Quantum theory of condensed matter II

Mesoscopic physics

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	Fri	10:00 - 12:00	H33
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Sheet 3

1. Coulomb blockade

Consider an electronic quantum dot connected by weak tunnelling junctions (e.g. two completely pinched off quantum point contacts in a two dimensional electron gas) to two reservoirs at electrochemical potentials $\mu_\mathrm{L} = \mu_0 + eV_\mathrm{bias}/2$ and $\mu_\mathrm{R} = \mu_0 - eV_\mathrm{bias}/2$, respectively, where e is the electron charge and V_bias is the bias voltage across the junction. As such, these reservoirs work as source and drain electrodes for the quantum dot. The energy levels of the latter are controlled via a third electrode: the gate. In particular, the energy of the mth excited state with N electrons on the quantum dot is given by $E(N, m) = E_0(N, m) + \alpha N e V_\mathrm{gate}$, where $E_0(m, N)$ denotes the spectrum of the isolated dot, V_gate is the potential of the gate electrode and α is a dimensionless parameter which determines the effectiveness of the gate electrode.

1. Using the fermionic statistics of the electrons and the energy conservation during the tunnelling process, determine under which conditions an electron can tunnel between the quantum dot and the left/right electrode. Express these conditions in terms of inequalities involving the gate and the bias voltages V_gate and V_bias, respectively.

   2 Points

2. Using the relations obtained at the previous point determine, in the plane V_gate,V_bias the regions in which the current is *not* flowing through the quantum dot. Make a sketch of the result. These areas goes under the name of Coulomb diamonds.

   2 Points

3. For $(V_\mathrm{gate}, V_\mathrm{bias})$ belonging to a Coulomb diamond the electron number is fixed on the quantum dot. For $V_\mathrm{bias} = 0$ prove that the gate voltages at which two neighboring Coulomb diamonds are touching each other, i.e. the electron number on the dot passes from N to $N + 1$ is $eV_\mathrm{gate} = E_0(N + 1, 0) - E_0(N, 0) - \mu_0$. What is the form of the (zero bias) conductance as a function of the gate voltage V_gate?

   2 Points

4. Prove that the highest bias associated to the Coulomb diamond corresponding to an occupation of N electrons is given by the equation $eV_\mathrm{bias} = E_0(N + 1, 0) - 2E_0(N, 0) + E_0(N - 1, 0)$. The expression on the right hand side is the so called addition energy of the quantum dot.

   2 Points

2. Electronic waveguide

Consider a narrow conductor etched out of a wide one, as shown in the figure 1. The wide conductor can be treated simply as a two-dimensional conductor.

1. Calculate the location of the Fermi energy relative to the bottom of the band E_S, assuming an effective mass $m^* = 0.07m$ with m the electron mass and an electron density of 5×10^{11} cm^{-2}.

   1 Point

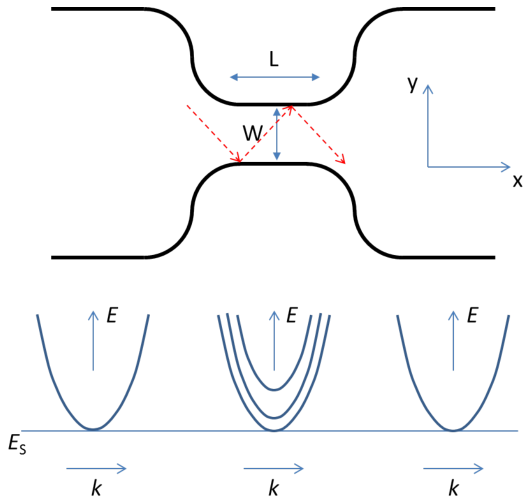

Figure 1: Schematic representation of the constriction used in exercises 2 and 3.

2. Plot the electronic density of states for the central region, assuming $W = 0.1\mu$m. Consider two possible realizations of the confining potential:

   a) Hard walls.

$$U(y) = \begin{cases} 0, & \text{for} \quad -W/2 < y > W/2 \\ \infty, & \text{otherwise} \end{cases}$$

   b) Harmonic confinement.

$$U(y) = \frac{1}{2} m\omega_0^2 y^2$$

   with ω_0 chosen such that $U(y = \pm W/2) = E_F - E_S$.

2 Points

3. A classical particle travelling in the etched conductor region will, in general, bounce a few times up and down before reaching again the wide conductor, as illustrated by the red-dashed arrows in figure 1. Is it possible to construct the quantum analogue for this dynamics? How?

1 Point

3. Filling of the Landau levels

Consider a narrow conductor etched out of a wide conductor as in Fig. 1 and assume a parabolic confining potential. Calculate the number of transverse modes as a function of the magnetic field, for the following cases:

a) assuming constant Fermi energy

b) assuming constant electron density

Use the results for the spectrum of the Landau levels obtained in the first exercise of the previous sheet and prove that if the Fermi energy remains constant then the conductor can be completely depleted as the magnetic field is increased, (see B.J. van Wees *et al.*, *Phys. Rev. B* **38**, 3625 (1988) for the experimental evidence), but if the electron density is assumed to remain constant then the number of modes cannot decrease to zero (at least one mode remains always occupied) as it is shown, for example in K.F. Berggren *et al.*, *Phys. Rev. B* **37**, 10118 (1988).

2 Points

Frohes Schaffen!