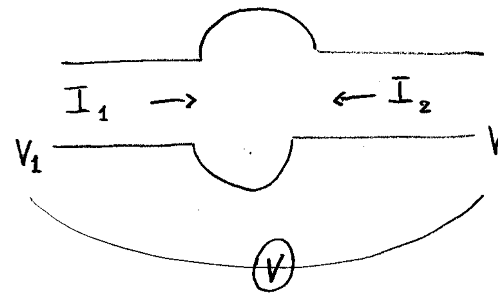
Coherence vs. Incoherence in multiterminal devices

i) two terminals

recall $G_{12} = \frac{2e^2}{h} T_{12}$

due to the reciprocity relations

$G_{12} = G_{21}$ Notice that for the 2 terminal dev. this is true $\forall B$!



$$I_1 = G_{12}(V_1 - V_2) = G_{12} V_1 - G_{12} V_2 \overset{\text{sum}}{\underset{\text{rule}}{=}} G_{21} V_1 - G_{12} V_2$$

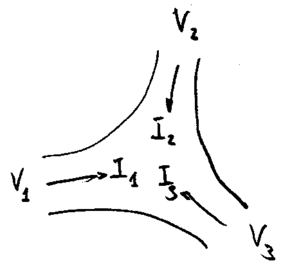
$$I_2 = G_{21}(V_2 - V_1) = G_{21} V_2 - G_{21} V_1 = G_{12} V_2 - G_{21} V_1 = -I_1$$

$$R_{12,12} = \frac{V_1 - V_2}{I_1} = \frac{1}{G_{21}}$$

$V_2 = 0$ is the reference voltage.

this definition of resistance reads: $R_{\alpha\beta,\gamma\delta} = \left.\frac{V_\gamma - V_\delta}{I_\alpha}\right|_{I_\sigma = 0,\ \sigma \neq \alpha,\beta}$

ii) three terminals



$$\underbrace{\begin{aligned} I_1 &= G_{12}(V_1 - V_2) + G_{13}(V_1 - V_3) \\ I_2 &= G_{21}(V_2 - V_1) + G_{23}(V_2 - V_3) \\ I_3 &= G_{31}(V_3 - V_1) + G_{32}(V_3 - V_2) \end{aligned}}_{} = \underbrace{\begin{aligned} &(G_{12} + G_{13}) V_1 - G_{12} V_2 - G_{13} V_3 \\ &- G_{21} V_1 + (G_{21} + G_{23}) V_2 - G_{23} V_3 \\ &- G_{31} V_1 - G_{32} V_2 + (G_{31} + G_{32}) V_3 \end{aligned}}_{}$$

interesting symmetric form, easy to remember. It involves all potential differences

Notice that if we use the sum rule $\sum_q T_{pq} = \sum_q T_{qp}$ we get to the physically simpler

$$I_1 = (G_{21} + G_{31}) V_1 - G_{12} V_2 - G_{13} V_3$$
$$I_2 = - G_{21} V_1 + (G_{12} + G_{32}) V_2 - G_{23} V_3$$
$$I_3 = - G_{31} V_1 - G_{32} V_2 + (G_{13} + G_{23}) V_3$$

Remember $G_{ij} = G_{i \leftarrow j}$ is the conductance relative to the transmission from contact j to contact i! The current is positive when pointing toward the scattering region.

$$I = \bar{g} V$$

in other terms $I_1 = \bar{g}_{11} V_1 + \bar{g}_{12} V_2 + \bar{g}_{13} V_3$ $\quad \bar{g}_{11} = G_{21} + G_{31}$. etc...

We set the reference voltage in $V_2 = 0$ and use the relation $I_2 = -I_1 - I_3$ (the scattering region is neither a source nor a drain of charge).

$$I_1 = (G_{21} + G_{31}) V_1 - G_{13} V_3$$
$$I_3 = - G_{31} V_1 + (G_{13} + G_{23}) V_3$$

We can solve this (2x2) system of equations to get

$$\begin{pmatrix} V_1 \\ V_3 \end{pmatrix} = R \begin{pmatrix} I_1 \\ I_3 \end{pmatrix} \qquad R = \begin{pmatrix} G_{21} + G_{31} & -G_{13} \\ -G_{31} & G_{13} + G_{23} \end{pmatrix}^{-1} = \frac{1}{\Delta} \begin{pmatrix} G_{13} + G_{23} & G_{13} \\ G_{31} & G_{21} + G_{31} \end{pmatrix}$$

$$\Delta = (G_{21} + G_{31})(G_{13} + G_{23}) - G_{31} G_{13} =$$
$$= G_{21} G_{13} + G_{21} G_{23} + G_{31} G_{23}$$

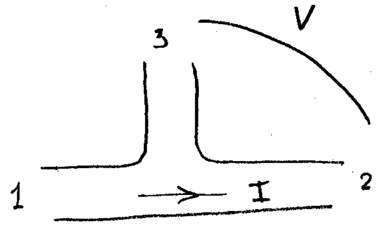
Thus:

$$V_1 = \frac{(G_{13} + G_{23}) I_1 + G_{13} I_3}{G_{21} G_{13} + G_{21} G_{23} + G_{31} G_{23}}$$

$$V_2 = 0$$

$$V_3 = \frac{G_{31} I_1 + (G_{21} + G_{31}) I_3}{G_{21} G_{13} + G_{21} G_{23} + G_{31} G_{23}}$$

Let's put $I_3 = 0$ => consider the contact + lead 3 as a voltage probe



$$R_{12,32} = \left.\frac{V_3 - V_2}{I_1}\right|_{I_3=0}$$

$$\boxed{R_{12,32} = \frac{G_{31}}{G_{21}G_{13} + G_{21}G_{23} + G_{31}G_{23}}}$$

Message: even if there is no current flowing in 3, the resistance depends on the transmission probabilities $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$! Even more strange is the two terminal resistance:

$$R_{12,12} = \frac{V_1 - V_2}{I_1} = \frac{V_1}{I_1}$$

$$\boxed{R_{12,12} = \frac{G_{13} + G_{23}}{G_{21}G_{13} + G_{21}G_{23} + G_{31}G_{23}}}$$

The presence of the voltage probe at 3 affects the 2 terminal resistance. We have here a signature of the non-local character of QM.

