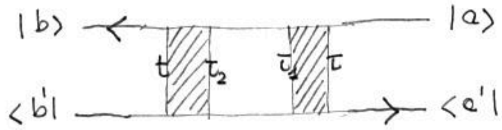
4.3.3 Diagrammatic representation of K_c

We have defined, cfr. (4.31), the correction kernel due to non-secular terms to be

$$K_c = - K^{(2)}_{sn} \left((K_0)_{nn} \right)^{-1} K^{(2)}_{ns}$$

This kernel can be seen as a sum of <u>reducible</u> fourth order diagrams. Specifically, let us consider the most generic reducible 4^{th} order graph:



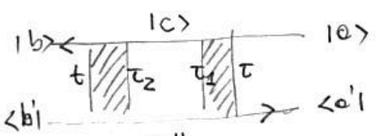
where  is a generic 2^{nd} order diagram.

This generic diagram corresponds to the analytical expression

$$\langle b| U_0(t,0) \Big[\int_0^t d\tau_2 \int_0^{\tau_2} d\tau_1 \int_0^{\tau_1} d\tau \; \mathrm{Tr}_{res} \Big\{ \mathcal{L}^I(t) \mathcal{L}^I(\tau_2) \; \mathrm{Tr}_{res} \Big\{ \mathcal{L}^I(\tau_1) \mathcal{L}^I(\tau) \left[U_0^+(\tau,0) |a\rangle\langle a'| U_0(\tau,0) \right] \otimes \rho_{res} \Big\} \otimes \rho_{res} \Big] U_0^+(t,0) |b'\rangle$$

$$= \langle b| U_0(t,0) \Big[\int_0^t d\tau_2 \int_0^{\tau_2} d\tau_1 \, \mathrm{Tr}_{res} \Big\{ \mathcal{L}^I(t) \mathcal{L}^I(\tau_2) \int_0^{\tau_1} d\tau \; K^{(2)}(\tau_1,\tau) \left[U_0^+(\tau,0) |a\rangle\langle a'| U_0(\tau,0) \right] \otimes \rho_{res} \; U_0^+(t,0) |b'\rangle$$

Since the third (from the right) Liouville operator is at time τ_2 one has to transform the result of the $K^{(2)}(\tau_1,\tau)$ operation and bring it in interaction picture at time τ_2.



$$= \sum_{cc'} \langle b| U_0(t,0) \Big[\int_0^t d\tau_2 \int_0^{\tau_2} d\tau_1 \; \mathrm{Tr}_{res} \Big\{ \mathcal{L}^I(t) \mathcal{L}^I(\tau_2) \, U_0^+(\tau_2,0) \overbrace{U_0(\tau_2,0) U_0^+(\tau_1,0)}^{U_0(\tau_2,\tau_1)} |c\rangle\langle c| U_0(\tau_1$$

$$\int_0^{\tau_1} d\tau \; K^{(2)}(\tau_1,\tau) \left[U_0^+(\tau,0) |a\rangle\langle a'| U_0(\tau,0) \right] \otimes \hat{\rho}_{res} \; U_0^+(\tau_1,0) |c'\rangle\langle c'| \overbrace{U_0(\tau_1,0) U_0^+(\tau_2,0)}^{U_0^+(\tau_2,\tau_1)} U_0(\tau_2,0) \Big\} U_0^+(t,0) |b'\rangle$$

$$= \sum_{cc'} \langle b| U_0(t,0) \, | \int_0^t d\tau_2 \, K^{(2)}(t,\tau_2) \Big[U_0^+(\tau_2,0) \int_0^{\tau_2} d\tau_1 \, U_0(\tau_2,\tau_1) |c\rangle$$

$$\langle c| U_0(\tau_1,0) \int_0^{\tau_1} d\tau \; K^{(2)}(\tau_1,\tau) \left[U_0^+(\tau,0) |a\rangle\langle a'| U_0(\tau,0) \right] U_0^+(\tau_1,0) |c'\rangle$$

$$\langle c'| U_0^+(\tau_2,\tau_1) \, U_0(\tau_2,0) \Big] \; U_0^+(t,0) |b'\rangle$$

One recognizes a sequence of nested operations:

i) $|a\rangle\langle a'| \longrightarrow U_0^\dagger(\tau,0)\,|a\rangle\langle a'|\,U_0(\tau,0)$ Schr → Interaction picture

ii) $\int_0^{\tau_1} d\tau\, K_I^{(2)}(\tau_1,\tau)[\quad] \longrightarrow$ application of the 2^{nd} order Kernel in interaction picture

iii) $U_0(\tau_1,0)[\quad]\,U_0^\dagger(\tau_1,0) \rightarrow$ Interaction → Schr at time τ_1

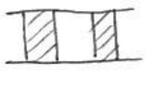
iv) $\int_0^{\tau_2} d\tau_1\, U_0(\tau_2,\tau_1)[\quad]\,U_0^\dagger(\tau_2,\tau_1) \rightarrow$ Evolution in the Schr. picture from τ_1 to τ_2

v) $U_0^\dagger(\tau_2,0)[\quad]\,U_0(\tau_2,0)$ Schr. → interaction picture at time τ_2

vi) $\int_0^{t} d\tau_2\, K_I^{(2)}(t,\tau_2)[\quad] \rightarrow$ application of the 2^{nd} order Kernel in interaction picture

vii) $U_0(t,0)[\quad]\,U_0^\dagger(t,0) \longrightarrow$ Back to the Schrödinger picture at time t.

