Universität Regensburg Winter Term 14/15

Quantum Theory of Condensed Matter II

Mesoscopic Physics

Prof. Ferdinand Evers
Dr. Andrea Donarini Fri 12:30, PHY 5.0.20

Sheet 6

1. Gaussian disorder

Consider the disordered potential

$$U(\mathbf{r}) = \int d\mathbf{x} v(\mathbf{x}) \delta\rho(\mathbf{r} + \mathbf{x})$$

where $\delta\rho(\mathbf{r}) = \sum_{\mathrm{i}} \delta(\mathbf{r} - \mathbf{r_i}) - n$ describes the spatial oscillation of the density of scattering centers. The latter are located in the positions $\mathbf{r_i}$ and n represents the average density of the scattering centers. Assume that the positions of the scattering centers are completely uncorrelated. In this exercise you will prove step by step that the probability P associated to a specific configuration $U(\mathbf{r})$ of the disorder is given by

$$P\{U(\mathbf{r})\} = \mathcal{N} \exp\left(-\frac{1}{2V} \sum_{\mathbf{k}} |U_{\mathrm{k}}|^2 / n v_{\mathrm{k}}^2\right)$$

in the limit in which $v(\mathbf{r} = 0) \to 0$ and $n \to \infty$, but $nv(0)^2$ remains constant. $\mathcal{N}$ is a normalization constant, while U_{k} and v_{k} are the Fourier components of the disorder potential $U(\mathbf{r})$ and the single (impurity) potential $v(\mathbf{r})$, respectively.

1. As a first step, show that, for uncorrelated scatterers -*i.e* their position is completely randomized - in the limit of infinite system size V it holds

$$\begin{aligned}
\langle \delta\rho(\mathbf{x_1}) \delta\rho(\mathbf{x_2}) \rangle &= \delta(\mathbf{x_1} - \mathbf{x_2}) n \\
\langle \delta\rho(\mathbf{x_1}) \delta\rho(\mathbf{x_2}) \delta\rho(\mathbf{x_3}) \delta\rho(\mathbf{x_4}) \rangle &= n \delta(\mathbf{x_1} - \mathbf{x_2}) \delta(\mathbf{x_1} - \mathbf{x_3}) \delta(\mathbf{x_1} - \mathbf{x_4}) \\
&\quad + n^2 (\delta(\mathbf{x_1} - \mathbf{x_2}) \delta(\mathbf{x_3} - \mathbf{x_4}) + \text{Permut.})
\end{aligned} \tag{1}$$

   Hint: The averages should be intended as normalized integrals over the impurity positions.

2. As a second step, prove that, in the limit introduced above, it holds:

$$\langle U(\mathbf{r_1}) U(\mathbf{r_2}) U(\mathbf{r_3}) U(\mathbf{r_4}) \rangle = \langle U(\mathbf{r_1}) U(\mathbf{r_2}) \rangle \langle U(\mathbf{r_3}) U(\mathbf{r_4}) \rangle + \text{Permut.}$$

   and calculate $\langle U(\mathbf{r}) U(\mathbf{r}') \rangle$.

3. Finally, use the representation of P as average of the product of δ-functions

$$P\{U(\mathbf{r})\} \propto \langle \Pi_{\mathbf{k}} \delta(U_{\mathbf{k}} - \delta\rho_{\mathbf{k}} v_{-\mathbf{k}}) \rangle$$

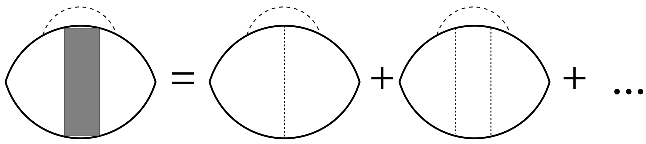
   Make usage of the known representation of the delta function in terms of Fourier transform. Expand in terms of $\delta\rho$ and in this way calculate the average (Cumulant expansion).

(6 Points)

2. Conductivity diagrams

Demonstrate that the following class of diagrams gives a non-divergent contribution to the conductivity:

1. Rewrite the digrams in terms of sum over momenta and sum them with the help of the geometrical series ($\longrightarrow$ Diffuson, compare withe the lecture).



2. Perform two of the three remaining sums using the techniques described in the lecture and show that the pole of the diffuson implies a non divergent contribution from the last summation.

(4 Points)

Frohes Schaffen!