Mesoscopic Physics

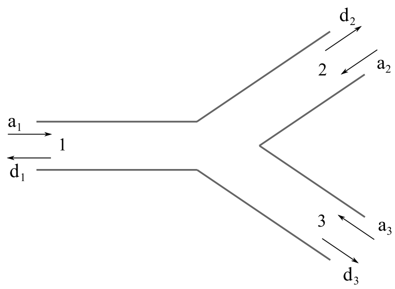
Dr. Andrea Donarini
Jürgen Wurm
Matthias Scheid

Room 3.1.26
Fridays at 10:15

Sheet 10

1. Mesoscopic beam splitter

Consider a device with three terminals, up/down symmetry and time reversal symmetry.



(a) • Show that the scattering matrix can be parametrized as

$$S = \begin{pmatrix} r_0 & t & t \\ t & r & r' \\ t & r' & r \end{pmatrix} \quad (1)$$

(b) • Assume real parameters and show that for nonzero t either

$$t^2 = \frac{1-r_0^2}{2}, \qquad r = -\frac{1+r_0}{2}, \qquad r' = \frac{1-r_0}{2} \quad (2)$$

or

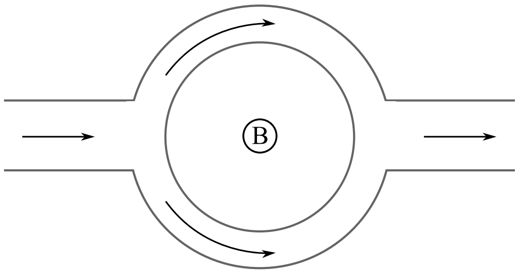
$$t^2 = \frac{1-r_0^2}{2}, \qquad r = \frac{1-r_0}{2}, \qquad r' = -\frac{1+r_0}{2} \quad (3)$$

has to hold. What is the maximum value for t^2?

(c) • Consider a fully symmetric system. What changes? Can r become zero?

2. Mesoscopic Aharonov-Bohm effect

The conductance through a loop which is pierced by a magnetic field B oscillates as a function of the field. Consider two identical beam splitters with scattering matrices as in equation (1) connected in series through a ring threaded by a magnetic field. Assume that the magnetic field is nonzero only in the middle region of the ring and the electrons do not feel a Lorentz force.



(a) Start with $B = 0$. Suppose that electrons acquire a phase φ when traversing either the upper or the lower branch of the ring (that is to say the "transmission" amplitude of one branch is $e^{i\varphi}$ just as it is e^{ikL} for propagation through a piece of free space with length L). Show that the total transmission amplitude $\tilde{t}$ is given by

$$\tilde{t}(\varphi) = 2\,t^2 e^{i\varphi} \frac{1-(r-r')^2 e^{2i\varphi}}{1-2\,(r^2+r'^2)\,e^{2i\varphi}+(r^2-r'^2)^2\,e^{4i\varphi}}\,. \tag{4}$$

You can use Maple or Mathematica for the algebra. Show that for real parameters, using the results of problem 1, that

$$T \equiv |\tilde{t}|^2 = \frac{(1-r_0^2)^2}{1-2\,r_0^2\cos(2\varphi)+r_0^4}\,. \tag{5}$$

Where are the conductance resonances as a function of φ? What does the resonance condition mean?

(b) If the magnetic field is finite, an electron moving clockwise through one of the arms acquires an additional phase ϕ, while an electron moving counterclockwise acquires an additional phase $-\phi$ with $2\,\phi = \oint \boldsymbol{A}\cdot d\boldsymbol{l} = 2\pi\Phi/\Phi_0$. Here Φ is the magnetic flux through the ring and $\Phi_0 = h/e$ is the magnetic flux quantum. Show that in this case one gets

$$\tilde{t}(\varphi,\phi) = 2\,t^2\cos(\phi)e^{i\varphi} \frac{1-(r-r')^2 e^{2i\varphi}}{1-2\,(r^2+r'^2\cos[2\phi])\,e^{2i\varphi}+(r^2-r'^2)^2\,e^{4i\varphi}}\,. \tag{6}$$

The transmission probability $|\tilde{t}(\varphi,\phi)|^2$ is an oscillating function of Φ. The fundamental frequency is given by Φ_0. These oscillations are called Aharonov-Bohm (AB) oscillations. However also higher harmonics with periods that are integer fractions of Φ_0 are present, for example the Altshuler-Aronov-Spivak (AAS) oscillations with period $\Phi_0/2$. What is the physical origin of the AB and the AAS oscillations? Plot $T(\varphi,\phi)$ for different parameters.

(c) • Consider the limit of a nearly closed ring $r_0 = 1-\Delta$ and $r' = \Delta/2$, $\Delta \ll 1$. Show that only the fundamental oscillation survives in leading order in Δ. Explain this observation.

3. Remember the numerical problem of last week!

Frohes Schaffen!