Let us now consider 2 limiting cases:

a) <u>Coherent limit</u> $G_{13}, G_{31}, G_{32}, G_{23} \ll G_{12}, G_{21}$. The basic idea is that the transmission to the 3rd contact is negligible.

$$R_{12,12} = \frac{\cancel{G_{13} + G_{23}}}{G_{21}(\cancel{G_{13}+G_{23}})} \simeq \frac{1}{G_{21}} \quad \text{as in the 2-Terminal system.}$$

The ratio between G_{31} and G_{23} is fixed by the voltage drop.

$$\frac{V_3 - V_2}{V_1 - V_3} = \frac{G_{31}}{G_{13} + G_{23} - G_{31}} = \frac{G_{31}}{\cancel{G_{31}} + G_{32} - \cancel{G_{31}}} = \frac{G_{31}}{G_{32}}$$

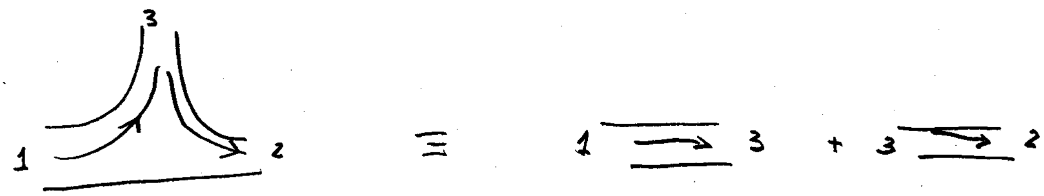
b) Incoherent limit $\quad G_{12}, G_{21} \ll G_{13}, G_{31}, G_{32}, G_{23}$ Now we can neglect the direct scattering between 1 and 2.

$$R_{12,12} = \frac{G_{13}}{G_{21}G_{13} + G_{23}(G_{21} + G_{31})} + \frac{G_{23}}{G_{21}G_{13} + G_{23}G_{21} + G_{23}G_{31}}$$

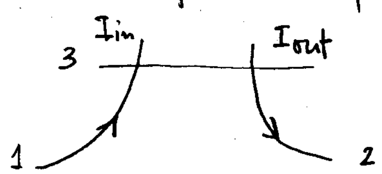
$$= \frac{G_{13}}{G_{21}G_{13} + G_{23}(G_{12} + G_{13})} + \frac{G_{23}}{G_{21}G_{13} + G_{23}G_{21} + G_{23}G_{31}}$$

$$\simeq \frac{\cancel{G_{13}}}{G_{23}\cancel{G_{13}}} + \frac{\cancel{G_{23}}}{\cancel{G_{23}}G_{31}} = \frac{1}{G_{23}} + \frac{1}{G_{31}}$$

$$= R_{32,32} + R_{13,13}$$ as two 2-terminal devices in series.

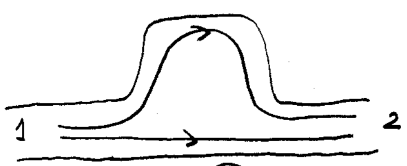


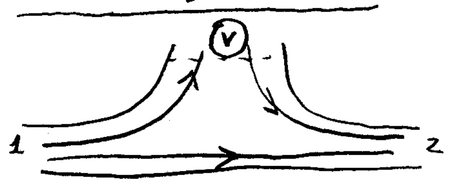
An invasive voltage probe is thus a source of decoherence: two different pictures of the phenomenon:

- 

$I_{in} = I_{out} \Rightarrow$ no net current

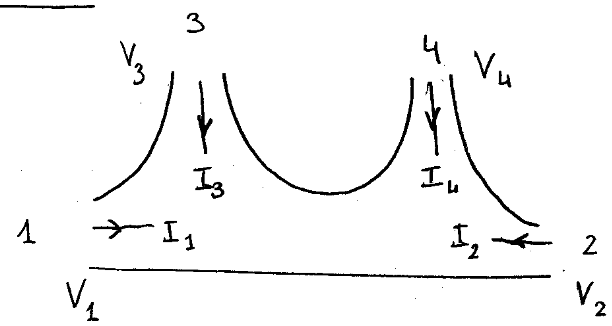
but: the phases of the incoming and outgoing electron are completely uncorrelated as there is decoherence in the contact, where we have a quasieq. electron gas.