Considering only the Schrödinger picture one can thus write

 $= \langle b|\int_0^t d\tau_2\, K_S^{(2)}(t-\tau_2)\left[\int_0^{\tau_2} d\tau_1\, U_0(\tau_2-\tau_1)\int_0^{\tau_1} d\tau\, K_S^{(2)}(\tau_1-\tau)\big[|a\rangle\langle a'|\big]\,U_0^\dagger(\tau_2-\tau_1)\right]|b'\rangle$

that is, 3 convoluted operations on the operator $|a\rangle\langle a'|$. In the Laplace space they will then result into the product of operations

$$K^{aa'}_{bb'} = \sum_{\substack{cc'\\dd'}} K^{(2)\,cc'}_{bb'}\, K^{nl\,dd'}_{0\,cc'}\, K^{(2)\,aa'}_{dd'}$$

In order to obtain the final identification of the Kernel $K^{nl\,dd'}_{0\,cc'}$ where nl stands for non-local we have to perform the Laplace transformation

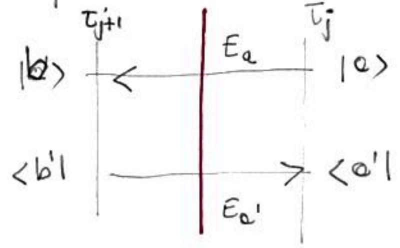
$$K^{nl\,dd'}_{0\;cc'} = \lim_{\lambda \to 0} \int_0^\infty d\tau \; e^{-\lambda\tau} \langle c | U_0(\tau,0) | d \rangle\langle d' | U_0^\dagger(\tau,0) | c' \rangle$$

$$= \lim_{\lambda \to 0} \int_0^\infty d\tau \; \delta_{cd}\,\delta_{d'c'}\; e^{-\frac{i}{\hbar}(E_d - E_{d'})\tau - \lambda\tau} = -\frac{\hbar}{i}\,\frac{1}{E_{d'} - E_d}\,\delta_{cd}\,\delta_{c'd'}$$

from which we obtain $K^{nl\,dd'}_{0\;cc'} = -\left(\left(K_0\right)^{dd'}_{cc'}\right)^{-1}$.

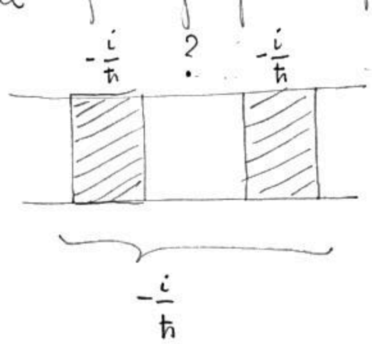
$$K_0{}^{dd'}_{cc'} = \delta_{cd}\,\delta_{c'd'}\,\left(-\frac{i}{\hbar}\right)(E_d - E_{d'}) \quad \Rightarrow \quad \left(K_0{}^{dd'}_{cc'}\right)^{-1} = \delta_{cd}\,\delta_{c'd'}\,\frac{\hbar}{i}\,\frac{1}{E_{d'} - E_d}.$$

Notice: • the form of the non-local kernel $(K_0^{nl})^{aa'}_{bb'}$ follows from the diagrammatic rules in energy space:



$$= -\frac{i}{\hbar}\,\frac{1}{E_{a'} - E_a}\,\delta_{ab}\,\delta_{a'b'}$$

The prefactor $-\frac{i}{\hbar}$ is wrong, but the diagram rules for the full graph imply:



$$-\frac{i}{\hbar} = \left(-\frac{i}{\hbar}\right) \times \left(-\frac{i}{\hbar}\right) \Rightarrow x = -\frac{\hbar}{i}$$

which means that in a global calculation it is properly calculated.

- The nl diagram does not diverge since it is only considered in the non-secular approximation.

4.4 Diagram grouping and physical interpretation

We start this section with a sum rule. Starting with the commutator structure of the time evolution Kernel it is obvious that the kinetic equation preserves the normalization condition. $\sum_a \rho_{red,aa}(t) = 1 \quad \forall t$. Which implies $\sum_{bb} \dot{\rho}_{red,bb} = 0$

The stationary limit of the dynamical equation reads (4.4)

$$0_{bb'} = -\frac{i}{\hbar}\sum_{aa'} \delta_{ab}\,\delta_{a'b'}\,(E_a - E_{a'})\,\rho^{stat}_{aa'} + \sum_{aa'} K^{aa'}_{bb'}\,\rho^{stat}_{aa'}$$

