hat{H}_R + \hat{H}_{int} + \hat{H}_S$.

SPECIAL CASE: "INFINITE" MESOSCOPIC SYSTEM

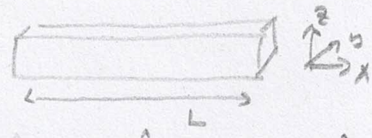
However, in the case in which the reservoirs are simply seen as electron sources, setting the equilibrium chemical potential μ_0, and finite size effects at the boundary with the reservoirs are not important, one can set $L \to \infty$ I.e., it is further possible to concentrate on the mesoscopic system alone $\boxed{\hat{H}_{tot} \to \hat{H}_S}$

This situation applies e.g. to a 1D-wire with $L \to \infty$ or to other quasi-one dimensional system (cf. Ch 1)

$\hat{H}_S = \hat{T} + \hat{U} + \hat{V}$, $\hat{U}$ confinement potential, $\hat{V}$ e-e interaction

4.1. CONDUCTANCE OF A BALLISTIC WIRE WITH $L \to \infty$

$\hat{H}_s$ is then the Hamiltonian for a system of N electrons in the wire:



$$\hat{T} = \sum_{i=1}^{N} \frac{\hat{p}_i^2}{2m}, \quad \hat{U} = \sum_{i=1}^{N} u(\hat{y}_i, \hat{z}_i), \quad \hat{V} = \frac{1}{2}\sum_{i \neq j} v_{ee}(\vec{r}_i - \vec{r}_j)$$

$\Rightarrow$ $$\hat{H}_S = \sum_{i=1}^{N} \hat{h}_i + \hat{V} \qquad (4.1)$$

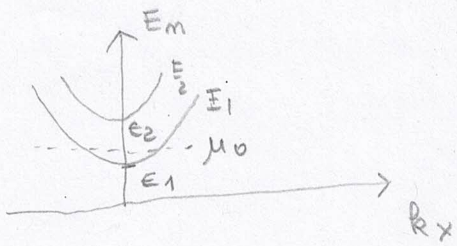
- We need the second quantization version of $\hat{H}_S$

$\Rightarrow$ a convenient single particle basis is the one of the non interacting single particle Hamiltonian

$$\hat{h} = \frac{\hat{p}^2}{2m} + u(\hat{y}, \hat{z}) \qquad (4.2)$$

As seen in ch. 1, the Schrödinger equation is solved by (cf. (1.8))

$$\begin{cases} \psi_{mk_x}(\vec{r}) = \phi_m(y,z)\, \dfrac{e^{ik_x x}}{\sqrt{L}} \\ E_m(k_x) = \epsilon_m + \dfrac{\hbar^2}{2m} k_x^2 \end{cases} \qquad (4.3)$$



$\epsilon_2 > \mu_0 \Rightarrow$ effective 1D system

$\hookrightarrow$ basis set $\{|m k_x \sigma\rangle\}$

with $\psi_{mk_x}(\vec{r}) = \langle \vec{r} | m k_x \rangle$, σ electron spin

- System Hamiltonian

$$\hat{H}_S = \hat{T} + \hat{U} + \hat{V} \equiv \hat{H}_{SP} + \hat{V}$$

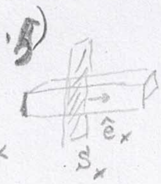
$$\hookrightarrow \begin{cases} \hat{H}_{SP} = \sum_{mk_x\sigma} E_m(k_x)\, \hat{c}^+_{mk_x\sigma}\, \hat{c}_{mk_x\sigma} & (4.4) \\ \hat{V} = \frac{1}{2} \sum_{\lambda\lambda'\nu\nu'} \vartheta_{\lambda\lambda'\nu\nu'}\, \hat{c}^+_{\lambda} \hat{c}^+_{\lambda'}\, \hat{c}_{\nu'} \hat{c}_{\nu} & \lambda = \{mk_x\sigma\} \end{cases}$$

total Hamiltonian in $\{|mk_x\sigma\rangle\}$ representation

- Current operator (position representation)

$$\hat{I}(x) = \sum_\sigma \int d\vec{S} \cdot \hat{\vec{J}}_\sigma(\vec{r}) = \sum_\sigma \int_{S_x} d\vec{r}_\perp\, \hat{e}_x \cdot \hat{\vec{J}}_\sigma(\vec{r}) \qquad (4.5)$$

