Universität Regensburg Winter Semester 09/10

Mesoscopic Physics

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Room 3.1.26
Fridays at 10:15

Sheet 11

1. Single particle Matsurbara Green function for non-interacting particles

Consider a system of non-interacting particles described by the Hamiltonian

$$H_0 = \sum_\nu \epsilon_\nu \, c_\nu^\dagger c_\nu \tag{1}$$

(a) • Use the creation and annihilation operators in the imaginary-time Heisenberg picture, $c_\nu^\dagger(\tau)$ and $c_\nu(\tau)$, to calculate the single particle Matsurbara Green function $\mathcal{G}_0$, which in general is defined as

$$\mathcal{G}(\nu\tau, \nu'\tau') = -\left\langle T_\tau \left[c_\nu(\tau) c_\nu^\dagger(\tau') \right] \right\rangle \, . \tag{2}$$

Perform the calculation for both free fermions and bosons.

(b) • Fourier transform the resulting $\mathcal{G}_{0,\mathrm{F/B}}(\nu, \tau - \tau')$ into the frequency representation. Compare the results with the retarded Green function.

2. Polarizability of free electrons

Calculate the retarded polarization function $\chi^R(\boldsymbol{q}, \omega)$ starting from the expression

$$\chi^R(\boldsymbol{q}, \omega) \equiv \chi_0(\boldsymbol{q}, iq_n)|_{iq_n = \omega + i\eta} \tag{3}$$

with the Matsurbara function $\chi_0 = C_{\rho\rho,0}$. Start calculating χ_0 in the imaginary time domain

$$\chi_0(\boldsymbol{q}, \tau) = -\frac{1}{V} \left\langle T_\tau \left[\rho(\boldsymbol{q}, \tau) \rho(-\boldsymbol{q}, 0) \right] \right\rangle_0 \tag{4}$$

and use Wick's theorem to split the average over four creation/annihilation operators into products of averages containing only two operators. After Fourier transformation in order to get the frequency dependent quantity, perform the summation over Matsurbara frequencies, proving that for $N \geq 2$

$$\frac{1}{\beta} \sum_{ik_n} \prod_{j=1}^{N} \frac{1}{z - z_j} = \sum_j \mathrm{Res}_j \left(\prod_{j=1}^{N} \frac{1}{z - z_j} \right) n_F(z_j) \tag{5}$$

with the fermi function $n_F(z) = [\exp(\beta z) + 1]^{-1}$.

Frohes Schaffen!