If we take $b'=b$ $\quad 0 = \sum_{aa'} K^{aa'}_{bb}\,\rho^{stat}_{aa'} = \sum_b\sum_{aa'} K^{aa'}_{bb}\,\rho^{stat}_{aa'} = \sum_{aa'}\left(\sum_b K^{aa'}_{bb}\right)\rho_{aa'}$

We obtain in this way the sum rule for the Kernel:

$$\left(\sum_b K^{aa'}_{bb}\right) = 0 \qquad \forall aa' \qquad (4.64)$$

Notice that the set of conditions (4.64) make the problem (4.4) undetermined and the condition $\sum_a \rho^{stat}_{aa}$ is needed to restore the uniqueness of the solution. Eq. (4.64) suggests that, by fixing for example $a=a'$, it should be possible to identify gain-loss partners which cancel away to yield (4.64).

4.4.1 Gain-loss pairs

Let us consider once again the AIM and restrict ourselves to the case of non-polarized or parallel polarized leads, such that spin coherences vanish and we can restrict to populations only.

The GME assumes, in terms of populations and rates the form:

$$
\begin{cases}
\dot{P}_0 = \underbrace{-\left(\sum_\sigma \Gamma^{0\to\sigma} + \Gamma^{0\to 2}\right)}_{K_{00}^{00}} P_0 + \sum_\sigma \underbrace{\Gamma^{\sigma\to 0}}_{K_{00}^{\sigma\sigma}} P_\sigma + \underbrace{\Gamma^{2\to 0}}_{K_{00}^{22}} P_2 \\[2ex]
\dot{P}_\sigma = \underbrace{-\left(\Gamma^{\sigma\to 0} + \Gamma^{\sigma\to\bar\sigma} + \Gamma^{\sigma\to 2}\right)}_{K_{\sigma\sigma}^{\sigma\sigma}} P_\sigma + \underbrace{\Gamma^{0\to\sigma}}_{K_{\sigma\sigma}^{00}} P_0 + \underbrace{\Gamma^{\bar\sigma\to\sigma}}_{K_{\sigma\sigma}^{\bar\sigma\bar\sigma}} P_{\bar\sigma} + \underbrace{\Gamma^{2\to\sigma}}_{K_{\sigma\sigma}^{22}} P_2 \\[2ex]
\dot{P}_2 = \underbrace{-\left(\sum_\sigma \Gamma^{2\to\sigma} + \Gamma^{2\to 0}\right)}_{K_{22}^{22}} P_2 + \sum_\sigma \underbrace{\Gamma^{\sigma\to 2}}_{K_{22}^{\sigma\sigma}} P_\sigma + \underbrace{\Gamma^{0\to 2}}_{K_{22}^{00}} P_0
\end{cases}
\qquad (4.65)
$$

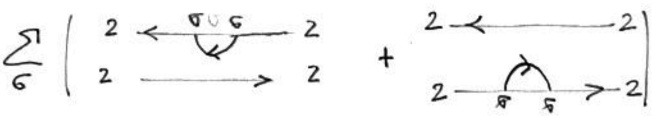
Where $P_{0/\sigma/2}$ is the probability of occupation of the corresponding AIM eigen-state $|0\rangle$, $|\sigma\rangle$, $|2\rangle$. $\Gamma^{\alpha\to\beta}$ is the rate of transfer of the probability from one state to another. In read we identify the different components of the Kernel. If now we write (4.64) for the special case $aa' = 22$

$$
\underbrace{K_{22}^{22}}_{\text{loss (from } |2\rangle)} = -\underbrace{\left(K_{00}^{22} + \sum_\sigma K_{\sigma\sigma}^{22}\right)}_{\text{gain (from } |0\rangle \text{ and from } |\sigma\rangle)} \qquad (4.66)
$$

The Kernel K_{22}^{22} enters the eq. (4.65) as depopulating level $|2\rangle$. On the other hand K_{00}^{22} populates $|0\rangle$ from $|2\rangle$ and $K_{\sigma\sigma}^{22}$ populates $|\sigma\rangle$ starting from $|2\rangle$. This relation is valid at each order. It is also valid in 2^{nd} order. Because $\left(\Gamma^{(2)}\right)^{2\to 0} = \left(K^{(2)}\right)_{00}^{22} = 0$

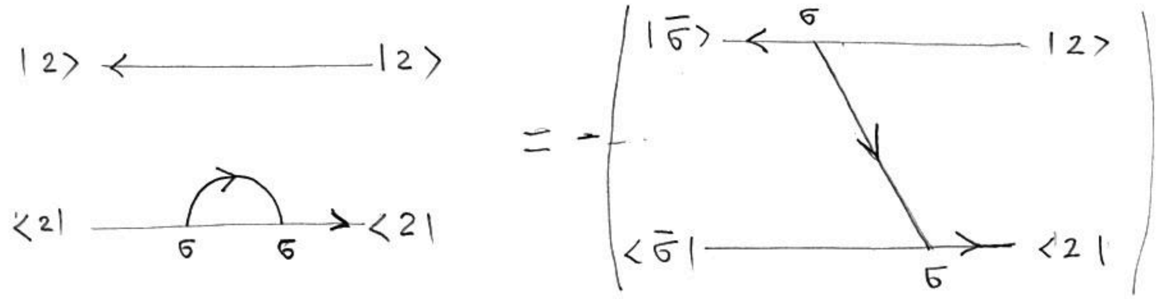
$\left(K^{(2)}\right)_{22}^{22} = -\sum_\sigma \left(K^{(2)}\right)_{\sigma\sigma}^{22}$ Diagrammatically one can write:

$\left(K^{(2)}\right)_{22}^{22} = \sum_\sigma$

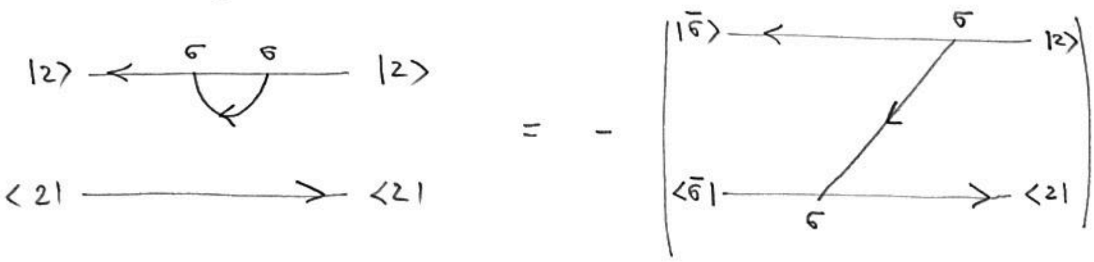


$\left(K^{(2)}\right)_{\sigma\sigma}^{22} =$

We can thus identify easily the gain-loss pairs:



and



Moving the last vertex from the upper ↔ lower contour only changes a sign. This operation is only possible if $b = b'$.

4.4.2 Diagram grouping

We wish to demonstrate that by evaluating the effective Kernel (4.31)

$$K_{eff}^{(4)} = K_{ss}^{(4)} + K_{c}^{(4)}$$

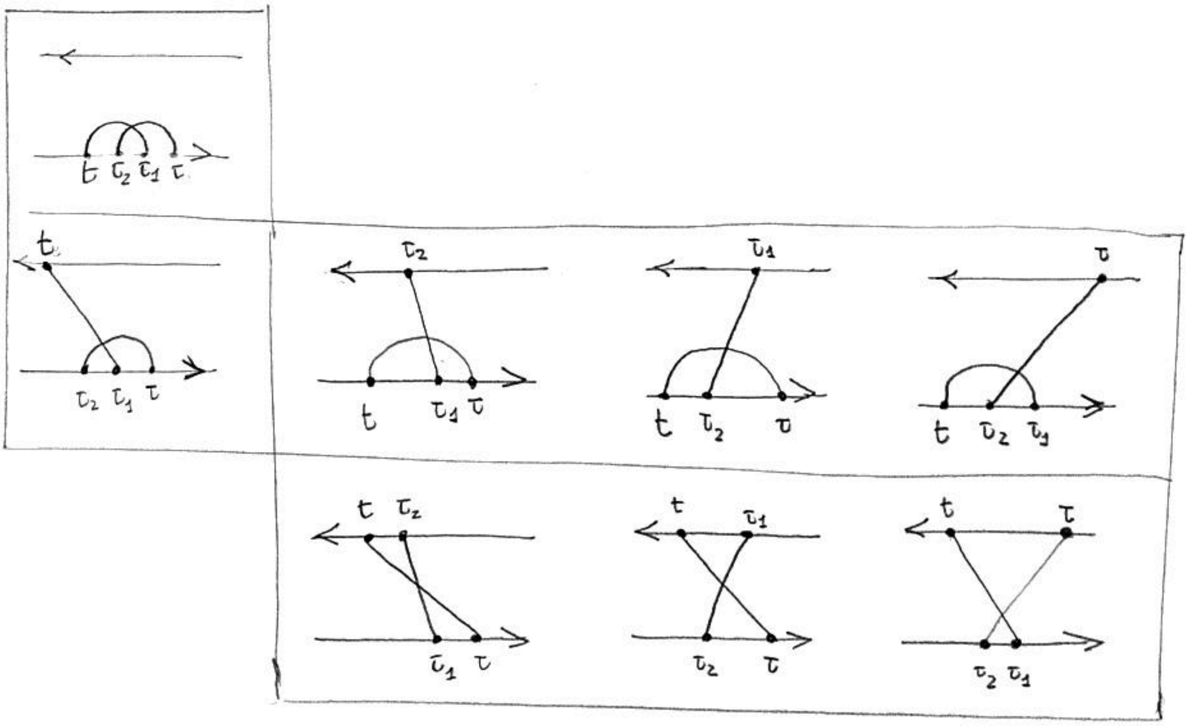
a possibility to group diagrams exists which allows analytical simplifications. The resulting grouping structure is shown in the figure below. The criteria are:

* For a given position of the four vertices → 3 distinct ways to connect them with fermion lines. => 3 classes A, B, C
* Number of vertices in the upper contour denotes the subclasses G.(0), G.(1), G.(2)
* The vertices can be placed in 1 + 4 + 3 <=> for 0, 1, 2 vertices on upper contour.