S_x crossection of the wire at x



with

$$\hat{\vec{J}}_\sigma(\vec{r}) = -\frac{ie\hbar}{2m} \left[\hat{\psi}^+_\sigma(\vec{r})\, \vec{\nabla} \hat{\psi}_\sigma(\vec{r}) - \vec{\nabla} \hat{\psi}^+_\sigma(\vec{r})\, \hat{\psi}_\sigma(\vec{r}) \right] \qquad (4.6)$$

current density operator in position representation (cf. exercise sheet and 2nd Q notes)

- Basis transformation

Since $\hat{H}$ is diagonal in $\{|mk_x\sigma\rangle\}$ basis, we need $\hat{I}(\vec{r})$ in that basis as well. I.e. we need the transformation relating $\hat{\psi}^+_\sigma(\vec{r})$ and $\hat{c}^+_{mk_x\sigma}$.

From

$$\mathbb{1} = \sum_{mk_x\sigma'} |mk_x\sigma'\rangle\langle mk_x\sigma'|$$

$$\hookrightarrow |\vec{r}\sigma\rangle = \sum_{mk_x\sigma'} |mk_x\sigma'\rangle \langle mk_x\sigma'|\vec{r}\sigma\rangle = \sum_{mk_x} \psi^*_{mk_x\sigma}(\vec{r})\, |mk_x\sigma\rangle$$

Hence

$$|\vec{r}\sigma\rangle = \hat{\psi}^{+}_{\sigma}(\vec{r})|0\rangle = \sum_{mk_x} \psi^{*}_{mk_x\sigma}(\vec{r})\,\hat{c}^{+}_{mk_x\sigma}|0\rangle$$

$$\hookrightarrow \boxed{\hat{\psi}^{+}_{\sigma}(\vec{r}) = \sum_{mk_x} \psi^{*}_{mk_x}(\vec{r})\,\hat{c}^{+}_{mk_x\sigma}} \quad (4.7)$$

$$\psi^{*}_{mk_x}(\vec{r}) = \langle mk_x|\vec{r}\rangle \quad \text{independent of } \sigma$$

· Current operator in energy basis

Using the expression (4.6) it follows from (4.7)

$$\boxed{\begin{aligned}\vec{J}_{\sigma}(\vec{r}) = \frac{-ie\hbar}{2m} \sum_{\substack{mm' \\ k_x k'_x}} \Big[& \psi^{*}_{mk_x\sigma}(\vec{r})\,\hat{c}^{+}_{mk_x\sigma}\,\vec{\nabla}\psi_{m'k'_x\sigma}(\vec{r})\,\hat{c}_{m'k'_x\sigma} \\ & - \vec{\nabla}\psi^{*}_{mk_x\sigma}(\vec{r})\,\hat{c}^{+}_{mk_x\sigma}\,\psi_{m'k'_x\sigma}(\vec{r})\,\hat{c}_{m'k'_x\sigma}\Big]\end{aligned}} \quad (4.8)$$

with $\psi_{mk_x\sigma}(\vec{r}) = \phi_m(y,z)\,\frac{e^{ik_x x}}{\sqrt{L}}$ $\qquad (y,z) = \vec{r}_\perp$

$$\Rightarrow \hat{\vec{J}}_{\sigma}(\vec{r})\cdot\hat{e}_x = \frac{-ie\hbar}{2mL} \sum_{\substack{mm' \\ k_x k'_x}} \Big[\phi^{*}_{m}(\vec{r}_\perp)\,\phi_{m'}(\vec{r}_\perp)\,e^{-i(k_x-k'_x)x}\,(ik'_x)$$

$$- (-ik_x)\,\phi^{*}_{m}(\vec{r}_\perp)\,\phi_{m'}(\vec{r}_\perp)\,e^{-i(k_x-k'_x)x}\Big]\,\hat{c}^{+}_{mk_x\sigma}\,\hat{c}_{m'k'_x\sigma}$$

$$\vec{\nabla}_{\vec{r}}\psi = (\partial_x\psi, \partial_y\psi, \partial_z\psi) = (\partial_x\psi, \partial_{\vec{r}_\perp}\psi)$$

