University of Regensburg | Winter Term 16/17

Quantum theory of condensed matter II

Mesoscopic physics

Prof. Milena Grifoni | Tue 8:00 - 10:00 9.2.01
| Fri 10:00 - 12:00 H33
PD Dr. Andrea Donarini | Fri 12:00 - 14:00 5.0.20

Sheet 5

1. Fisher-Lee relation

Given a scattering problem, with a scattering region connected to several leads, the Fisher-Lee relation connects the (retarded) Green's function of the full system G^{R} to the elements of its scattering matrix. Consider a 2 terminal device as the one in the figure and restrict yourself to a single transversal mode per lead. Starting from the eigenfunction representation of the retarded Green's function, prove that:

$$G^{\mathrm{R}}(x,x';E) = t_E G_0^{\mathrm{R}}(x,x';E)\theta(x-x_{\mathrm{R}}) + [1 + r_E e^{-2ik_E x'}]G_0^{\mathrm{R}}(x,x';E)\theta(x_{\mathrm{L}}-x), \tag{1}$$

where x_{L} and x_{R} define the borders of the scattering region, $x' < x_{\mathrm{L}}$, $G_0^{\mathrm{R}}(x,x';E)$ is the free particle Green's function, and t_E and r_E are the transmission and reflection amplitudes associated to the scattering region.

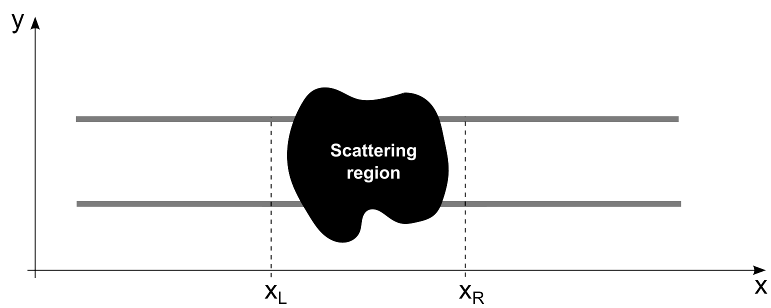

Figure 1:

Note that $k_E = \sqrt{\frac{2mE}{\hbar^2}}$. **(4 Points)**

2. Resonant tunnelling

In e.g., resonant tunnelling heterostructures one can make quantum-well systems which to a good approximation can be described by a 1D model of free electrons with two tunnelling barriers. Here we simplify it somewhat further by representing the tunnelling barriers by delta functions situated at $x_L = -d/2$ and $x_R = d/2$. The Hamiltonian is then given by:

$$H = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + U_0[\delta(x-x_1) + \delta(x-x_2)]. \tag{2}$$

1. With the help of the Dyson equation show that the retarded Green's function, for $x' < x_L < x_R < x$, reads:

$$G^{\mathrm{R}}(x,x';E) = \frac{e^{ik_E(x-x')}}{iv_E}\left[1 + \alpha(e^{-ikx_L}, e^{-ikx_R})\begin{pmatrix} 1-\alpha & -\alpha e^{i\theta} \\ -\alpha e^{i\theta} & 1-\alpha \end{pmatrix}^{-1}\begin{pmatrix} e^{ikx_L} \\ e^{ikx_R} \end{pmatrix}\right],$$

where $v_E = \frac{\hbar k_E}{m} = \sqrt{\frac{2E}{m}}$, $\alpha = \frac{U_0}{i\hbar v_E}$ and $\theta = k_E d$.

(4 Points)

2. From the expression of the Green's function just calculated, derive which is the transmission through a double delta barrier.

(2 Points)

Frohes Schaffen!