- 



probe as which path detector. The voltmeter detects the presence of electric charges at the opening of the probe
→ collapses the wave functions

iii) four terminals



In the matrix form

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = \begin{pmatrix} G_{21}+G_{31}+G_{41}, & -G_{12}, & -G_{13}, & -G_{14} \\ -G_{21}, & G_{12}+G_{32}+G_{42}, & -G_{23}, & -G_{24} \\ -G_{31}, & -G_{32}, & G_{13}+G_{23}+G_{43}, & -G_{34} \\ -G_{41}, & -G_{42}, & -G_{43}, & G_{14}+G_{24}+G_{34} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix}$$

As before, we put $V_2 = 0$ $\quad I_2 = -I_1 - I_3 - I_4$

$$\begin{pmatrix} I_1 \\ I_3 \\ I_4 \end{pmatrix} = \bar{g} \begin{pmatrix} V_1 \\ V_3 \\ V_4 \end{pmatrix}$$ the elements of $\bar{g}$ can be read from the matrix above

$$\bar{g} = \begin{pmatrix} G_{21}+G_{31}+G_{41}, & -G_{13}, & -G_{14} \\ -G_{31}, & G_{13}+G_{23}+G_{43}, & -G_{34} \\ -G_{41}, & -G_{43}, & +G_{14}+G_{24}+G_{34} \end{pmatrix} =$$

$$= \begin{pmatrix} \bar{g}_{11}, & -G_{13}, & -G_{14} \\ -G_{31}, & \bar{g}_{33}, & -G_{34} \\ -G_{41}, & -G_{43}, & \bar{g}_{44} \end{pmatrix}$$

$$\bar{g}^{-1} = \frac{1}{\Delta} \begin{pmatrix} \bar{g}_{33}\bar{g}_{44} - G_{43}G_{34}, & G_{31}\bar{g}_{44} + G_{41}G_{34}, & +G_{31}G_{43} + G_{41}\bar{g}_{33} \\ G_{13}\bar{g}_{33} + G_{43}G_{14}, & \bar{g}_{11}\bar{g}_{44} - G_{41}G_{14}, & -\bar{g}_{11}G_{43} + G_{41}G_{13} \\ G_{13}G_{34} + \bar{g}_{33}G_{14}, & +\bar{g}_{11}G_{34} + G_{31}G_{14}, & \bar{g}_{11}\bar{g}_{33} - G_{31}G_{13} \end{pmatrix}^T$$

$$\Delta = \bar{g}_{11}\bar{g}_{33}\bar{g}_{44} - G_{31}G_{43}G_{14} - G_{13}G_{34}G_{41} - \bar{g}_{33}G_{41}G_{14} - \bar{g}_{11}G_{34}G_{43} - \bar{g}_{44}G_{31}G_{13}$$

$$V_1 = \frac{1}{\Delta}\Big[\left(\bar{\mathcal{G}}_{33}\bar{\mathcal{G}}_{44} - G_{43}G_{34}\right) I_1 + \left(G_{13}\bar{\mathcal{G}}_{44} + G_{43}G_{14}\right) I_3 + \left(G_{13}G_{34} + \bar{\mathcal{G}}_{33}G_{14}\right) I_4\Big]$$

$$V_2 = 0$$

$$V_3 = \frac{1}{\Delta}\Big[-\left(G_{31}\bar{\mathcal{G}}_{44} + G_{41}G_{34}\right) I_1 + \left(\bar{\mathcal{G}}_{11}\bar{\mathcal{G}}_{44} - G_{41}G_{14}\right) I_3 + \left(\bar{\mathcal{G}}_{11}G_{34} + G_{31}G_{14}\right) I_4\Big]$$

$$V_4 = \frac{1}{\Delta}\Big[\left(G_{31}G_{43} + G_{41}\bar{\mathcal{G}}_{33}\right) I_1 + \left(\bar{\mathcal{G}}_{11}G_{43} + G_{41}G_{13}\right) I_3 + \left(\bar{\mathcal{G}}_{11}\bar{\mathcal{G}}_{33} - G_{31}G_{13}\right) I_4\Big]$$

Let's assume $I_3 = I_4 = 0$, that is introducing voltage probes.

$$I_1 = I$$

$$V_3 - V_4 = \frac{I}{\Delta}\Big[\left(G_{31}\bar{\mathcal{G}}_{44} + G_{41}G_{34}\right) - \left(G_{31}G_{43} + G_{41}\bar{\mathcal{G}}_{33}\right)\Big]$$

$$= \frac{I}{\Delta}\Big[\left(G_{31}G_{14} + G_{31}G_{24} + G_{31}G_{34} + G_{41}G_{34}\right)$$

$$- \left(G_{31}G_{43} + G_{41}G_{13} + G_{41}G_{23} + G_{41}G_{43}\right)\Big]$$

$$= \frac{I}{\Delta}\Big[\cancel{G_{31}G_{41}} + G_{31}G_{42} + \cancel{G_{31}G_{43}} + \cancel{G_{41}G_{34}} - \cancel{G_{31}G_{43}} - \cancel{G_{41}G_{31}} - G_{41}G_{32} - \cancel{G_{41}G_{34}}\Big]$$

$$= \frac{I}{\Delta}\left(G_{31}G_{42} - G_{41}G_{32}\right)$$

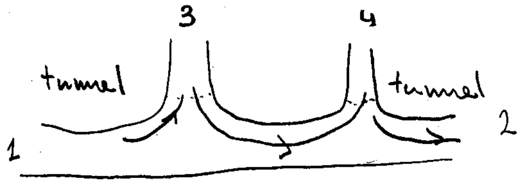
$$\boxed{\begin{aligned} R_{12,12} &= \frac{\bar{\mathcal{G}}_{33}\bar{\mathcal{G}}_{44} - G_{34}G_{43}}{\Delta} \\ R_{12,34} &= \frac{G_{31}G_{42} - G_{41}G_{32}}{\Delta} \end{aligned}}$$

the name $\bar{\mathcal{G}}_{22}$ could more physically be replaced by $\bar{\mathcal{G}}_{33}$ and the same $\bar{\mathcal{G}}_{33} \to \bar{\mathcal{G}}_{44}$

