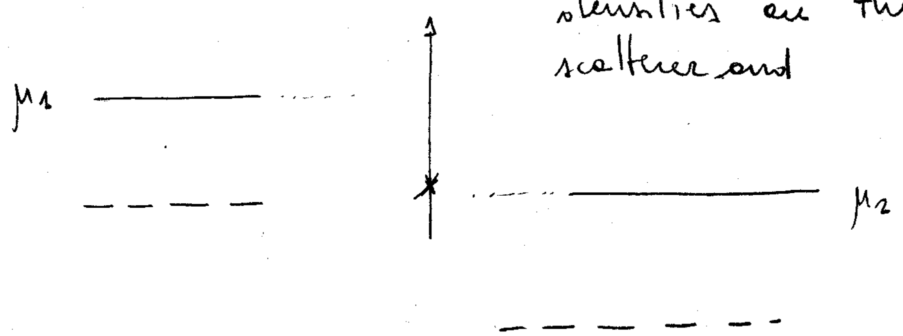
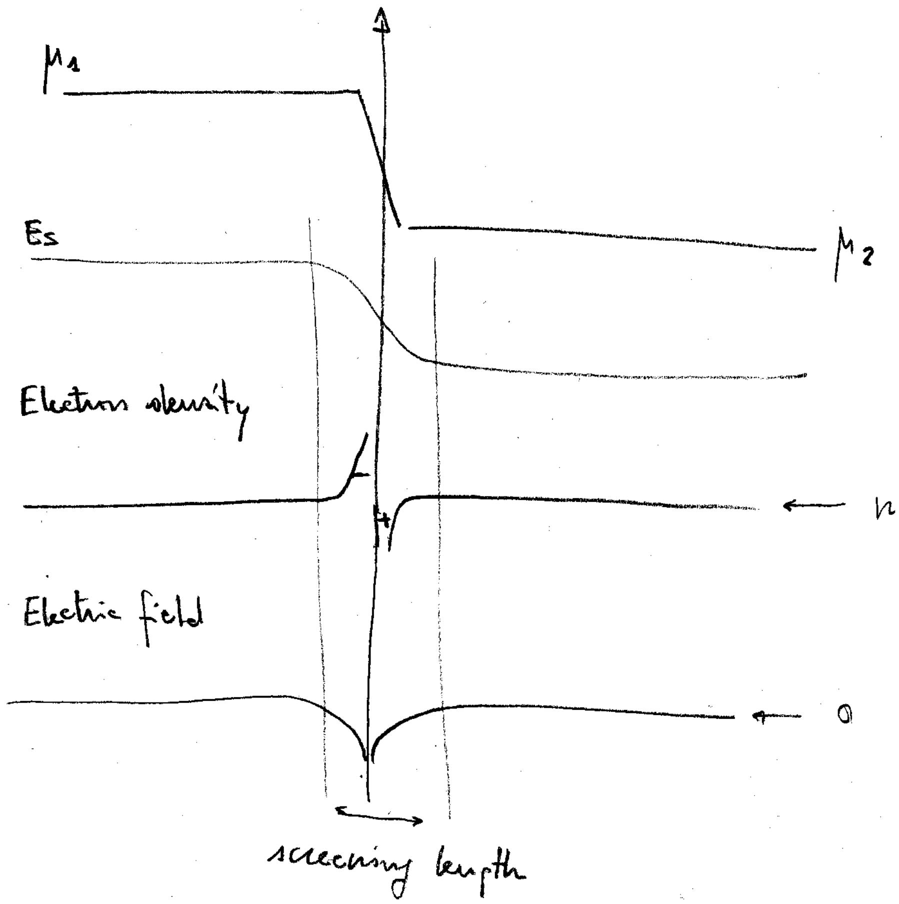
Resistivity dipoles

The fact that we are imposing different electrochemical potentials far to the left and to the right imposes a corresponding difference between the bottom of the conduction bands. In fact the electron densities are the same far from the scatterer and



$n = N_s(\mu - E_s)$.
density of states.

But what about the matching? A gradient in E_s is associated to an electric field which implies a charge (better a dipole since the electric field . A local density of electrons can arise only if μ and E_s are not parallel. The point is that $E_s(x)$ is smoother than $\mu(x)$.