Integrating over $\vec{r}_\perp$ yields $\hat{I}_\sigma(x)$:

$$\hat{I}_\sigma(x) = -\frac{ie\hbar}{2mL} \sum_{\substack{m m' \\ k_x k'_x}} \int d\vec{r}_\perp \, \phi^*_m(\vec{r}_\perp)\phi_{m'}(\vec{r}_\perp) \, .$$

$$\cdot (ik'_x + ik_x) \, e^{-i(k_x - k'_x)x} \, \hat{c}^+_{m k_x \sigma} \hat{c}_{m' k'_x \sigma}$$

use orthogonality of transverse modes

$$\int d\vec{r}_\perp \, \phi^*_m(\vec{r}_\perp)\phi_{m'}(\vec{r}_\perp) = \langle m | m' \rangle = \delta_{mm'}$$

and set $k_x - k'_x = q$, $k_x = k$, $k'_x = k'$

$$\Rightarrow \boxed{\hat{I}_\sigma(x) = \frac{e\hbar}{2mL} \sum_k \sum_q \sum_m (2k+q) \, e^{-iqx} \, \hat{c}^+_{mk\sigma} \, \hat{c}_{m(k+q)\sigma}} \quad (4.9)$$

note; We have not included the spin so far in the treatment of the current in the Kubo formalism

It holds $\hat{I}(x) = \sum_\sigma \hat{I}_\sigma(x)$

note: $\langle k m \sigma | \hat{i}(x) | k+q m \sigma' \rangle = \frac{e\hbar}{2mL} (2k+q) \, e^{-iqx} \, \delta_{mm} \, \delta_{\sigma\sigma'}$

$\equiv i_{\lambda\lambda'}(x)$ in basis $\{|\lambda\rangle\}$ with $|\lambda\rangle = |m k \sigma\rangle$

(4.9b)

THE NON INTERACTING CASE $\hat{V}=0$

- Current - current correlation function ; the case of non interacting electrons $\hat{V}=0$

$$\chi^R_{II}(t) = -\frac{i}{\hbar}\,\theta(t)\,\langle [\hat{I}(x,t), \hat{I}(x',0)] \rangle_0 \qquad (4.10)$$

$\rho_{I(x)I(x')}$

where the time evolution is in the interaction picture

$$\hat{I}(x,t) = e^{i\hat{H}t/\hbar}\,\hat{I}(x)\,e^{-i\hat{H}t/\hbar}, \qquad \hat{H}=\hat{H}_{sp} \quad (\hat{V}=0)$$

i) evaluation of $\hat{I}(x,t)$

First we need to provide $\hat{c}_\alpha(t) = e^{i\hat{H}t/\hbar}\,\hat{c}_\alpha\,e^{-i\hat{H}t/\hbar}$

Such time evolution easily comes from the Heisenberg equation for non interacting fermions

$$\begin{cases} \dot{\hat{c}}_\alpha(t) = \frac{i}{\hbar}[\hat{H}, \hat{c}_\alpha(t)] = \frac{i}{\hbar} e^{i\hat{H}t/\hbar}[\hat{H},\hat{c}_\alpha]e^{-i\hat{H}t/\hbar} \\ \hat{H} = \sum_\lambda \varepsilon_\lambda \hat{c}^+_\lambda \hat{c}_\lambda \end{cases}$$

from $[\hat{H},\hat{c}_\alpha] = \sum_\lambda \varepsilon_\lambda [\hat{c}^+_\lambda \hat{c}_\lambda, \hat{c}_\alpha] = -\varepsilon_\alpha \hat{c}_\alpha$

$\hookrightarrow \dot{\hat{c}}_\alpha(t) = -\frac{i}{\hbar}\varepsilon_\alpha\,\hat{c}_\alpha(t)$

$\hookrightarrow \hat{c}_\alpha(t) = e^{-i/\hbar\,\varepsilon_\alpha t}\,\hat{c}_\alpha \qquad (4.11a)$

and $\hat{c}^+_\alpha(t) = e^{i/\hbar\,\varepsilon_\alpha t}\,\hat{c}^+_\alpha \qquad (4.11b)$

$$\hookrightarrow \quad \hat{I}_\sigma(x,t) = \frac{e\hbar}{2mL}\sum_\sigma\sum_k\sum_q\sum_n (2k+q)\, e^{-iqx}\, \hat{c}^\dagger_{nk\sigma}(t)\, \hat{c}_{n(k+q)\sigma}(t)$$