Let us discuss the particular case of weakly intercoupled probes:

$$G_{34} \sim 0$$

This is a realistic model since the transmission between the probes involves a tunnelling barrier



$$\Delta = \bar{G}_{11}\bar{G}_{33}\bar{G}_{44} - \cancel{G_{31}G_{43}G_{14}} - \cancel{G_{13}G_{34}G_{41}} - \bar{G}_{33}G_{41}G_{14} - \cancel{\bar{G}_{11}G_{34}G_{43}} - \bar{G}_{44}G_{31}G_{13}$$

$$= (G_{21}+G_{31}+G_{41})(G_{13}+G_{23})(G_{14}+G_{24}) - (G_{13}+G_{23})G_{41}G_{14} - (G_{14}+G_{24})G_{31}G_{13}$$

$$= G_{21}(G_{13}+G_{23})(G_{14}+G_{24}) + \cancel{G_{31}G_{13}(G_{14}+G_{24})} + G_{31}G_{23}(G_{14}+G_{24})$$

$$+ \cancel{G_{41}G_{14}(G_{13}+G_{23})} + G_{41}G_{24}(G_{13}+G_{23}) - \cancel{G_{41}G_{14}(G_{13}+G_{23})} -$$

$$- \cancel{G_{31}G_{13}(G_{14}+G_{24})} =$$

$$= G_{21}\bar{G}_{33}\bar{G}_{44} + G_{31}G_{23}\bar{G}_{44} + G_{41}G_{42}\bar{G}_{33}$$

$$R_{12,12} = \frac{\bar{G}_{33}\bar{G}_{44}}{G_{21}\bar{G}_{33}\bar{G}_{44}\left(1 + \frac{G_{23}G_{31}}{G_{21}\bar{G}_{33}} + \frac{G_{24}G_{41}}{G_{21}\bar{G}_{44}}\right)}$$

In the coherent limit

$$G_{21} \gg G_{23},\ G_{32},\ G_{31},\ G_{13},\ G_{24},\ G_{42},\ G_{41},\ G_{14}$$

$R_{12,12} = \dfrac{1}{G_{21}}$ as in the 2-terminal device.

$$R_{12,34} = \frac{1}{G_{21}} \cdot \frac{G_{31}G_{42} - G_{41}G_{32}}{\bar{g}_{33}\,\bar{g}_{44}} \cdot \frac{1}{1 + \frac{G_{23}G_{31}}{G_{21}\,\bar{g}_{33}} + \frac{G_{24}G_{41}}{G_{21}\,\bar{g}_{44}}}$$

in the coherent limit

$$\boxed{R_{12,34} = \frac{1}{G_{21}} \cdot \frac{G_{31}G_{42} - G_{41}G_{32}}{(G_{31}+G_{32})(G_{41}+G_{42})}}$$

Notice that this result implies that the 4-probe resistance can be negative! Only if the 2 probes are symmetric we can recover the Landauer formula in its original form:

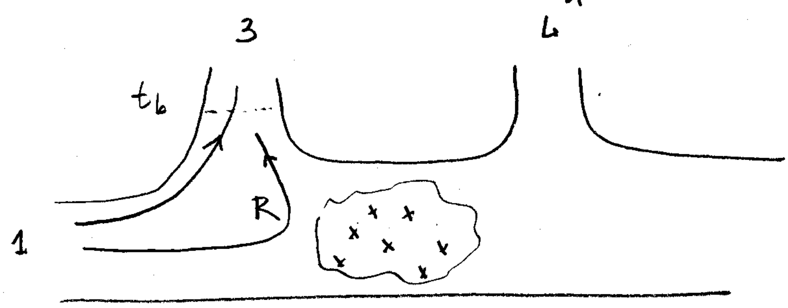
let's denote:

$$G_{12} = G_{21} = G = \frac{2e^2}{h}T \qquad G_{34} = T\,t_b^2\,\frac{2e^2}{h} \sim 0$$

$$G_{31} = G_{42} = (1+R)\,t_b\,\frac{2e^2}{h}$$

$$G_{41} = G_{32} = T\cdot t_b\,\frac{2e^2}{h}$$

since it is of II order in t_b.



$$R_{12,34} = \frac{1}{G}\;\frac{(1+R)^2\cancel{t_b^2} - T^2\cancel{t_b^2}}{\left((1+R)\cancel{t_b}+T\cancel{t_b}\right)\left((1+R)\cancel{t_b}+T\cancel{t_b}\right)} =$$

$$= \frac{1}{G}\;\frac{(1+R)^2 - (1-R)^2}{(1+R+T)^2} = \frac{1}{G}\;\frac{\cancel{2}\cdot\cancel{2}R}{\cancel{4}} = \frac{R}{G}$$

$$= \frac{h}{2e^2}\;\frac{1-T}{T}\,. \Leftarrow$$ this is the original Landauer formula

The asymmetric coupling that we already studied at the level of the effective electrochemical potential can be captured here by setting