The way of taking into account selfconsistently of the relations between $\mu, E_s, n, \vec{E}$ is the Poisson equation

$$\nabla^2 (\delta E_s) = -\frac{e^2 (\delta n)}{\varepsilon d} = -\frac{e^2 N_s (\delta\mu - \delta E_s)}{\varepsilon d}$$

variation with respect of the unperturbed band bottom

ε is the dielectric constant

d is the thickness of the 2DEG

$$(\nabla^2 - \beta^2)\, \delta E_s = -\beta^2 \delta\mu \qquad \boxed{\beta^{-1} = \sqrt{\frac{\varepsilon d}{e^2 N_s}}}$$ is the screening length.

First we solve $(\nabla^2 - \beta^2)\, \delta E_s = \delta(x)$ — S is the solution and it decay exponentially over x $\frac{e^{-\beta|x|}}{x}$

$$\delta E_s \sim \delta\mu \otimes S$$ where $\otimes$ means convolution.

$$\beta^{-1} = \frac{1}{2}\sqrt{\frac{\varepsilon d}{e^2} \cdot \frac{4\pi\hbar^2}{m}} = \sqrt{\frac{d\, a_B}{4}} \qquad a_B = \frac{4\pi\varepsilon\hbar^2}{m e^2}$$

$a_B \sim 10$ nm in GaAs

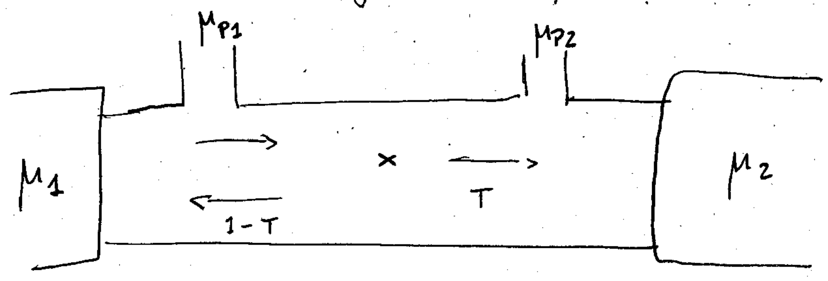
$d \sim 10$ nm

$\beta^{-1} \sim 5$ nm.

Notice that in metals β^{-1} is much shorter due to the high density of states.

VOLTAGE PROBES AND MULTITERMINAL DEVICES

The question is: can we measure the resistance generated by a mesoscopic scatterer of transmission probability T ?
In principle yes, with voltage probes: i.e. contacts through which current cannot flow (floating contacts)



$$\mu_{P1} - \mu_{P2} = (1-T)\,\Delta\mu \qquad \Delta\mu \equiv \mu_1 - \mu_2$$

$$I = \frac{2e}{h} M T \Delta\mu$$

$$R_{4t} = \frac{\mu_{P1} - \mu_{P2}}{I} = \frac{h}{2e^2 M}\,\frac{1-T}{T} \; .$$

But there are at least 3 problems:

1 * mesoscopic probes are invasive (but it can be minimized)

2 * mesoscopic probes are seldom identical. They can for example couple differently to the left or right movers

$$\begin{rcases} \mu_{P1} \simeq \mu^- = (1-T)\,\Delta\mu \\ \mu_{P2} \simeq \mu^+ = T\,\Delta\mu \end{rcases} \Rightarrow R_{4t} = \frac{h}{2e^2 M}\,\frac{1-2T}{T}$$

or

$$\begin{rcases} \mu_{P1} \simeq \mu^+ = \Delta\mu \\ \mu_{P2} \simeq \mu^- = 0 \end{rcases} \Rightarrow R_{4t} = \frac{h}{2e^2 M}\,\frac{1}{T}$$

Also this second problem is not so relevant, at least for $T \ll 1$ since $1 \sim 1-2T$. The point is nevertheless that, in that case the four and two terminal resistance would be difficult to distinguish.

3.4 <u>INTERFERENCE EFFECTS</u> : if $T \ll 1 \Rightarrow$ the electrochemical potentials for $+k$ and $-k$ states are both nearly equal at the left and right of the scatterer. $\Rightarrow$ $+k$ and $-k$ can be mixed.

The probe could see a "node" of the interference between $+k$ and $-k$ states. We could $\Rightarrow$ measure for example a e-chemical potential also to the LEFT of the scatterer, even if both states have the same occupation.

