Universität Regensburg Winter Semester 09/10

Mesoscopic Physics

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Room 3.1.26
Fridays at 10:15

Sheet 12

1. Matsubara Green function and distribution functions

Use the Matsurbara Green function to find the Fermi-Dirac distribution by showing that

$$\langle c_\nu^\dagger c_\nu \rangle = n_F(\varepsilon_\nu)\,. \tag{1}$$

How would you calculate $\langle c_\nu c_\nu^\dagger \rangle$?

2. Single impurity scattering

the Dyson equation in the imaginary time domain for otherwise free electron that scatter against an external potential is given by

$$\mathcal{G}(b,a) = \mathcal{G}^0(b,a) + \sum_{\sigma_1} \int d\boldsymbol{r}_1 \int_0^\beta d\tau_1\, \mathcal{G}(b,\boldsymbol{r}_1,\sigma_1,\tau_1)\, V(\boldsymbol{r}_1,\sigma_1,\tau_1)\, \mathcal{G}^0(\boldsymbol{r}_1,\sigma_1,\tau_1,a) \tag{2}$$

with $a \equiv (\boldsymbol{r},\sigma,\tau)$ and $b \equiv (\boldsymbol{r}',\sigma',\tau')$. Suppose now that the electrons are confined in 1D and that the external potential is $V(x) = V_0\delta(x)$.
Show that the solution to the Dyson equation in the frequency domain is in this case

$$\mathcal{G}_\sigma(x,x',ik) = \mathcal{G}^0_\sigma(x,x',ik) + \mathcal{G}^0_\sigma(x,0,ik)\frac{V_0}{1 - V_0\mathcal{G}^0_\sigma(0,0,ik)}\mathcal{G}^0_\sigma(0,x',ik)\,. \tag{3}$$

Hint: Solve for $\mathcal{G}_\sigma(0,x',ik)$ first and insert that into the Dyson equation for $\mathcal{G}_\sigma(x,x',ik)$.
From this show furthermore that the retarded Green function can for $x < 0$ be written as

$$G^R_\sigma(x,x',\omega) = t\,G^{0R}_\sigma(x,x',\omega)\theta(x) + \left[1 + re^{i\phi(x,x')}\right] G^{0R}_\sigma(x,x',\omega)\theta(-x)\,. \tag{4}$$

Herefore use the expression for the unperturbed retarded Green function

$$G^{0R}_\sigma(x,x',\omega) = -\frac{i}{v_\omega}e^{ik_\omega|x-x'|} \tag{5}$$

with $k_\omega = \sqrt{2m(\mu+\hbar\omega)}/\hbar$ and the chemical potential μ. Discuss r, t and ϕ.

Frohes Schaffen!