Universität Regensburg Winter Semester 09/10

Mesoscopic Physics

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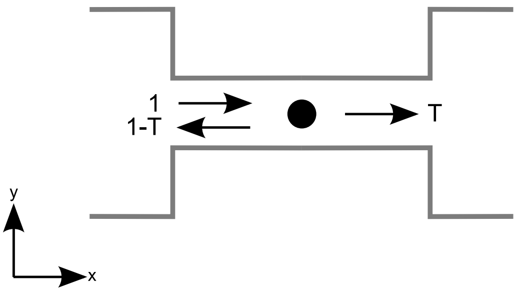
Sheet 7

1. Resistivity dipoles in a mesoscopic conductor

• Consider a mesoscopic device with two leads, two contacts and a scatterer in the middle at $x = 0$, which electrons can pass with an energy independent probability T. We set the chemical potential in the right contact to zero and in the left contact to μ.



Assume a symmetric arrangement and sketch the profile of the average chemical potential. Now ignore the contacts and approximate the chemical potential profile as

(a) $$\bar{\mu}(x) \approx \begin{cases} \mu\,(1 - T/2) & x < 0 \\ \mu\,T/2 & x \geq 0 \end{cases}$$

(b) $$\bar{\mu}(x) \approx \begin{cases} \mu\,(1 - T/2) & x < -L/2 \\ \mu\,[x\,(T-1)\,/L + 1/2] & -L/2 \leq x \leq L/2 \\ \mu\,T/2 & x > L/2 \end{cases}$$

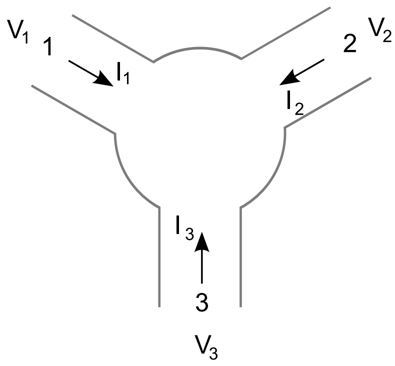
The electrostatic potential V will follow this profile, however it will be continuous. It is obtained via the Poisson's equation from the charge density n. In a very simple model we ignore the quantization in y-direction and assume that n has a spacial dependence only in x-direction. Prove that under these conditions the Poisson's equation reads

$$\partial_x^2\, V(x) = -\frac{e\,n(x)}{\epsilon\, d}\,, \tag{1}$$

where d is the extension of the potential well in the z direction. Since the density of states in a 2DEG is a constant $N_s = m/(\pi\hbar^2)$, n depends on the electrostatic potential in a simple way $n(x) = eN_s(\bar{\mu}(x) - V(x))$. Solve Eq. (1) to obtain both the electrostatic potential $V(x)$ and the two dimensional charge distribution $n(x)$. In case (b), how does L influence the charge distribution?

2. Landauer-Büttiker formalism for a three terminal device

Consider a scattering system with three leads that are connected to contacts.



(a) Use the Landauer Büttiker formula to set up a matrix equation that connects the currents to the voltages. For convenience set $V_2 = 0$ and remember that current conservation tells you $I_2 = -I_1 - I_3$, so that it is enough to consider

$$\begin{pmatrix} I_1 \\ I_3 \end{pmatrix} = \mathcal{G} \begin{pmatrix} V_1 \\ V_3 \end{pmatrix}$$

and solve this equation for V_1 and V_3 with given currents I_1 and I_3.

(b) From now on assume that contact 3 is used as a voltage probe ($I_3 = 0$) and calculate the resistances $R_{12,32}$ and $R_{12,12}$. The multiterminal resistances are defined as

$$R_{\alpha\beta,\gamma\delta} = \frac{V_\gamma - V_\delta}{I_{\alpha\to\beta}}$$

$I_{\alpha\to\,\beta}$ is the current flowing from contact α to contact β. In other terms by fixing to zero the currents in all contacts different from contact α or β. Is there anything in the resulting expressions that you would not have expected naively?

(c) Use the Onsager relations for the conductance to show explicitly that the reciprocity relation $R_{12,12}(B) = R_{12,12}(-B)$ holds.

(d) In the coherent limit $G_{i3}, G_{3i} \ll G_{12}, G_{21}$ for $i \in \{1, 2\}$. What is $R_{12,12}$ in this case? Calculate $R_{12,12}$ also in the incoherent limit $G_{i3}, G_{3i} \gg G_{12}, G_{21}$ for $i \in \{1, 2\}$. To which physical situations could the two limits correspond?

(e) Consider a set up with the same number of modes in each lead and with reflectionless contacts:

$$G_{13} = G_{31} = \frac{2e^2}{h} TN \quad \text{and} \quad G_{12} = G_{21} = \frac{2e^2}{h}(1-T)N$$

and calculate $R_{12,12}$. N is the number of modes in the leads and $T \leq 1$. Calculate the *invasiveness* α of the voltage probe

$$\alpha = \frac{R_p}{R_c},$$

where $R_c = \frac{h}{2e^2}\frac{1}{N}$ is the contact resistance and $R_p = R_{12,12} - R_c$ is the resistance that is due to the voltage probe.

Hint: Use the sum rule $\sum\limits_{i,i\neq j} G_{ij} = \sum\limits_{i,i\neq j} G_{ji}$ for fixed j.

Frohes Schaffen!