Quantum theory of condensed matter II

Mesoscopic physics

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	Fri	10:00 - 12:00	H33
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Sheet 1

1. Charging energy

The charging energy required to charge a capacitor with capacitance C by the charge of a single electron is $E_C = e^2/(2C)$. Let us assume a simple parallel-plate model for a capacitor, $C = \epsilon A/d$, where $\epsilon = \epsilon_r\epsilon_0$, typically ϵ_r =10 and ϵ_0 =8.85 pF/m, A is the area of the plates and d their separation. We can also set $d = 2$ nm, a typical thickness of the oxide layer ("tunnel contact") formed between two metals. Assuming a square plate $A = w^2$, estimate a width w of the junction which would correspond to the charging energy E_C being equal to 1 K. Estimate also the corresponding capacitances. How should these scales change so that charging effects would be observable at room temperature, i.e., $E_C/k_B \approx 300$ K? Remember that $e = 1.6 \times 10^{-19}$ C and $k_B = 1.38 \times 10^{-23}$ J/K.

(4 Points)

2. Density of states in reduced dimensions

The density of states (DOS) is a basic quantity useful to characterize a physical system. Fingerprints of the shape of the density of states $\rho(E)$ are observed both in the equilibrium state and non-equilibrium dynamics. The DOS is defined as:

$$\rho(E) = \frac{1}{V_d}\sum_i \delta(E - \epsilon_i), \tag{1}$$

where i denotes the set of quantum numbers characterizing the system. E.g. for free electrons in semiconductors, it is

$$\epsilon_i = \epsilon_{\vec{k}\sigma} = \frac{\hbar^2|\vec{k}|^2}{2m^*}, \tag{2}$$

with m^* the effective mass, $\vec{k}$ and σ the electron wave number and the spin.

1. Using eqautions (1) and (2), calculate the DOS for an electron gas in zero (quantum dot), one (quantum wire), two (2DEG), and three (bulk) dimensions. Sketch the function $\rho(E)$.
2. Do you recognize any relation between dimensionality d, dispersion relation and energy dependence of $\rho(E)$?
3. What changes in the DOS if the electrons follow a linear dispersion relation instead of the quadratic one given in equation (2)?

(6 Points)

3. Question on a scientific paper

In nanoelectronics measurements, the most typically measured observable is either the current as a function of voltage or the (linear) conductance as a function of some control parameter, such as the gate voltage or a magnetic field. Sometimes the voltage is measured as a function of current. Consult the papers Smit *et al., Nature*, **419**, 906 (2002), Kummeth *et al., Nature*, **452**, 448 (2008) and Heersche *et al., Nature*, **446**, 56 (2007), and determine **what the measured observables and the control parameters were** in the reported experiments. Note that the size of the typically measured nanoelectronic structures ranges from a few nanometers to some micrometers. On the other hand, the measurement equipment is in our everyday scale (from some tens of cm to meters). Argue why and in which case the latter can show some information about the former.

(4 Points)

Frohes Schaffen!