[A]

$$G_{12} = G_{21} = G = \frac{2e^2}{h} T$$

$$G_{31} = G_{42} = t_b \frac{2e^2}{h}$$

$$G_{41} = G_{32} = 0$$

3 coupled to forward
4 coupled to backward

$$R_{12,34} = \frac{1}{G} \frac{t_b^2}{t_b \cdot t_b} = \frac{1}{G} = \frac{h}{2e^2} \frac{1}{T}$$

[B] Coupling inverted

$$G_{12} = G_{21} = \frac{2e^2}{h} T = G$$

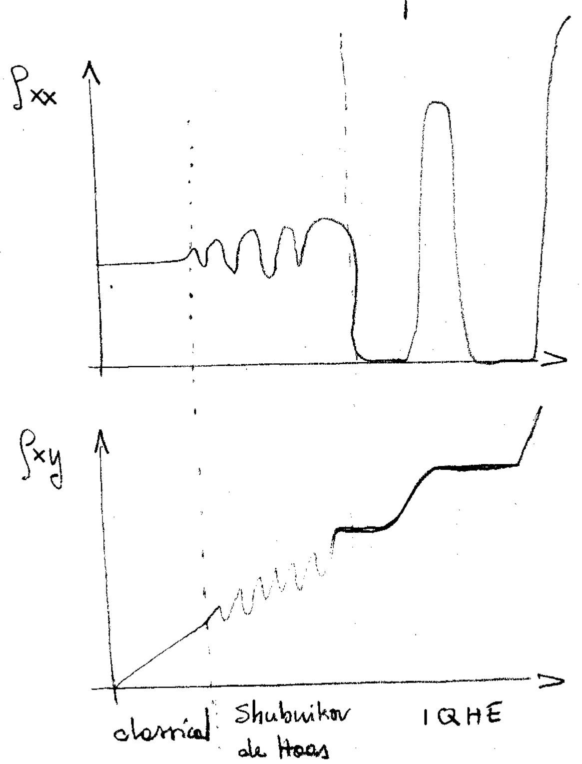
$$G_{31} = G_{42} = R\, t_b \frac{2e^2}{h}$$

$$G_{41} = G_{32} = T\, t_b \frac{2e^2}{h}$$

$$R_{12,34} = \frac{1}{G} \frac{R^2 - T^2}{(R+T)^2} = \frac{h}{2e^2} \frac{1-2T}{T} .$$

Integer Quantum Hall Effect (IQHE)

The facts and the regimes:



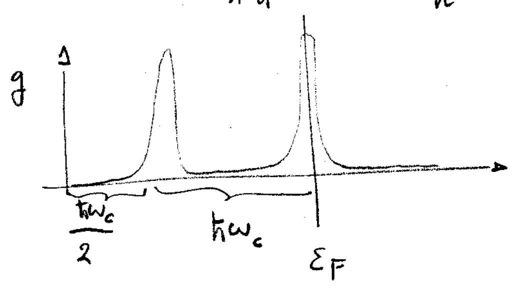
We have shown already that in the low B limit the resistivity can be understood with classical arguments

$$\rho(B) = \rho(0) \begin{pmatrix} 1 & \omega_c \tau_m \\ -\omega_c \tau_m & 1 \end{pmatrix}$$

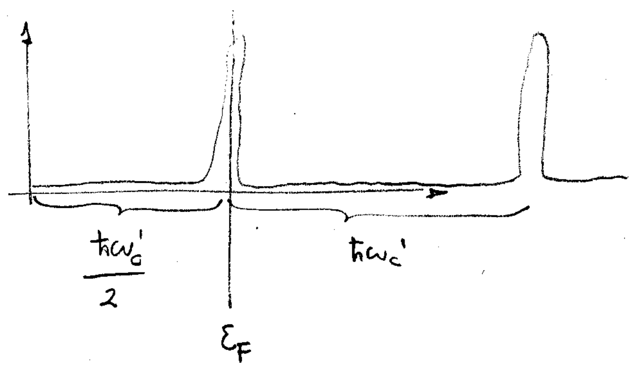
where τ_m is the momentum relaxation time. This regime holds until $\omega_c \tau_m \approx 1$

Due to quantization of the cyclotron motion Landau levels are appearing with corresponding oscillation in the density of states.

$$g(\varepsilon) = \frac{m}{\pi \hbar^2} \hbar\omega_c \sum_n \delta(\varepsilon - \varepsilon_n) \qquad \varepsilon_n = \hbar\omega_c \left(n + \tfrac{1}{2}\right)$$



As the magnetic field increases the # of occupied Landau levels changes



$\omega_c' > \omega_c$

Each time ε_F is at the center of a Landau level, ρ_{xx} goes through a maximum. To understand it we can use Einstein formula

$$\rho(0) = e^2 g(\varepsilon_F) D$$

Suppose that at a certain magnetic field B_N there are N Landau levels filled. The relation between N and B_N is given in terms of density of electrons n_e

$$n_e = \int_0^{\frac{1}{2}\hbar\omega_c + (N-1)\hbar\omega_c} d\varepsilon \; g(\varepsilon_F) = N \frac{m}{\pi\hbar^2} \cancel{\hbar}\omega_c = \qquad \omega_c = \frac{eB_N}{m}$$