hence

$$\boxed{\hat{I}(x,t) = \frac{e\hbar}{2mL}\sum_k\sum_q\sum_{n\sigma} (2k+q)\, e^{-iqx}\, e^{\frac{i}{\hbar}(\mathcal{E}(k)-\mathcal{E}(k+q))t}\, \hat{c}^\dagger_{nk\sigma}\, \hat{c}_{n(k+q)\sigma}} \tag{4.12}$$

$$E_n(k) = \epsilon_n + \frac{\hbar^2 k^2}{2m} \equiv \mathcal{E}_n + \mathcal{E}(k)$$

ii) Evaluation of $\langle [\hat{c}^\dagger_{nk\sigma}\hat{c}_{n(k+q)\sigma}, \hat{c}^\dagger_{nk'\sigma'}\hat{c}_{n(k'+q')\sigma'}]\rangle_0$

At this point we are left with the evaluation of $\langle \cdots \rangle_0$ in Eq.( ). To this extent one only needs to concentrate on the operatorial part, which enters explicitly in the anticommutator. It holds in general for fermions

$$[\hat{c}^\dagger_\nu \hat{c}_\mu, \hat{c}^\dagger_{\nu'}\hat{c}_{\mu'}] = \hat{c}^\dagger_\nu \hat{c}_{\mu'}\,\delta_{\mu\nu'} - \hat{c}^\dagger_{\nu'}\hat{c}_\mu\,\delta_{\nu\mu'} \tag{4.13}$$

$$\hookrightarrow \langle [\underbrace{\hat{c}^\dagger_{nk\sigma}}_{\nu}\,\underbrace{\hat{c}_{n(k+q)\sigma}}_{\mu}, \underbrace{c^\dagger_{nk'\sigma'}}_{\nu'}\,\underbrace{c_{n(k'+q')\sigma'}}_{\mu'}]\rangle_0$$

$$= \langle c^\dagger_{nk\sigma}\, c_{n(k'+q')\sigma}\rangle_0\, \delta_{nn}\, \delta_{k+q,k'}\, \delta_{\sigma\sigma'} - \langle \hat{c}^\dagger_{nk'\sigma}\, c_{n(k+q)\sigma}\rangle\, \delta_{nn}\, \delta_{k,k'+q'}\, \delta_{\sigma\sigma'}$$

$$= f(E_n(k))\, \delta_{k,k'+q'}\, \delta_{k+q,k'}\, \delta_{\sigma\sigma'} - f(E_n(k'))\, \delta_{k',k+q}\, \delta_{k,k'+q'}\, \delta_{\sigma\sigma'}$$

where we used

$$\langle \hat{c}^{\dagger}_{\lambda} \hat{c}_{\lambda'} \rangle_0 = f(\varepsilon_\lambda)\, \delta_{\lambda\lambda'}$$

and $$f(\varepsilon_\lambda) = \frac{1}{e^{\beta(\varepsilon_\lambda - \mu_0)} + 1}$$ the Fermi function (4.14)

Further from the Kronecker relations it follows: $k - k' = q' = -q$, $\sigma' = \sigma$

$$\Rightarrow \begin{cases} k' = k + q \\ 2k' + q' = 2k + 2q - q = 2k + q \end{cases}$$

iii) Final form of $\chi^R_{II}(t)$, $\tilde{\chi}^R_{II}(\omega)$

From the above results we finally find

$$\chi^R_{II}(t) = -\frac{i\theta(t)}{\hbar} \underbrace{2}_{\text{spin } \sum_\sigma} \left(\frac{e\hbar}{2mL}\right)^2 \sum_{kqm} e^{-iq(x-x')} (2k+q)^2 e^{\frac{i}{\hbar}(\varepsilon(k) - \varepsilon(k+q))t} \left[f(E_m(k)) - f(E_m(k+q)) \right]$$

$$= \chi^R_{II}(t, x - x') \quad (4.14)$$

Taking the Fourier transform

$$\boxed{\tilde{\chi}^R_{II}(\omega) = +\frac{1}{\hbar}\, 2 \left(\frac{e\hbar}{2mL}\right)^2 \sum_{kqm} (2k+q)^2 e^{-iq(x-x')} \frac{f(E_m(k)) - f(E_m(k+q))}{\frac{1}{\hbar}(\varepsilon(k) - \varepsilon(k+q)) + \omega + i\eta}}$$