A.(0)

A.(1)

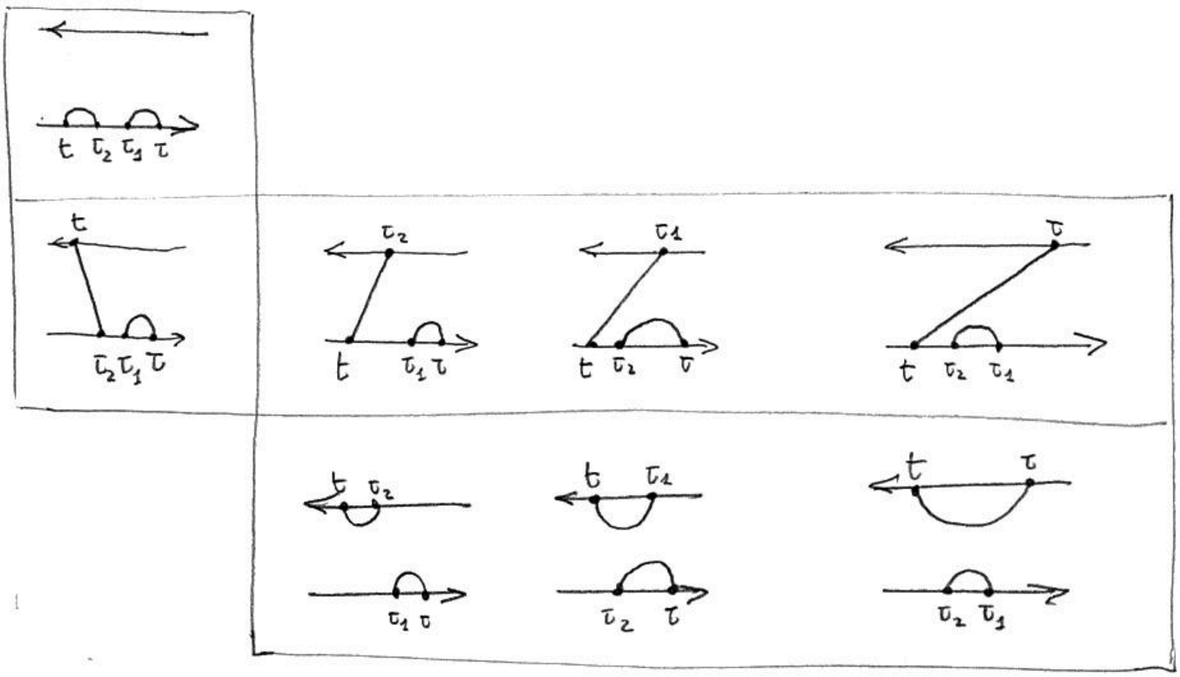
A.(2)



B.(0)

B.(1)

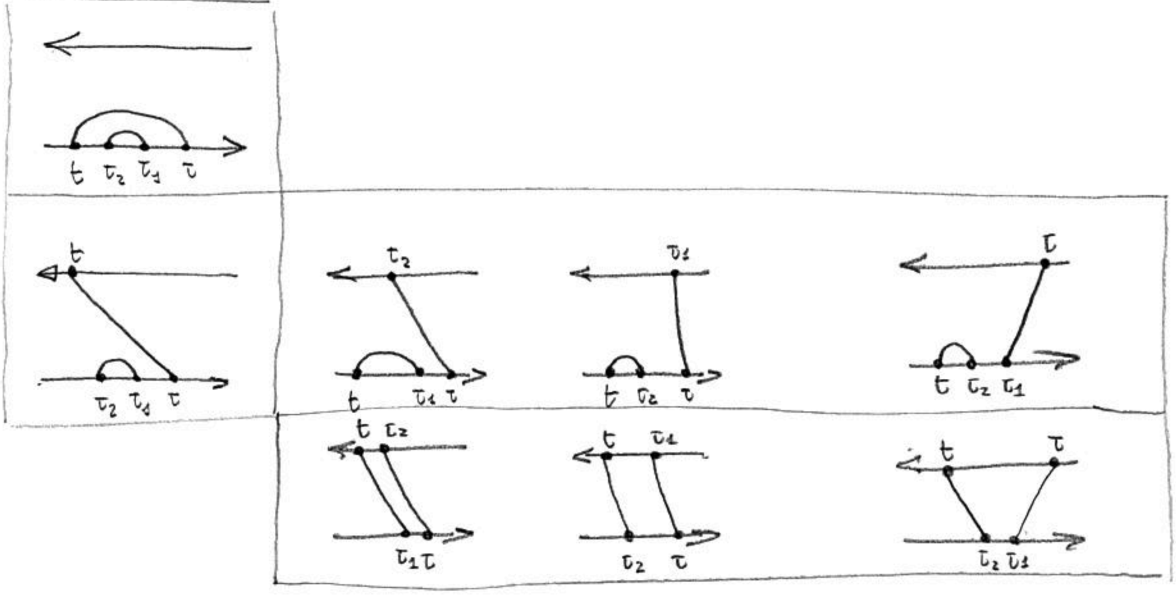
B.(2)



C.(0)

C.(1)

C.(2)



* One obtains => $3\times 8 = 24$ diagrams of which 16 are irreducible and 8 reducible.