To fix the ideas: let us take a simple k $+k, -k$

No interference

$$\rho = \frac{1}{2}\left[|{+k}\rangle\langle{+k}| + |{-k}\rangle\langle{-k}|\right]$$

$$\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

INTERFERENCE

$$\rho = \frac{1}{2}\left[\left(|{+k}\rangle + |{-k}\rangle\right)\left(\langle{+k}| + \langle{-k}|\right)\right]$$

$$\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

The solution to the problem comes from the Büttiker formalism.

BÜTTIKER FORMULA (1985)

In order to interpret results of four terminal devices with voltage probes in the mesoscopic regime, Büttiker devised a very elegant and simple method. The basic idea is to TREAT ALL TERMINALS ON EQUAL FOOTING.

<u>2 terminal</u> $\quad I = \frac{2e}{h} \bar{T} [\mu_1 - \mu_2]$

$\downarrow$

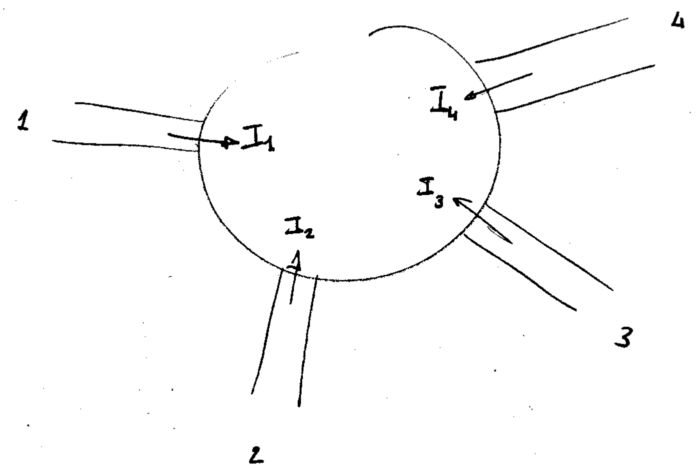
general $\quad I_p = \frac{2e}{h} \sum_q [\bar{T}_{q\leftarrow p} \mu_p - \bar{T}_{p\leftarrow q} \mu_q]$

$\bar{T}$ is the transmission function. It is in general a function of the energy $T\,M(E)$ at least. If $\Delta\mu < \Delta E_c$ $M(E)$ is constant. We call ~ this regime linear response.

in other terms:

$$I_p = \sum_q [G_{qp} V_p - G_{pq} V_q] \qquad G_{pq} = \frac{2e^2}{h} \bar{T}_{p\leftarrow q}$$

N.B. The currents are all flowing <u>from</u> the contact



If $\quad V_p = V_q \quad \forall\, p\, q \quad \Rightarrow \quad I_p = 0 \quad \forall p$ this observation implies immediately

$$0 = \sum_q (G_{qp} - G_{pq}) \Rightarrow \sum_q G_{qp} = \sum_q G_{pq}$$

$$I_p = \sum_q \left(G_{pq} V_p - G_{pq} V_q \right) = \sum_q G_{pq} \left(V_p - V_q \right)$$

Another important relation (that we will prove later) involves the magnetic field

$$\left[G_{qp} \right]_{+B} = \left[G_{pq} \right]_{-B}$$

intuitively we can justify it by saying that all paths go into their (time) reverse by reverting the sign of the magnetic field.