$$= N \frac{\cancel{m}}{\pi\hbar} \cdot \frac{eB_N}{\cancel{m}}$$

$$\Rightarrow \quad B_N = \frac{1}{N} \frac{n_e \pi\hbar}{e}$$

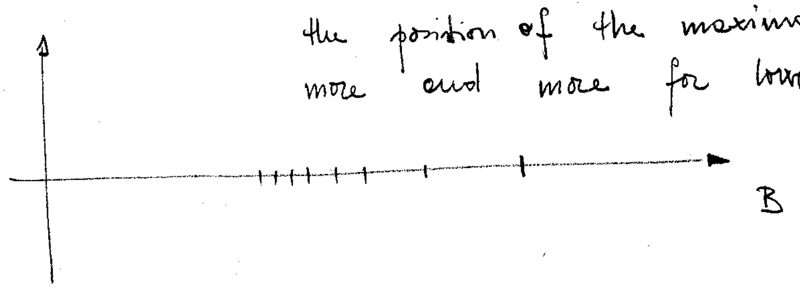
$$B_{N-1} = \frac{1}{N+1} \frac{n_e 2h}{e}$$

$$\Rightarrow \quad B_N - B_{N+1} = \left(\frac{1}{N} - \frac{1}{N+1}\right) n_e \frac{2h}{e} = \left(\frac{1}{N} - \frac{1}{N+1}\right) \Delta$$

Let's assume $n_e = 10^{11}/\text{cm}^2 \simeq 10^{15}/\text{m}^2 \qquad \Delta = 10^{15} \dfrac{\cancel{2} \cdot 6 \cdot 10^{-34}}{\cancel{2} \cdot 10^{-19}} \approx 1\,T$

Say that we get a maximum at $B_0 = 10T$ this corresponds to $N = \dfrac{B_0}{\Delta} = 10$ filled Landau levels. $\Rightarrow B_{11} = B_{10} \cdot \dfrac{10}{11}$

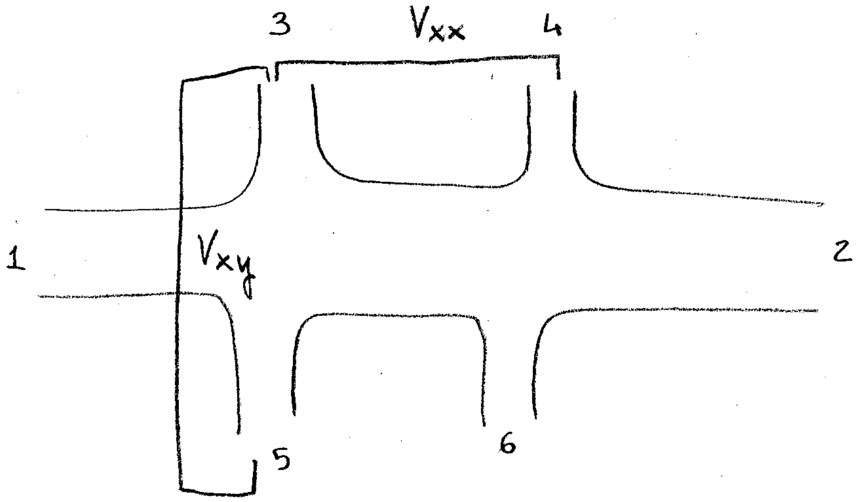
$$B_{12} = \frac{\cancel{11}}{12} \cdot \frac{10}{\cancel{11}} B_{10}$$ and so on.



the position of the maxima are accumulating more and more for lower magnetic fields.

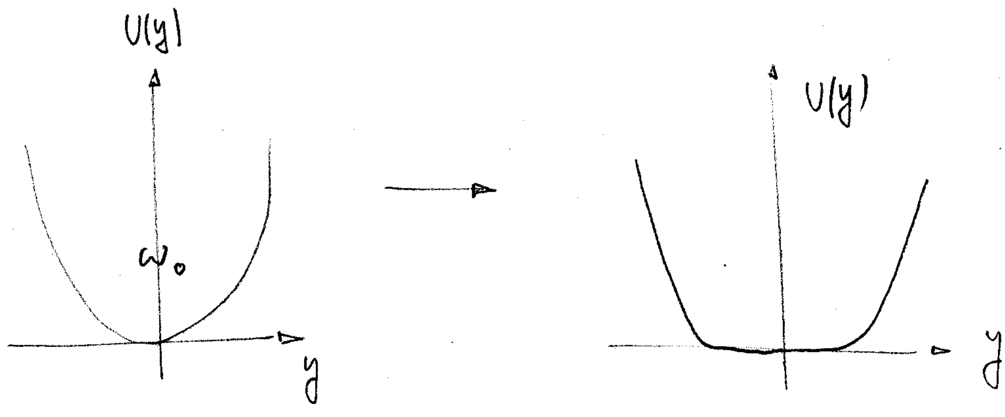
For higher magnetic fields at some point (that will become clear later) the ρ_{xx} is periodically vanishing and ρ_{xy} becomes correspondingly quantized in units of $\frac{h}{2e^2}$ with a precision which is specified in units per million.

* complete suppression of momentum relaxation
* => mean free path of millimeters
* origin is the SPATIAL SEPARATION OF LEFT AND RIGHT MOVERS.



Notice that the system is not a superconductor: we are speaking about $R_{12,34} = R_{xx}$!