* Notice further that there are "standalone" diagrams (s) and groups of three diagrams (t). In the groups G.(1)(s) the last (left most) vertex is on the upper contour.

The groups are defined as following:

A - fermionic lines are crossing

C - (apart from C.(0)) earliest (right most) vertex on upper contour
connects to " " " on lower " .

B - the others.

Group B and C contain reducible diagrams.

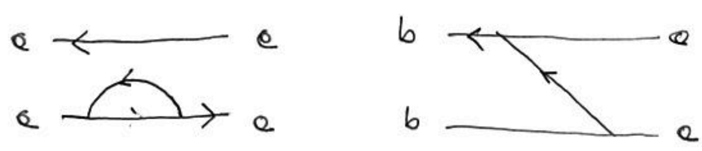
* Under assumption $b' = b$ G.(0) is the gain-loss pair of G.(1)(s).

=> G.(0)(s) = - G.(1)(s)

* Likewise each diagram in G.(1)(t) has a gain-loss pair in G.(2)(t).

=> G.(1)(t) = - G.(2)(t)

* Notice that in general they contribute to a different Kernel element
For example, in 2nd order



$$K^{ee\,(2)}_{ee} = - K^{ee\,(2)}_{bb}$$

4.4.3 Diagram summations

We have seen that the analytical expression related to a diagram splits in two parts: a product of TME and an energy-dependent function.
It now holds that within any group G.(x), $x \in 0,1,2$ all the diagrams have the <u>same topology</u> => for fixed sequence of states the members of the group have the same product of TMEs.

=> the energy dependent parts can be summed up to a new energy-dependent function $\tilde{G}_{(x)}$

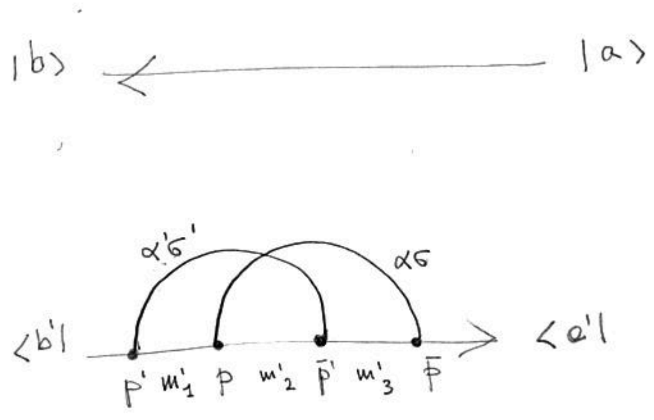
Specifically, an arbitrary element of the effectively secular Kernel can be written as:

$$\left(K^{(4)}_{eff}\right)^{aa'}_{bb'} = \sum_{x \in \{0,1,2\}} \left[\left(A_{(x)}\right)^{aa'}_{bb'} + \left(B_{(x)}\right)^{aa'}_{bb'} + \left(C_{(x)}\right)^{aa'}_{bb'} + h.c.\begin{pmatrix} a \leftrightarrow a' \\ b \leftrightarrow b' \end{pmatrix} \right] \qquad (4.67)$$

The nine contributions to eq. (4.67) read: [118 - 120]

$$\left(A_{(0)}\right)^{aa'}_{bb'} = -\sum_{m_1' m_2' m_3'} \sum_{pp'} \sum_{\alpha\alpha'} \left(\sum_{\sigma\sigma'} T^{\bar{p}}_{\alpha\sigma}(a', m_3') \, T^{\bar{p}'}_{\alpha'\sigma'}(m_3', m_2') \, T^{p}_{\alpha\sigma}(m_2', m_1') \, T^{p'}_{\alpha'\sigma'}(m_1', b') \right)$$

$$\cdot \tilde{A}_{(0)}\left(E_{am_1'} - p'eV_{\alpha'}, E_{am_3'} - peV_\alpha, E_{am_2'} - peV_\alpha - p'eV_{\alpha'}\right) \delta_{ab} \qquad (4.68)$$



Note: all energy differencies in the argument of $\hat{A}_{(0)}$ can be read off from the 3 possible cuts in the diagram where $\quad E_{ab} = E_a - E_b \quad$ and

$$\tilde{A}_{(0)}(\mu,\mu',\Delta) := {}^{+}X^{--}_{++}(\mu,\mu',\Delta) \tag{4.69}$$