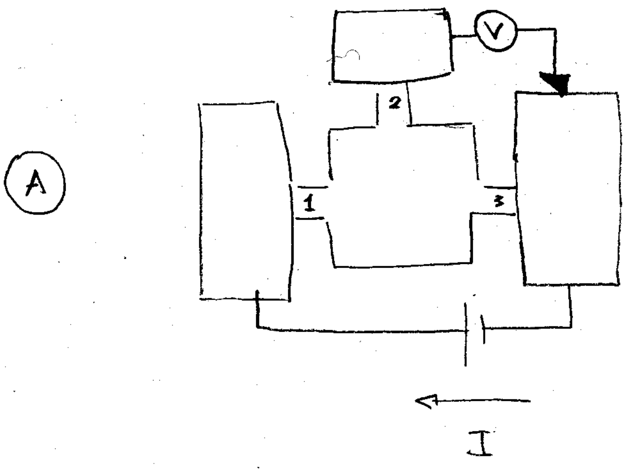
The potential of a voltage probe

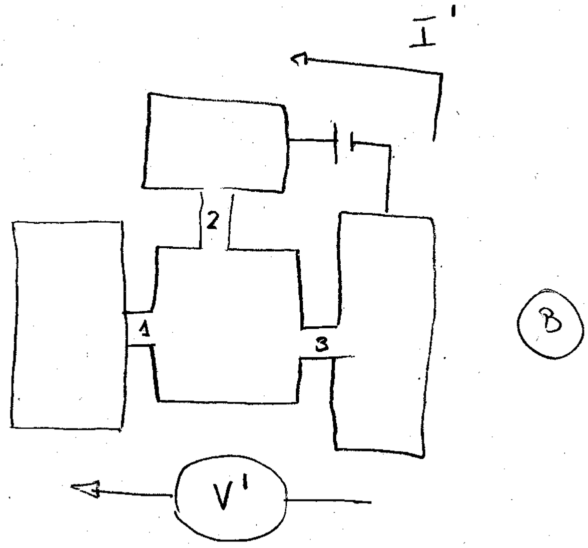
$$0 = I_p = \sum_q G_{pq} \left(V_p - V_q \right) = \sum_{q \neq p} G_{pq} \left(V_p - V_q \right)$$

$$\Rightarrow \quad V_p = \frac{\sum_{q \neq p} G_{pq} V_q}{\sum_{q \neq p} G_{pq}}$$

If $B = 0$ $\quad G_{qp} = G_{pq}$ and the Büttiker formula reduces to the Kirchhoff law for a set of conductors all connected between each others.

Three-terminal device





In a generic 3 terminal device we have 3 voltages and 3 currents. Using the Büttiker formula we connect them:

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} G_{12}+G_{13} & -G_{12} & -G_{13} \\ -G_{21} & G_{21}+G_{23} & -G_{23} \\ -G_{31} & -G_{32} & G_{31}+G_{32} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$I_1+I_2+I_3=0$ → I need only to know I_1 and I_2 (for example...)
I can also eliminate 1 bias by measuring the energies from there.

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} G_{12}+G_{13} & -G_{12} \\ -G_{21} & G_{21}+G_{23} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

By inverting we obtain:

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

Where $$[R] = \begin{pmatrix} G_{12}+G_{13} & -G_{12} \\ -G_{21} & G_{21}+G_{23} \end{pmatrix}^{-1} = \frac{1}{(G_{12}+G_{13})(G_{21}+G_{23}) - G_{21}G_{12}} \begin{pmatrix} G_{21}+G_{23} & G_{12} \\ G_{21} & G_{12}+G_{23} \end{pmatrix}$$

$$= \frac{1}{G_{12}G_{23}+G_{13}G_{21}+G_{13}G_{23}} \begin{pmatrix} G_{21}+G_{23} & G_{12} \\ G_{21} & G_{12}+G_{23} \end{pmatrix}$$

(A) $I_2 = 0 \Rightarrow -I_1 = +I_3 = I \qquad R_A = \dfrac{V_2 - V_3}{I}$

$V_3 = 0 \Rightarrow R_A = \dfrac{V_2}{I_1}\Big|_{I_2=0} = R_{21} = \dfrac{G_{21}}{G_{12}G_{23} + G_{13}G_{21} + G_{13}G_{23}}$

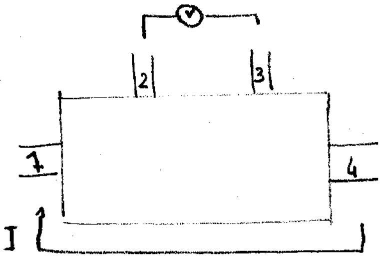
?

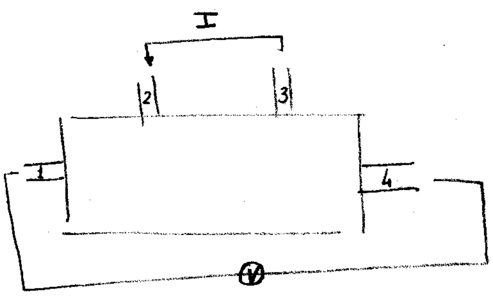
(B) $I_1 = 0$

$V_3 = 0 \qquad R_B = \dfrac{V_1}{I_3}\Big|_{I_1=0} = R_B = \dfrac{V_1}{+I_2}\Big|_{I_1=0} = +R_{12}$

Four terminal device

(A)





(B)

As a first step we set one of the voltages to zero (say V_4)

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} G_{12}+G_{13}+G_{14} & -G_{12} & -G_{13} \\ -G_{21} & G_{21}+G_{23}+G_{24} & -G_{23} \\ -G_{31} & -G_{32} & G_{31}+G_{32}+G_{34} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

by inversion

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} \qquad R = [G]^{-1}$$

$$R_A^{4t} = \frac{V}{I} = \frac{V_2 - V_3}{I_1}\Big|_{I_2=I_3=0} = R_{21} - R_{31} =$$

$$R_B^{4t} = \frac{V}{I} = \frac{V_1 - (0)}{I_2}\Big|_{\substack{I_2=-I_3 \\ I_1=0}} = R_{12} - R_{13}$$

We are thus left with the calculation of G_{ij}, in other terms of T_{ij}.