We have already encountered the special separation of the left and right movers while treating the magneto-electric subbands. In a wide Hall bar, nevertheless, the confinement is not parabolic



Magneto-electric subbands at high magnetic field

In absence of confining potential: $\vec{B} = B\hat{e}_z$

$$\psi_{n,k}(x,y) = \frac{1}{\sqrt{L}} e^{ikx} u_n(q+q_k) \equiv |n,k\rangle$$

$$E(n,k) = E_s + \left(n+\frac{1}{2}\right)\hbar\omega_c$$

$$u_n(q) = \exp\left(-q^2/2\right) H_n(q)$$ $H_n(q)$ is the Hermite polinomial.

$$q = \sqrt{m\omega_c/\hbar}\, y \quad \text{and} \quad q_k = \sqrt{m\omega_c/\hbar}\, y_k$$

$$y_k \equiv \frac{\hbar k}{eB} \quad \text{and} \quad \omega_c \equiv \frac{|e|B}{m}$$

If ω_c is big, i.e. to high magnetic fields, we can treat the confining potential as a perturbation.

$$E(n,k) \approx E_s + \left(n+\tfrac{1}{2}\right)\hbar\omega_c + \langle n,k|\, U(y)\, |n,k\rangle$$

Each state $|n,k\rangle$ is centered around a different y_k and with a spacial extent $\sim \sqrt{\frac{\hbar}{m\omega_c}}$. $\Rightarrow$ if ω_c is high enough we can treat $U(y)$ as a "local" constant.

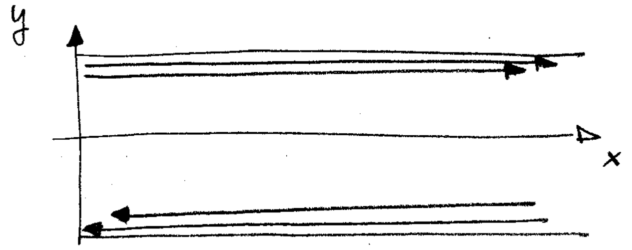
$$E(n,k) \approx E_s + \left(n+\tfrac{1}{2}\right)\hbar\omega_c + U(-y_k)$$

The effect is that:

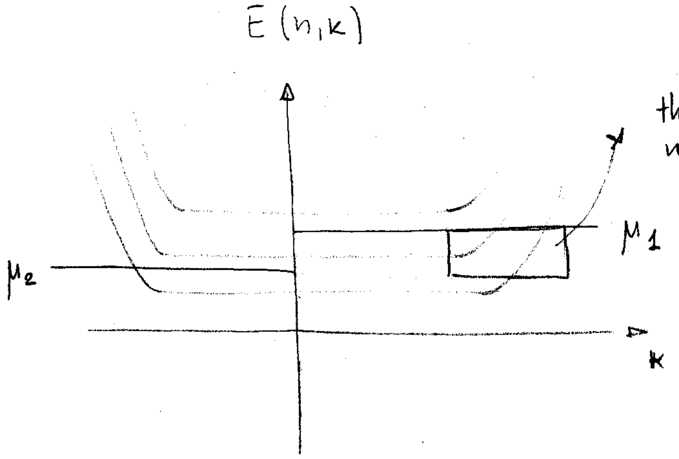
- near the center we have dispersionless Landau Levels
- close to the edges, states acquire a dispersion. These states are called edge states.

Velocity of the edge states:

$$v(n,k) = \frac{1}{\hbar}\frac{\partial E(n,k)}{\partial k} = \frac{1}{\hbar}\frac{\partial V(y_k)}{\partial k} = -\frac{1}{\hbar}\frac{\partial (y)}{\partial y}\frac{\partial y_k}{\partial k} = -\frac{1}{eB}\frac{\partial V(y)}{\partial y} \qquad e<0!$$



the states on the right (y>0) edge flow to the right (x>0), while states on the left edge flow towards the left.



these are the states responsible for the net current

What is usually giving resistance is the momentum relaxation that is here inhibited since the states are spatially separated.

As a consequence the states with $k>0$ are in equilibrium with the lead $\mu_1 = \mu_L$ and μ_L is the potential of the "right side" of the Hall bar. $\mu_2 = \mu_R$ is the potential of the "left side" of the Hall bar.

$$\Rightarrow \quad V_{xx} = 0 \qquad V_H = \mu_L - \mu_R .$$

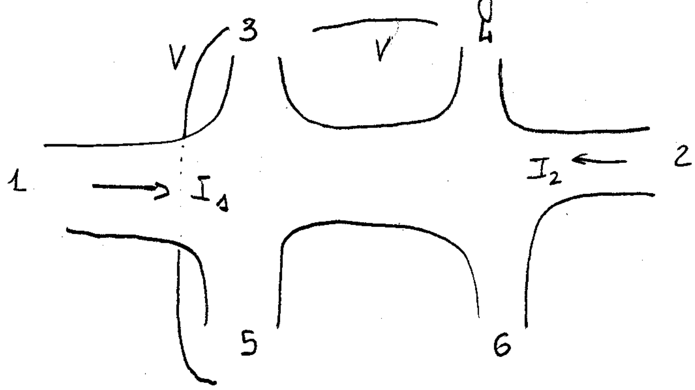
Since the transport is here completely ballistic $\Rightarrow$ the resistance is simply given by the contact resistence and it depends only on the number of modes:

$$R_c = \frac{h}{2e^2 M} \simeq \frac{13\ k\Omega}{M} = R_{xy} .$$

M is the number of bulk Landau levels below the Fermi energy.
(in fact, as usual, the Landauer formula is only valid if the number of modes does not change in the bias window $\mu_1 > E > \mu_2$).

