Universität Regensburg — Winter Semester 09/10

Mesoscopic Physics

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Room 5.01.01
Wednesdays at 15:15

Sheet 6

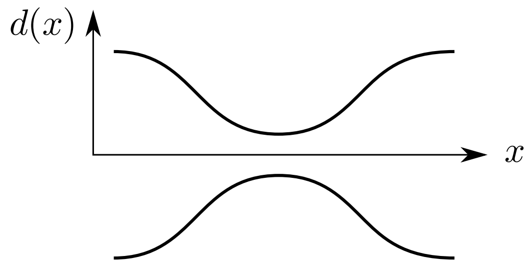
1. Adiabatic quantum point contact

A quantum point contact as shown in the picture can be described by the two-dimensional Schrödinger equation

$$-\frac{\hbar^2}{2m}\Delta\psi(x,y) = E\psi(x,y) \tag{1}$$

with the boundary condition

$$\psi(x, \pm d(x)) = 0 \tag{2}$$



(a) Expand the wavefunction $\psi(x,y)$ in terms of the local, transverse basis wavefunctions $\phi_n(x,y) = \sqrt{\frac{1}{d(x)}}\sin\left(\frac{n\pi}{2d(x)}(y+d(x))\right)$, which obviously fulfill the boundary condition (2). Use the expansion

$$\sum_{n=1}^{\infty} c_n(x)\,\phi_n(x,y) \tag{3}$$

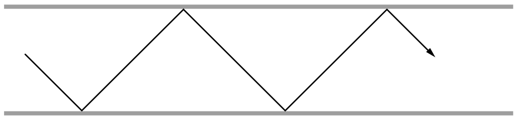
and the Schrödinger equation (1) to derive a set of equations for the $c_n(x)$. Is the above expansion exact or an approximation?

(b) Show that in the adiabitic limit $d'(x)/d(x)$, $d''(x)/d(x) \ll 1$, there is no mixing between the different states n.
(*Hint:* Show that the terms that couple different c_n are proportional to $d'(x)/d(x)$ and $d''(x)/d(x)$ respectively. You can use maple or mathematica to solve the integrals.)
Can you estimate the conductance of the quantum point contact?

2. Motion in wire with hard walls

(a) • Calculate the wavefunctions of the transverse modes and the corresponding energy spectrum of a lead with infinitely hard walls (infinitely high potential well). How does the hard wall confinement differ from the parabolic confinement discussed in class?

(b) • Classically the motion in such a wire would be zigzag like:



How can one describe this kind of orbits quantum mechanically? Construct a state whose expectation value of the position operator follows a zigzag like motion as in the picture.
(*Hint:* In transverse direction use two of the modes calculated in (a). In longitudinal direction, you can use e. g. a Gaussian wavepacket.)

3. Contact resistance for finite sized contacts

In the lecture the resistance of a ballistic conductor with two very large contacts was calculated under the assumption that in the contacts infinitely many transverse modes are occupied. The result was

$$R_{\mathrm{c}} = \frac{h}{2e^2} M \,, \tag{4}$$

where M was the number of occupied transverse modes in the conductor.
Now calculate the resistance taking into account that in a real contact the number of occupied transverse modes is a (large but finite) number N. To allow for a nonzero current the electrochemical potential of left and rightgoing charge carries has to differ by a small amount $\delta\mu$. Further assume reflectionless contact as in the lecture.

Frohes Schaffen!