Universität Regensburg Winter Term 14/15

Quantum Theory of Condensed Matter II

Mesoscopic Physics

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Sheet 10

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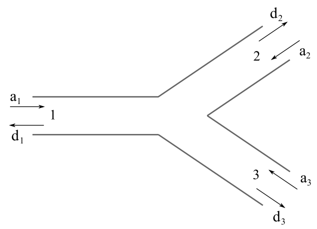
1. Mesoscopic beam splitter

Consider a device with three terminals, up/down symmetry and time reversal symmetry.



1. Show that the scattering matrix can be parametrized as

$$S = \begin{pmatrix} r_0 & t & t \\ t & r & r' \\ t & r' & r \end{pmatrix} \tag{1}$$

2. Assume real parameters and show that for nonzero t either

$$t^2 = \frac{1-r_0^2}{2}, \qquad r = -\frac{1+r_0}{2}, \qquad r' = \frac{1-r_0}{2}$$

or

$$t^2 = \frac{1-r_0^2}{2}, \qquad r = \frac{1-r_0}{2}, \qquad r' = -\frac{1+r_0}{2}$$

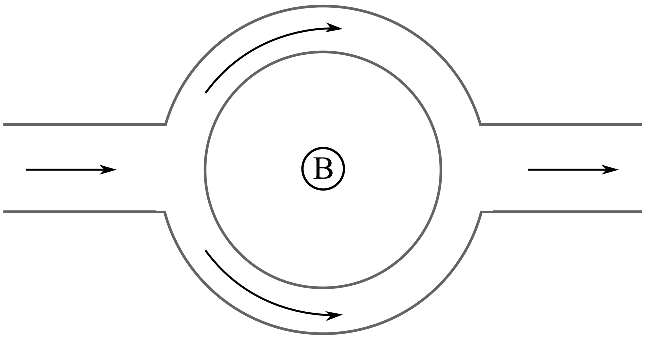
has to hold. What is the maximum value for t^2?

3. Consider a fully symmetric system. What changes? Can r become zero?

(6 Points)

2. Mesoscopic Aharonov-Bohm effect

The conductance through a loop which is pierced by a magnetic field B oscillates as a function of the field. Consider two identical beam splitters with scattering matrices as in equation (1) connected in series through a ring threaded by a magnetic field. Assume that the magnetic field is nonzero only in the middle region of the ring and the electrons do not feel a Lorentz force.



1. Start with $B = 0$. Suppose that electrons acquire a phase φ when traversing either the upper or the lower branch of the ring (that is to say the "transmission" amplitude of one branch is $e^{i\varphi}$ just as it is e^{ikL} for propagation through a piece of free space with length L). Show that the total transmission amplitude $\tilde{t}$ is given by
$$\tilde{t}(\varphi) = 2\,t^2 e^{i\varphi} \frac{1-(r-r')^2 e^{2i\varphi}}{1-2\,(r^2+r'^2)\,e^{2i\varphi}+(r^2-r'^2)^2\,e^{4i\varphi}}\,.$$
You can use Maple or Mathematica for the algebra. Show that for real parameters, using the results of problem 1, that
$$T \equiv |\tilde{t}|^2 = \frac{(1-r_0^2)^2}{1-2\,r_0^2\cos(2\varphi)+r_0^4}\,.$$
Where are the conductance resonances as a function of φ? What does the resonance condition mean?

2. If the magnetic field is finite, an electron moving clockwise through one of the arms acquires an additional phase ϕ, while an electron moving counterclockwise acquires an additional phase $-\phi$ with $2\,\phi = \oint \boldsymbol{A}\cdot d\boldsymbol{l} = 2\pi\Phi/\Phi_0$. Here Φ is the magnetic flux through the ring and $\Phi_0 = h/e$ is the magnetic flux quantum. Show that in this case one gets
$$\tilde{t}(\varphi,\phi) = 2\,t^2\cos(\phi)e^{i\varphi} \frac{1-(r-r')^2 e^{2i\varphi}}{1-2\,(r^2+r'^2\cos[2\phi])\,e^{2i\varphi}+(r^2-r'^2)^2\,e^{4i\varphi}}\,.$$
The transmission probability $|\tilde{t}(\varphi,\phi)|^2$ is an oscillating function of Φ. The fundamental frequency is given by Φ_0. These oscillations are called Aharonov-Bohm (AB) oscillations. However also higher harmonics with periods that are integer fractions of Φ_0 are present, for example the Altshuler-Aronov-Spivak (AAS) oscillations with period $\Phi_0/2$. What is the physical origin of the AB and the AAS oscillations? Plot $T(\varphi,\phi)$ for different parameters.

3. Consider the limit of a nearly closed ring $r_0 = 1-\Delta$ and $r' = \Delta/2$, $\Delta \ll 1$. Show that only the fundamental oscillation survives in leading order in Δ. Explain this observation.

(6 Points)

3. Origin of the memory

First, shortly answer the following questions:

1. Why are the constituting equations of Mechanics, be it classical Newtonian, be it Quantum, local in the time coordinate?

2. Is this still true also for Relativistic Mechanics?

3. What about Electrodynamics?

Consider now the following toy model:

$$\dot{x}_t = -\gamma x_t - \tilde{\gamma} y_t$$
$$\dot{y}_t = -\tilde{\alpha} x_t - \alpha y_t$$

where the notation x_t and y_t simply indicates that x and y are functions of the time. The goal is to write an equation of motion for the quantity x. First assume $\tilde{\alpha} = 0$, and deduce that the equation of motion for x has no memory. As a second step assume $\tilde{\alpha} \neq 0, \quad t > 0$. Prove that the quantity x fulfills the equation (with memory):

$$\dot{x}_t = -\gamma x_t - \tilde{\gamma} y_t^{\text{homo}} + \tilde{\gamma}\tilde{\alpha} \int_0^t \mathrm{d}s M(t-s) x_s$$

where y_t^{homo} is the solution of the homogeneous equation for y_t and $M(t) = \exp(-\alpha t)$. Finally justify the Markov approximation in the limit $\alpha \gg \gamma$.

Frohe Weihnachten!