Quantum Theory of Condensed Matter II

Mesoscopic Physics

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Sheet 8

1. Nanostructure in quasi-static non-equilibrium

Consider a nanostructure in which a scattering region is connected by single-transverse-channel reflectionless leads to a left and right contact at chemical potentials μ_L and μ_R respectively (See Fig. 1).

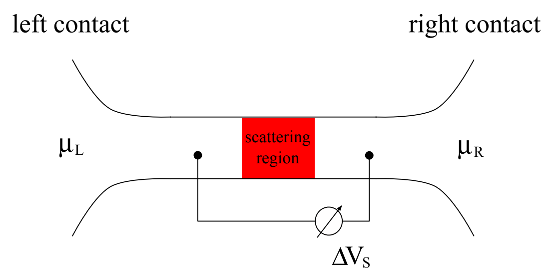

Figure 1: Schematic set up of a 4 points measurement

1. Calculate the voltage drop ΔV_S near to the scatterer if its transmission coefficient T is known. Prove the relation:
$$e\Delta V_S = (1 - T)(\mu_L - \mu_R)$$
Hint: Assume the transmission coefficient T to be energy independent within the bias window $[\mu_R, \mu_L]$. The left and right movers are in local equilibrium in every point of the nanostructure and thus possess local chemical potentials $\mu_-(x)$ and $\mu_+(x)$ respectively. In equilibrium, at zero bias, the density of left and right movers is the same and the same holds for the associated chemical potentials. Calculate the excess in the density left and right movers on both sides of the scatterer when a bias is applied and from the latter the associated local chemical potentials. The voltage drop is given by $e\Delta V_S = \mu_-(L) - \mu_-(R) = \mu_+(L) - \mu_+(R)$.
2. Determine the 4-point resistance $R_{4pt} = e\Delta V_S/I$. How does it compare to the standard definition $R = (\mu_L - \mu_R)/I$? Where is the rest of the potential drop happening? Hint: Use the Landauer formula for calculating the current through the device.
3. We have just demonstrated that the nanostructure has a finite resistance, thus it is dissipating energy. Nevertheless all processes discussed so far are elastic. Where is the energy dissipated?

(6 Points)

2. Transmission matrix of a single barrier

The scattering matrix S connects the amplitudes of the incoming and outgoing scattering states. The transmission matrix M connects instead the amplitudes of the states on one side of the scatterer to the ones on the other side.

Formally:

$$\begin{pmatrix} O_l \\ O_r \end{pmatrix} = S \begin{pmatrix} I_l \\ I_r \end{pmatrix} \qquad \begin{pmatrix} I_r \\ O_r \end{pmatrix} = M \begin{pmatrix} O_l \\ I_l \end{pmatrix}$$

where O_α (I_α) is the amplitude of the outgoing (ingoing) scattering state in the lead α ($\alpha = l, r$).

1. We parametrize the scattering matrix S in the form

$$S = \begin{pmatrix} r_l & t_{lr} \\ t_{rl} & r_r \end{pmatrix}$$

   Prove that the M matrix takes the form:

$$M = \begin{pmatrix} t_{lr}^{-1} & -r_l t_{lr}^{-1} \\ r_r t_{lr}^{-1} & -\det(S) t_{lr}^{-1} \end{pmatrix}$$

2. Prove that the determinant of a scattering matrix is a complex number of modulus 1. Hint: use the fact that the flux of particle is conserved in a scattering event.

(4 Points)

3. Transmission of a double barrier

Consider now the case of a double scatterer. Assume each of the sattering regions charaterized by the scattering matrix S_i ($i = 1, 2$).

1. Prove that the transmission matrix for the composed nanostructure can be written as the matrix multiplication of the two transmission matrices. Is the order in the multiplication important?
2. Calculte the tunneling probability through the entire nanostrucure. Prove that it reads:

$$T_{12} = \frac{T_1 T_2}{1 - 2\sqrt{R_1 R_2} \cos\theta + R_1 R_2}$$

   where $T_i = |t_{lr}^{(i)}|^2$ is the transmission probability through the i-th scattering region and $R_i = 1 - T_i = |r_\alpha^{(i)}|^2$ with $\alpha = l, r$ and $\theta = \arg(r_r^{(2)} r_l^{(1)}) - \arg[\det(S_1)\det(S_2)]$. Hint: Start by identifying which element of the transmission matrix is needed to calculate the transmission probability. Evaluate in the second step the composite transmission matrix and calculte the result.
3. Interpret the result found in the previous point. In particular, which is the meaning of the $\cos\theta$ term appearing in the denominator?

(6 Points)

Frohes Schaffen!