Reciprocity

We have just proven that the interchange of the voltage and the current probes leads to different definitions of the resistance

$$R_A^{3t} = R_{21} \qquad\qquad R_A^{4t} = R_{21} - R_{31}$$

?

$$R_B^{3t} = R_{12} \qquad\qquad R_B^{4t} = R_{12} - R_{13}$$

But, are these resistances connected to each other? The answer is yes and we take as given the relation

$$G_{pq}(B) = G_{qp}(-B)$$ [we will prove it later]

If $B = 0 \Rightarrow G_{pq} = G_{qp}$. From the expression of $[R]^{-1}$ it follows immediately that $[R^{-1}]_{pq} = [R^{-1}]_{qp}$ if $q \neq p$. $\Rightarrow \{[R]^{-1}\}^T = [R]^{-1}$.

But $[R^{-1}]^T = [R^T]^{-1}$. $\quad [R^{-1}]^T R^T = [RR^{-1}]^T = \mathbb{1}^T = \mathbb{1} \quad \Rightarrow R = R^T$

$$R^T [R^{-1}]^T = [R^{-1}R]^T = \mathbb{1}^T = \mathbb{1}.$$

It follows that, for $B = 0$ the resistances of the cases A and B are the same.

$B \neq 0 \quad \Rightarrow \quad [R^{-1}]_{pq}(B) = [R^{-1}]_{qp}(-B)$ for $q \neq p$.

$$q = p: \quad [R^{-1}]_{qq}^{(B)} = \sum_{p\neq q} G_{qp}(B) = \sum_{p\neq q} G_{pq}(-B) = \underbrace{\sum_{p\neq q} G_{qp}(-B)}_{\text{see pag. 108}}$$

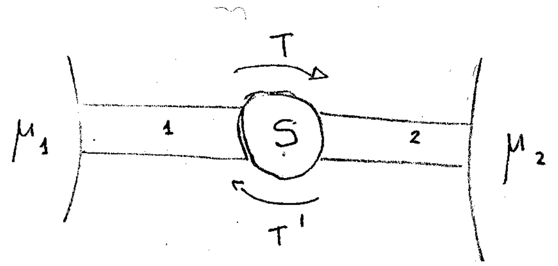
$$= [R^{-1}]_{qq}(-B).$$

$$\Rightarrow \quad [R^{-1}](B) = [R^{-1}]^T(-B) \quad \Rightarrow \quad \boxed{R(B) = R^T(-B)}$$

RECIPROCITY RELATIONS

Linear response ?

The Landauer formula as we have derived it gives the conductance of a device. It is thus explicitly a formula valid for "small biases". But what do we mean by small ? Let us reconsider the derivation, now using the current distributions :



Landauer $\quad I = \frac{2e}{h} \mathcal{T} \Delta\mu \quad (*) \qquad \mathcal{T} = TM.$

$$\left.\begin{aligned} i_1^+(E) &= \frac{2e}{h} M(E) f_1(E) \\ i_2^-(E) &= -\frac{2e}{h} M'(E) f_2(E) \end{aligned}\right\} \text{ reflectionless contacts}$$

$$\left.\begin{aligned} i_1^-(E) &= -(1-T)\, i_1^+ + T' i_2^- \\ i_2^+(E) &= T i_1^+ - (1-T')\, i_2^- \end{aligned}\right\} \text{ where also } T, T' \text{ are energy dependent}$$

$$I = \int dE \;\; i_1^+(E) + i_1^-(E) = \frac{2e}{h} \int dE \;\; T(E) M(E) f_1(E) - T'(E) M'(E) f_2(E)$$