Analogously one writes, for the other contributions

$$\left(A_{(1)}\right)^{aa'}_{bb'} = + \sum_{m_1' m_2'} \sum_{pp'} \sum_{\alpha\alpha'} \left(\sum_{\sigma,\sigma'} T^{p}_{\alpha\sigma}(b,a)\, T^{\bar{p}'}_{\alpha'\sigma'}(a',m_2')\, T^{\bar{p}}_{\alpha\sigma}(m_2',m_1')\, T^{p'}_{\alpha'\sigma'}(m_1',b') \right)$$

$$\times\ \tilde{A}_{(1)}\left(E_{am_2'} - p'eV_{\alpha'},\ E_{ba} + peV_\alpha,\ E_{am_1'} - peV_\alpha - p'eV_{\alpha'};\ E_{aa'},\ E_{bb'}\right)$$

$$\left(A_{(2)}\right)^{aa'}_{bb'} = - \sum_{mm'} \sum_{pp'} \sum_{\alpha\alpha'} \left(\sum_{\sigma,\sigma'} T^{p}_{\alpha\sigma}(b,m)\, T^{p'}_{\alpha'\sigma'}(m,a)\, T^{\bar{p}}_{\alpha\sigma}(a',m')\, T^{\bar{p}'}_{\alpha'\sigma'}(m',b') \right)$$

$$\times\ \tilde{A}_{(2)}\left(E_{am'} - peV_\alpha,\ E_{ma} + p'eV_{\alpha'},\ E_{ab'} - peV_\alpha - p'eV_{\alpha'};\ E_{aa'}\right) \tag{4.70}$$

where

$$\tilde{A}_{(2)}(\mu,\mu',\Delta;\nu) := {}^{+}D^{--}_{++}(\mu'+\Delta,\mu,\Delta) + {}^{+}D^{-+}_{++}(\mu'+\Delta,\mu,\mu+\mu') + {}^{+}X^{-+}_{++}(\mu'+\Delta,\mu'+\nu,\mu+\mu')$$

$$\tilde{A}_1(\mu,\mu',\Delta;\nu,\nu') := \tilde{A}_0(\nu'-\mu',\mu,\Delta) + \tilde{A}_2(\mu,\mu',\Delta;\nu) \tag{4.71}$$

- $\tilde{A}_{(2)}$ has 3 contributions in terms of functions with 3 energy arguments. This reflects 3 graphs with each 3 possible cuts.

- $\tilde{A}_{(1)}$ has 4 possible contributions. Importantly, $\tilde{A}_{(1)}$ can be written in terms of $\tilde{A}_{(0)}$ and $\tilde{A}_2$. This reflects the gain-loss pairing (the sign is absorbed in the definition of $A_{(x)}$)

- The additional arguments ν, ν' in $\tilde{A}_{(2)}$ and $\tilde{A}_{(1)}$ remark the energy modification in the gain-loss pairing transformation if $E_b \neq E_{b'}$ and/or $E_a \neq E_{a'}$.

For completeness we report also the contributions for the B and C groups:

$$\left(B_{(0)}\right)^{aa'}_{bb'} = + \sum_{m_1' m_2' m_3'} \sum_{pp'} \sum_{\alpha\alpha'} \left(\sum_{\sigma\sigma'} T^{\bar{p}'}_{\alpha'\sigma'}(m_3', m_2') T^{p'}_{\alpha'\sigma'}(m_3', m_2') T^{\bar{p}}_{\alpha\sigma}(m_2', m_1') T^{p}_{\alpha\sigma}(m_1', b') \delta_{ab} \right.$$

$$\times \tilde{B}_{(0)} \left(E_{am_1'} - peV_\alpha , E_{am_3'} - p'eV_{\alpha'}, E_{am_2'} \right)$$

$$\left(B_{(1)}\right)^{aa'}_{bb'} = - \sum_{m_1' m_2'} \sum_{pp'} \sum_{\alpha\alpha'} \left(\sum_{\sigma\sigma'} T^{p}_{\alpha\sigma}(b,a) T^{\bar{p}'}_{\alpha'\sigma'}(a', m_2') T^{p'}_{\alpha'\sigma'}(m_2', m_1') T^{\bar{p}}_{\alpha\sigma}(m_1', b') \right)$$

$$\times \tilde{B}_{(1)} \left(E_{am_2'} - p'eV_{\alpha'}, E_{ba} + peV_\alpha , E_{am_1'} ; E_{aa'}, E_{bb'} \right)$$

$$\left(B_{(2)}\right)^{aa'}_{bb'} = + \sum_{m, m'} \sum_{p, p'} \sum_{\alpha, \alpha'} \left(\sum_{\sigma\sigma'} T^{\bar{p}}_{\alpha\sigma}(b,m) T^{p}_{\alpha\sigma}(m, a') T^{\bar{p}'}_{\alpha'\sigma'}(a', m') T^{p'}_{\alpha'\sigma'}(m', b') \right)$$

$$\times \tilde{B}_{(2)} \left(E_{am'} - p'eV_{\alpha'}, E_{ma} + peV_\alpha , E_{ab'} ; E_{aa'} \right)$$