Application of the Büttiker formula

6 - terminal measuring scheme



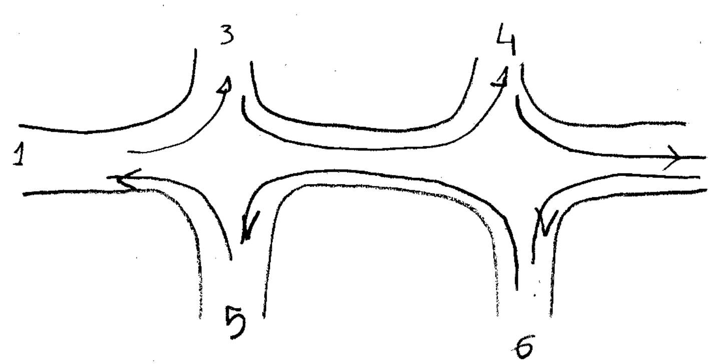
1,2 source and drain

3,4,5,6 voltage probes

$I_1 = -I_2 \qquad I_3 = I_4 = I_5 = I_6 = 0$

$$\underbrace{R_{12,12} = \frac{V_1 - V_2}{I_1}}_{\text{contact + sample + probes resistance}} , \quad \underbrace{R_{12,34} = \frac{V_3 - V_4}{I_1}}_{\text{longitudinal sample resistance, } \rho_{xx}} \quad \underbrace{R_{12,35} = \frac{V_3 - V_5}{I_1}}_{\text{transverse, Hall resistance } \rho_{xy}}$$

Since scattering is inhibited and left and right movers are spacially separated, the only conductances which are different from zero when ε_F is BETWEEN bulk Landau levels.



G_{31}, G_{43}, G_{24}

G_{62}, G_{56}, G_{15}

The sum rules imply then $G_{15} = G_{31}$, $G_{43} = G_{24}$, $G_{24} = G_{62}$

$G_{62} = G_{56}$, $G_{56} = G_{15}$ $\quad G_{31} = G_{43}$

It follows that

$$G_{15} = G_{31} = G_{43} = G_{24} = G_{62} = G_{56} = G = \frac{2e^2}{h} N$$ (N is the # of occupied bulk subbands)

The Landauer-Büttiker equations read:

$$I_1 = G_{15}(V_1 - V_5)$$
$$I_2 = G_{24}(V_2 - V_4)$$
$$\left.\begin{aligned} I_3 &= G_{31}(V_3 - V_1) = 0 \\ I_4 &= G_{43}(V_4 - V_3) = 0 \\ I_5 &= G_{56}(V_5 - V_6) = 0 \\ I_6 &= G_{62}(V_6 - V_2) = 0 \end{aligned}\right\} \quad \begin{aligned} V_1 &= V_3 = V_4 \\ V_5 &= V_6 = V_2 \end{aligned}$$

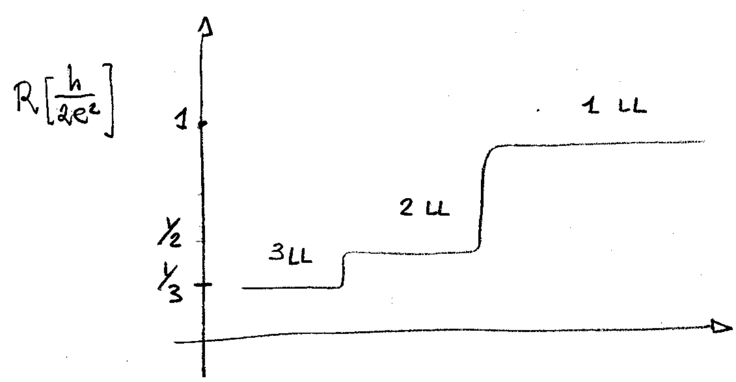
$$\Rightarrow \quad I_1 = G(V_1 - V_2)$$

$$R_{12,12} = \frac{V_1 - V_2}{I_1} = \frac{1}{G}$$

$$R_{12,34} = \frac{V_3 - V_4}{I_1} = 0$$

$$R_{12,35} = \frac{V_3 - V_5}{I_1} = \frac{V_1 - V_2}{I_1} = \frac{1}{G} = R_{12,12}$$

the total longitudinal resistance is quantized, like in a QPC. The sample resistance vanishes. The Hall resistance is the same as the contact resistance.



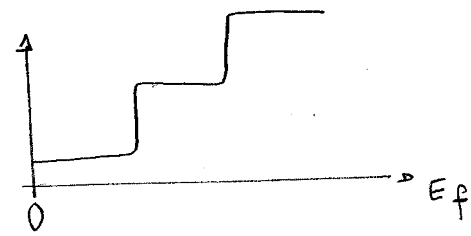
in absence of Zeeman splitting.

Why should the Fermi energy stay between Landau levels?

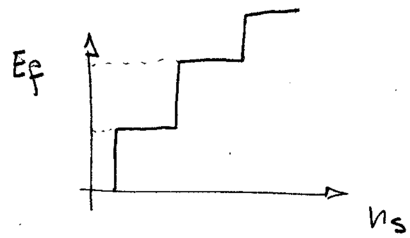
$$n_e = \int_{-\infty}^{E_F} g_s(E,B)\, dE$$

This formula gives the density of electrons as a function of the Fermi energy and the density of states.

n_e =



In case the density of states is δ-like. We can read the graph in an alternative way by exchanging n_e and E_f. This time



The Fermi energy jumps from one to the following Landau level as a function of the electron density. => the conditions for IQHE would never be satisfied. The solution comes from the existance of localized which arise due to disorder. They are giving a broadening to the density of states and thus allow a smoothern transition of the Fermi energy between different Landau levels. In some sense this is the reason for the accuracy of the quantization. Disorder is helping it!