In order to obtain (*) we need at first sight:

$$\left.\begin{aligned} &* \;\; T(E) = T'(E) \\ &* \;\; M(E) = M'(E) \end{aligned}\right\} \text{ or at least } \mathcal{T}(E) = \mathcal{T}'(E) \qquad \mathcal{T} = TM.$$

$*$ Temp $\to 0$

In reality (*) is more general

$$\underline{\underline{1}} \quad \int_{-\infty}^{+\infty} \left[f_1(E) - f_2(E)\right] dE = \lim_{L \to -\infty} \int_L^{\infty} dE \left[f(E-\mu_1) - f(E-\mu_2)\right] = \lim_{L \to -\infty} \int_{L-\mu_1}^{\infty} - \int_{L-\mu_2}^{\infty} f(E)\, dE$$

$$= \lim_{L \to -\infty} \int_{L-\mu_1}^{L-\mu_2} f(E)\, dE = \mu_1 - \mu_2 \, .$$

It follows that if $\bar{T}$ does not depend on E => (*) is valid $\forall$ bias and temperature
N.B. at some point μ_i is also affecting $\bar{T}$, but this is another reason.

3 Still in the limit $\bar{T}(E) = \bar{T}'(E)$.

$$I = \frac{2e}{h}\int dE\ \bar{T}(E)\left[f_1(E) - f_2(E)\right] = \frac{2e}{h}\int dE\ \bar{T}(E)\ f(E-\mu_1) - f(E-\mu_2)$$

$$= \frac{2e}{h}\int dE\ \bar{T}(E)\int_{\mu_2}^{\mu_1} dE'\ f'(E-E') = \frac{2e}{h}\int_{\mu_2}^{\mu_1} dE' \int_{-\infty}^{+\infty} dE\ \bar{T}(E)\left[-f'(E-E')\right]$$

$$= \frac{1}{e}\int_{\mu_2}^{\mu_1} dE'\ \hat{G}(E') \qquad \text{where} \qquad \hat{G}(E') = \frac{2e^2}{h}\int_{-\infty}^{+\infty} dE'\ \bar{T}(E)\left[-f'(E-E')\right]$$

Thus (*) is valid under the looser condition that $\hat{G}(E')$ is constant in the intervall between μ_2 and μ_1. The energy scale of the variations of $\bar{T}(E)$ is due to coherent scattering processes and is called correlation energy ε_c. f' is a peak of width given by the temperature:

* $k_BT < \varepsilon_c$ => (*) is valid under the condition $\Delta\mu \ll \varepsilon_c$
* $\varepsilon_c < k_BT$ => " $\Delta\mu \ll k_BT$

summarizing: $\Delta\mu \ll \varepsilon_c + k_BT$

In the case $k_BT < \varepsilon_c$ it is justified $T \to 0$. But it these conditions $-f'(E-E') = \delta(E-E')$ => $\hat{G}(E') = \frac{2e^2}{h}\bar{T}(E')$. E' is in the window between μ_2 and μ_1. E_f is also there and $\bar{T}(E') \simeq \bar{T}(E_f)$. In the other case we can linearize in $\frac{\mu_i - \varepsilon_f}{k_BT}$ and obtain

$$I = \frac{2e}{h}\int dE\ \bar{T}(E)\left[-f_0'(E)\right](\mu_1-\mu_2) = \hat{G}(\varepsilon_F)\frac{(\mu_1-\mu_2)}{e}.$$

Multiterminal devices

The extension of the Landauer to multiterminal is straightforward also for current densities:

$$I_p = \int i_p(E)\, dE$$

$$i_p(E) = \frac{2e}{h} \sum_q \left[T_{qp}(E)\, f_p(E) - T_{pq}(E)\, f_q(E) \right]$$

Under the no current condition at equilibrium

$$\sum_q T_{qp}(E) = \sum_q T_{pq}(E) \quad \forall E$$

$$\Rightarrow \quad i_p(E) = \frac{2e}{h} \sum_q T_{pq}(E) \left[f_p(E) - f_q(E) \right]$$

Under the same assumptions presented for the 2 terminal device

$$I_p = \sum_q G_{pq} (V_p - V_q)$$

$$G_{pq} = \frac{2e^2}{h} \int dE\, T_{pq}(E) \left(- \frac{\partial f_0}{\partial E} \right) \simeq \frac{2e^2}{h} T_{pq}(\varepsilon_F)$$

Naively: why not $\quad i_p(E) = \frac{2e}{h} \sum_q \left[T_{qp}(E)\, f_p (1 - f_q) - T_{pq}\, f_q (1 - f_p) \right]$