(4.15

iv) Homogeneous system

↳ DC-Conductance

$$G = \lim_{\omega \to 0} -\left(\frac{\mathrm{Im}\, \tilde{\chi}^{R}_{II}(\omega)}{\omega} \right) \qquad (4.16)$$

From (4.15) we immediately get, if we set $x = x'$, that the imaginary contribution solely comes from $\mathrm{Im} \frac{1}{x + i\eta} = -\pi\,\delta(x)$

$$\mathrm{Im}\, \tilde{\chi}_{II}(\omega) = -\pi \; 2\, \frac{1}{\hbar} \left(\frac{e\hbar}{2mL} \right)^2 \sum_{kqn} (2k+q)^2 \left[f(E_n(k)) - f(E_n(k+q)) \right] \hbar\, \delta(\varepsilon_k - \varepsilon_{k+q} + \hbar\omega) \qquad (4.17)$$

notice $\frac{1}{\hbar}\hbar$

$$\Rightarrow \boxed{G = \lim_{\omega \to 0} \frac{\pi}{\omega}\, 2 \left(\frac{e\hbar}{2mL} \right)^2 \sum_{kqn} (2k+q)^2 \left(-\frac{\partial f(E_n(k))}{\partial \varepsilon_k} \right) \hbar\omega\, \delta(\varepsilon_k - \varepsilon_{k+q})} \qquad (4.18)$$

where we used

$$E_n(k) = \epsilon_n + \varepsilon(k) = \epsilon_n + \varepsilon(k+q) + \hbar\omega = E_n(k+q) - \omega$$

$$f(E_n(k+q)) = f(E_n(k) + \hbar\omega) = f(E_n(k)) + \frac{\partial f}{\partial \varepsilon_k}(E_n(k))\,\hbar\omega$$

Hence
$$\boxed{G = 2\pi\hbar \left(\frac{e\hbar}{2mL} \right)^2 \sum_{kqn} (2k+q)^2 \left(-\frac{\partial f(E_n(k))}{\partial \varepsilon_k} \right) \delta(\varepsilon_k - \varepsilon_{k+q})} \qquad (4.18b)$$

or $\quad G = 2\pi\hbar \sum_{kqn} |\langle kn | \hat{i}(x) | k+q\, n \rangle|^2 \left(-\frac{\partial f}{\partial \varepsilon_k}(E_n(k)) \right) \delta(\varepsilon_k - \varepsilon_{k+q})$

- $\varepsilon(k) = \frac{\hbar^2 k^2}{2m}, \quad \frac{d\varepsilon}{dk} = \frac{\hbar^2 |k|}{m} \quad \Rightarrow \delta(\varepsilon_k - \varepsilon_{k+q})$

and $\delta(f(x) - f(x_0)) = \delta(x - x_0) \frac{1}{f'(x_0)}$

$$\Rightarrow \delta(\varepsilon_k - \varepsilon_{k+q}) = \delta(|k| - |k+q|) \frac{1}{d\varepsilon/dk}$$

This yields with $\sum_q = \frac{1}{2\pi}\int dq$

$$G = 2\pi\hbar \left(\frac{e\hbar}{2mL}\right)^2 \left(\frac{L}{2\pi}\right) \sum_{km} \int dq \; (2k+q)^2 \left(-\frac{\partial f}{\partial \varepsilon_k}\right) \frac{1}{\partial\varepsilon/\partial k}\, \delta(q)$$

$$= 2\pi\hbar \left(\frac{e\hbar}{2mL}\right)^2 \left(\frac{L}{2\pi}\right) \sum_{km} (2k)^2 \left(-\frac{\partial f}{\partial \varepsilon_k}\right) \frac{1}{\partial\varepsilon/\partial k}$$

$$= 2\pi\hbar \left(\frac{e\hbar}{2m}\right)^2 \frac{1}{2\pi L} \sum_{km} (2k)^2 \frac{1}{\frac{\hbar^2 |k|}{m}} \left(-\frac{\partial f}{\partial \varepsilon_k}\right)$$

and hence

$$G = 2\pi\hbar \left(\frac{e\hbar}{m}\right)^2 \frac{1}{2\pi L} \frac{4m}{\hbar^2} \sum_{km} \frac{k^2}{|k|} \left(-\frac{\partial f}{\partial \varepsilon_k}\right) \qquad (4.19)$$