$$\left(C_{(0)}\right)^{aa'}_{bb'} = + \sum_{m_1' m_2' m_3'} \sum_{pp'} \sum_{\alpha\alpha'} \left(\sum_{\sigma\sigma'} T^{\bar{p}'}_{\alpha'\sigma'}(a', m_3') T^{\bar{p}}_{\alpha\sigma}(m_3', m_2') T^{p}_{\alpha\sigma}(m_2', m_1') T^{b'}_{\alpha'\sigma'}(m_1', b') \delta_{ab} \right.$$

$$\times \tilde{C}_{(0)} \left(E_{am_1'} - p'eV_{\alpha'}, E_{am_3'} - p'eV_{\alpha'}, E_{am_2'} - peV_\alpha - p'eV_{\alpha'} \right)$$

$$\left(C_{(1)}\right)^{aa'}_{bb'} = - \sum_{m_1' m_2'} \sum_{pp'} \sum_{\alpha\alpha'} \left(\sum_{\sigma\sigma'} T^{p'}_{\alpha'\sigma'}(b,a) T^{\bar{p}'}_{\alpha'\sigma'}(a', m_2') T^{\bar{p}}_{\alpha\sigma}(m_2', m_1') T^{p}_{\alpha\sigma}(m_1', b') \right)$$

$$\times \tilde{C}_{(1)} \left(E_{am_2'} - p'eV_{\alpha'}, E_{ba} + p'eV_{\alpha'} , E_{am_1'} - peV_\alpha - p'eV_{\alpha'} ; E_{aa'}, E_{bb'} \right)$$

$$\left(C_{(2)}\right)^{aa'}_{bb'} = + \sum_{m, m'} \sum_{pp'} \sum_{\alpha\alpha'} \left(\sum_{\sigma\sigma'} T^{p}_{\alpha\sigma}(b,m) T^{p'}_{\alpha'\sigma'}(m,a) T^{\bar{p}'}_{\alpha'\sigma'}(a', m') T^{\bar{p}}_{\alpha\sigma}(m', b') \right)$$

$$\times \tilde{C}_{(2)} \left(E_{am'} - p'eV_{\alpha'}, E_{ma} + p'eV_{\alpha'}, E_{ab'} - peV_\alpha - p'eV_{\alpha'} ; E_{aa'} \right) \tag{4.72}$$

where for the functions $\tilde{B}_{(x)}$ and $\tilde{C}_{(x)}$ we have used

$$\tilde{B}_{(0)}(\mu,\mu',\Delta) := (\delta_{\Delta,0}-1)\, \Upsilon_+^-(\mu)\,\frac{i\hbar}{\Delta}\,\Upsilon_+^-(\mu')$$

$$\tilde{B}_{(1)}(\mu,\mu',\Delta;\nu,\nu') := \tilde{B}_{(0)}(\nu'-\mu',\mu,\Delta) + \tilde{B}_{(2)}(\mu,\mu',\Delta;\nu)$$

$$\tilde{B}_{(2)}(\mu,\mu',\Delta;\nu) := {}^+X^{+-}_{++}(\mu'+\Delta,\mu,\mu+\mu') + {}^+D^{+-}_{++}(\mu'+\Delta,\mu'+\nu,\mu+\mu')$$
$$+ (\delta_{\Delta,0}-1)\,\Upsilon^-_+(\mu)\,\{i\hbar/\Delta\}\,\Upsilon^+_+(\mu'+\Delta)$$

$$\tilde{C}_{(0)}(\mu,\mu',\Delta) := {}^+D^{--}_{++}(\mu,\mu',\Delta)$$

$$\tilde{C}_{(1)}(\mu,\mu',\Delta;\nu,\nu') := \tilde{C}_{0}(\nu'-\mu',\mu,\Delta) + \tilde{C}_{(2)}(\mu,\mu',\Delta;\nu)$$

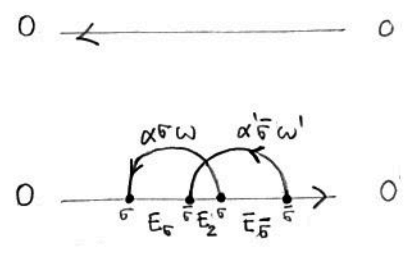
$$\tilde{C}_{(2)}(\mu,\mu',\Delta;\nu) := {}^+X^{--}_{++}(\mu'+\Delta,\mu,\Delta) + (\delta_{\mu,-\mu'}-1)\,\Upsilon^-_+(\mu'+\Delta)\,\frac{i\hbar}{\mu+\mu'}\left(\Upsilon^-_+(\mu)+\Upsilon^+_+(\mu'+\nu)\right) \tag{4,73}$$

From the structure of these functions one can further notice that:

- Apart from $\tilde{C}_{(0)}$ all of them include reducible components
- The reducible components vanish when the free propagation on the two contours is associated to states with the same energy.

To conclude we would like to prove exemplarily a pair-loop pairing relation for an irreducible 4th order diagram. For example, one can take the two diagrams associated to the AIM:

A



B