University of Regensburg Winter Term 16/17

Quantum theory of condensed matter II

Mesoscopic physics

Prof. Milena Grifoni	Tue	8:00 - 10:00	9.2.01
	Fri	10:00 - 12:00	H33
PD Dr. Andrea Donarini	Fri	12:00 - 14:00	5.0.20

Sheet 2

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1. Landau levels

Consider a 2 dimensional conductor confined to the xy plane in presence of a uniform magnetic field $\vec{B} = B\vec{e}_z$ pointing in the z direction.

1. Prove that the Schrödinger equation for the system can be written in the form:

$$\left(\frac{(p_x + eBy)^2}{2m} + \frac{p_y^2}{2m}\right)\Psi(x,y) = E\Psi(x,y) \tag{1}$$

   where $p_x = -i\hbar\frac{\partial}{\partial x}$ and $p_y = -i\hbar\frac{\partial}{\partial y}$.
   Hint: Use the minimal coupling $\vec{p} \to \vec{p} - e\vec{A}$ for the description of the magnetic field, where $\vec{A}$ is the vector potential. **2 Points**

2. Using the translational invariance of the Hamiltonian (1) in the x direction one can make the Ansatz

$$\Psi(x,y) = \frac{1}{\sqrt{L}}e^{ikx}\chi_k(y),$$

   where L is the (large, see next point for a quantitative estimate) size of the conductor, both in the x and y direction. Prove that the function $\chi_k(y)$ should solve the equation of a quantum harmonic oscillator of mass m and frequency $\omega_c = \frac{|e|B}{m}$ centered around the point $y_k = -\frac{\hbar k}{eB}$. Write the corresponding eigenvalues $E_{n,k}$ and also the eigenfunctions $\chi_{n,k}(y)$ in terms of the Hermite polynomials. **4 Points**

3. Prove that the solutions obtained above is only valid in the limit $L \gg \sqrt{\hbar/(m\omega_c)}$ and $L \gg |\hbar k/(eB)|$, thus giving a quantitative meaning to the condition on L assumed in the previous point. **2 Points**

4. Which is the group velocity associated to the state $\chi_{n,k}(y)$? How does this result compare with the classical picture of the orbits of a charged particle in a magnetic field? **1 Point**

5. Now consider the system considered before but with an additional parabolic confinement $U(y) = \frac{1}{2}m\omega_0^2 y^2$. How are the solutions of the Schrdinger equation modified by the additional confinement? Which is the group velocity associated to the state of the system?

4 Points

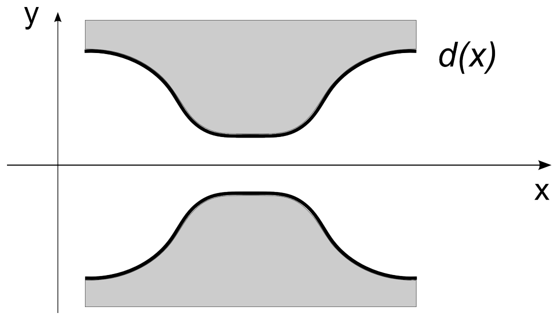
2. Adiabatic quantum point contact (to be discussed in class)

A quantum point contact as shown in the picture below can be described by the two-dimensional Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x,y) = E\psi(x,y) \tag{2}$$

with the boundary condition

$$\psi(x, \pm d(x)) = 0 \tag{3}$$



Make the following Ansatz for the wavefunction of the system:

$$\psi(x,y) = \sum_{n=1}^{\infty} c_n(x)\,\phi_n(y;x)$$

where

$$\phi_n(y;x) = \sqrt{\frac{1}{d(x)}}\,\sin\left(\frac{n\pi}{2d(x)}(y+d(x))\right),$$

are a set of local, basis wave functions for the transverse direction which obviously fulfill the boundary condition (3).

1. Derive a set of equations for the functions $c_n(x)$, by inserting the Ansatz for $\psi(x,y)$ in the Schrödinger equation (2), and by projecting it on the basis state $\phi_n(y;x)$.

2. Under which conditions for the function $d(x)$ are the equations for $c_n(x)$ and $c_m(x)$ with $m \neq n$ independent? Give a physical interpretation of the result.

Frohes Schaffen!