use finally

$$\sum_k |k| \left(-\frac{\partial f}{\partial \varepsilon_k}\right) = \frac{L}{2\pi} \int_{-\infty}^{+\infty} dk\, |k| \left(-\frac{\partial f}{\partial \varepsilon_k}\right) = \frac{2L}{2\pi} \int_0^\infty dk\, k \left(-\frac{\partial f}{\partial \varepsilon_k}\right)$$

$$= 2\frac{L}{2\pi}\frac{m}{\hbar^2} \int_0^\infty d\varepsilon \left(-\frac{\partial f}{\partial \varepsilon}\right) = \frac{2L}{2\pi}\frac{m}{\hbar^2} f(\varepsilon_n)$$

$$d\varepsilon = \frac{\hbar^2}{m} k\, dk$$

$$f = f(E_n(k)) = \frac{1}{e^{\beta(E_n(k)-\mu_0)} + 1}, \qquad E_n(k) = \varepsilon_n + \varepsilon$$

$$\Rightarrow \quad G = 2\pi\hbar \left(\frac{e\hbar}{m}\right)^2 \frac{1}{2\pi L} \cdot \frac{m}{\hbar^2} \frac{2L}{2\pi} \frac{m}{\hbar^2} \sum_n f(\varepsilon_n) = \frac{2e^2}{h} \sum_n f(\varepsilon_n)$$

We thus obtain the final important result

$$G = \frac{2e^2}{h} \sum_m f(\varepsilon_m) \qquad (4.20)$$

with $f(\varepsilon_m) = \frac{1}{e^{\beta(\varepsilon_m - \mu_0)} + 1}$

In the limit of $T \to 0$ we thus recover the famous result that the conductance is quantized for a quasionedimensional, non interacting wire

$$\lim_{T \to 0} \boxed{G = \frac{2e^2}{h} \sum_m \Theta(\mu_0 - \varepsilon_m)} \qquad (4.21)$$

Note: Landauer formula in disguise

By looking at Eq. (4.9) and (4.9b) we recognize

$$\hat{I}(x) = \sum_{\lambda\lambda'} i_{\lambda\lambda'}(x)\, \hat{c}^{+}_{\lambda} \hat{c}_{\lambda'} \quad (4.9)$$

where, in the chosen basis $\{|\lambda\rangle\}$, $|\lambda\rangle = |nk\sigma\rangle$, is

$$i_{\lambda\lambda'}(x) = i_{nk\sigma\, n(k+q)\sigma'}(x) = \frac{e\hbar}{2mL}(2k+q)\, e^{-iqx}\, \delta_{nn}\, \delta_{\sigma\sigma'} \quad (4.9b)$$

If we now examine Eq. (4.18b) we immediately read out

$$\boxed{G = 2\pi\hbar \sum_{\lambda\lambda'} |i_{\lambda\lambda'}(x)|^2 \left(\frac{-\partial f}{\partial E_\lambda}\right) \delta(E_\lambda - E_{\lambda'})} \quad (4.22)$$

This formula holds true for a generic noninteracting system (cf next page) and, as discussed in the next section, is the Landauer formula for the conductance of a mesoscopic system adiabatically connected to reservoirs ($L \to \infty$) in disguise

Proof of (4.22): conductance of noninteracting system ($L\to\infty$)

Use Eq. (4.a) and $\hat{c}_\nu(t) = e^{-iE_\nu t/\hbar}\,\hat{c}_\nu$. Use basis $\{|\lambda\sigma\rangle\}$.

i) $$\langle[\hat{I}(x',t),\hat{I}(x',0)]\rangle_0 = \sum_{\sigma\sigma'}\sum_{\nu\nu'} i_{\nu\nu'}(x,m)\sum_{\lambda\lambda'} i_{\lambda\lambda'}(x')\,e^{i(E_\lambda-E_{\lambda'})t/\hbar}\,\langle[\hat{c}^+_{\lambda\sigma}\hat{c}_{\lambda'\sigma'},\hat{c}^+_{\nu}\hat{c}_{\nu'}]\rangle_0$$

$$=\sum_{\sigma}\sum_{\lambda\lambda'} |i_{\lambda\lambda'}(x')|^2\,e^{i(E_\lambda-E_{\lambda'})t/\hbar}\,[f(E_\lambda)-f(E_{\lambda'})]$$

