# Quantum Theory of Condensed Matter II

Mesoscopic Physics

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## Sheet 7

### 1. Thouless energy

The Thouless energy for a system with linear dimension $L$ is defined as $E_c = \hbar/\tau_L$, where $\tau_L$ is the time it takes to a particle to diffuse from one side to the other of the system. Prove the following relation

$$g_L = \frac{E_c}{\Delta_L}$$

where $g_L$ is the dimensionless conductance and $\Delta_L$ the average distance between two energy levels.

**(3 Points)**

### 2. The 2 dimensional scaling function for symplectic symmetry.

The asymptotic form of the scaling function $\beta$ for 2 dimensional systems with spin-orbit scattering (symplectic case) reads

$$\beta(g) = \begin{cases} 1/2\pi g & : \quad g \gg 1 \\ \ln(g/g_0) & : \quad g \ll 1 \end{cases}$$

Describe the scaling behaviour of $g_L$ with increasing system size depending of the initial value $g_\ell$. Assume that the entire qualitative behaviour of $\beta$ can be correctly extracted out of minimal extrapolations from the asymptotic limits.

**(3 Points)**

### 3. Quantum corrections to the conductivity in 1, 2 and 3 dimensions

As shown in the lecture, the leading quantum corrections to the conductivity can be written as:

$$\delta\sigma_{\mathrm{WL}} = -\frac{e^2}{\pi} D \int \frac{d\mathbf{q}}{(2\pi)^d} \frac{1}{Dq^2 - i\omega}$$

1. Show that:

$$\delta\sigma \approx \begin{cases} -\frac{\sigma_0}{(k_{\mathrm{F}}\ell)^2} & d = 3 \\ -\frac{e^2}{2\pi\hbar} \ln\left(\frac{L_\omega}{\ell}\right) & d = 2 \\ -\frac{e^2}{2\pi\hbar} L_\omega & d = 1 \end{cases}$$

   where $L_\omega = \left(\frac{D}{-i\omega}\right)^{1/2}$ and $\sigma_0 = e^2 \nu D$ and the upper limit of the radial integrals is set to $\pi/(2\ell)$.

2. $\delta\sigma$ is the result of a perturbation theory. Which is the dimensionless small parameter used to perform the calculation? In the answer the same for all dimensions?

**(4 Points)**

## Frohes Schaffen!