if $x=x'$

where we used $\langle\hat{c}^+_{\lambda\sigma}\hat{c}_{\lambda'\sigma'}\rangle_0 = \delta_{\lambda\lambda'}f(E_\lambda)\,\delta_{\sigma\sigma'}$

$$i_{\lambda\lambda'}(x') = i^*_{\lambda'\lambda}(x')$$

ii) Remember $\chi^R_{II}(t) = -\frac{i}{\hbar}\theta(t)\langle[\hat{I}(x,t),I(x,0)]\rangle_0$

iii) Fourier transform $\to \tilde{\chi}^R_{II}(\omega) \to \mathrm{Im}\,\tilde{\chi}^R_{II}(\omega) \to \tilde{G}(\omega) = -\frac{\mathrm{Im}\,\tilde{\chi}(\omega)}{\hbar\omega}$

$$\tilde{G}(\omega) = -\overset{\text{spin}}{2}\,\frac{1}{\hbar\omega}\,\mathrm{Im}\sum_{\lambda\lambda'}\frac{|i_{\lambda\lambda'}(x)|^2}{\left(\omega+i\eta+\frac{E_\lambda}{\hbar}-\frac{E_{\lambda'}}{\hbar}\right)}\,[f(E_\lambda)-f(E_{\lambda'})]$$

iii) DC-limit yields the conductance

$$\boxed{\lim_{\omega\to0}\tilde{G}(\omega) = G = 2\pi\hbar\sum_{\lambda\lambda'}|i_{\lambda\lambda'}(x)|^2\left(-\frac{\partial f(E_\lambda)}{\partial E_\lambda}\right)\delta(E_\lambda-E_{\lambda'})} \quad (4.22)$$

$\Rightarrow$ we just need $|i_{\lambda\lambda'}(x)|^2$

notice that this expression is actually valid for other noninteracting systems since we only assumed that $\hat{V}_{ee}=0$ to i) get a trivial time dependence of $\hat{I}(x,t)$, ii) obtain that $\langle c^+_\nu c_\mu\rangle_0 = f(E_\nu)\,\delta_{\nu\mu}$

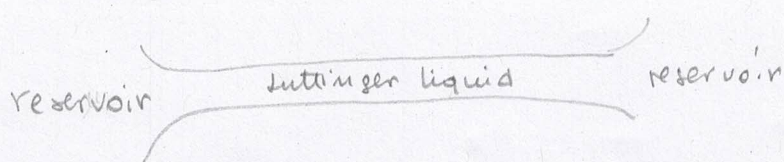
Note: Effects of e-e interactions?

The 1D interacting electron gas is very different from its higher dimensional counterparts. The main difference is the breakdown of the Fermi liquid theory.

The Fermi liquid theory enables to describe the elementary excitations of an interacting e-gas in 2D and in 3D in terms of quasiparticles.

In 1D the elementary excitations are the Tomonaga-Luttinger excitations, a sort of density waves, of collective nature (similar to sound waves in a crystal).

Within such model, the conductance at zero temperature still remains $G = \frac{e^2}{h}$ for a single mode (no spin) attached to noninteracting reservoirs



Maslov & Stone arXiv: cond-mat/9603106v1

2) To see Luttinger liquid behavior, impurities or tunneling barriers are needed

Realization of 1D conductors

- Semiconductor wires

Obtained by confinement of a 2D electron gas
(e.g. by cleaved edge overgrowth)

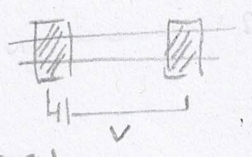
Tunneling experiments between two wire have been argued to show spin-charge separation characteristic of Luttinger liquids
(Tserkovnyak et al. PRB 68, 125312 (2003)

- Carbon nanotubes

diameter ~ 1 nm ⇒ only 1 relevant transverse mode

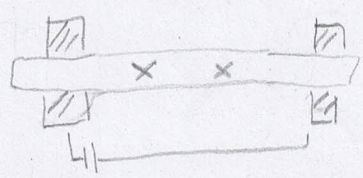
spin + valley degrees of freedom ⇒ $G = \frac{4e^2}{h}$

Also here Tomonaga-Luttinger liquid behavior has been seen in transport experiments measuring
tunneling density of states
or in presence of tunneling barriers



Bockrath et al, Nature 397, 598 (1999)

or of defects



Postma et al
Science 293, 76 (2003)