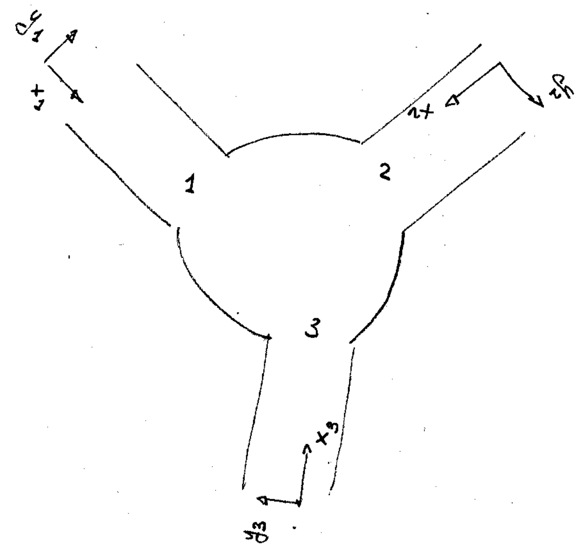
the answer is: in the case of reflectionless contact it is meaningless since the contacts are absorbing ALL. If the contact do NOT have this property $\Rightarrow$ the previous correction is definitively not enough. One should start to consider the weak coupling. Or the non equilibrium Green's function approach, in principle able to handle the transition.

Scattering states

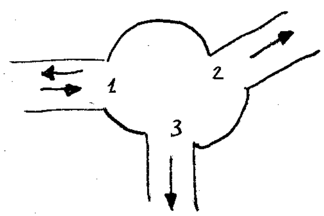
The idea that guides us to understand this kind of transport (the COHERENT TRANSPORT) is the one of SCATTERING STATE:

The wave function in lead p due to a scattering state (q, k) is given by:

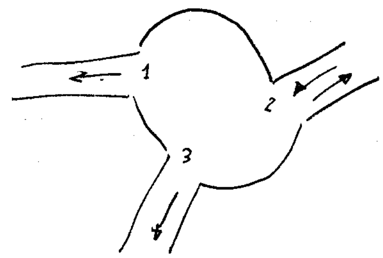
$$\Psi_p(q) = \delta_{pq}\, \chi_p^+(y_p)\, e^{ik^+x_p} + s'_{pq}\, \chi_p^-(y_p)\, e^{ik^-x_p}$$



$|1, k\rangle$



$|2, k\rangle$



The current associated to the scattering state (q, k) per unit energy in lead p

$$i_p(q) = \frac{2e}{h}\left(\delta_{pq} - T_{pq}\right)$$

(Assuming 1 mode per lead)

In conductors with reflectionless contacts the states $|q,k|$ are in equilibrium with contact q =>

$$I_p = \int \sum_q f_q(E)\, i_p(E)\, dE$$

which is the current generated in lead p due to all scattering states coming with whatever energy E from whatever contact q. Notice that we are treating a non-equilibrium problem as a special combination of equilibrium ones!

$$I_p = \frac{2e}{h} \int \left[f_p - \sum_q T_{pq} f_q \right] dE$$

The transmission coefficients obey the sum rule

$$\sum_q T_{pq} = \sum_q T_{qp} = 1$$

this part we know — this we will prove later.

In case of multiple modes in the leads:

$$\overline{T}_{pq} = \sum_{m \in p} \sum_{n \in q} T_{mn}$$

The presence of a magnetic field does not disturb this picture in terms of scattering states. In fact

[A] It is always possible to write the vector potential in such a way that $A \sim \hat{x}_q B y_q$ $\forall$ lead q.

Appendix E of H.U. Baranger and A. D. Stone (1989) Phys. Rev. B 40 8169.

$\boxed{B}$ The transverse modes are independent

$$\vec{J} = \frac{e}{2m}\left(\psi[(\vec{p}-e\vec{A})\psi^*] + \psi^*[(\vec{p}-e\vec{A})\psi]\right)$$

$$\vec{p} = -i\hbar\vec{\nabla}$$

$$I = \frac{e}{2m}\int[\psi(p_x - eA_x)\psi^* + \psi^*(p_x - eA_x)\psi]dy$$

$$\psi = \psi_i + \psi_s \qquad \text{where}$$

$$\psi_i = \frac{1}{\sqrt{L}}\chi^+(y)\exp[ik^+x]$$

$$\psi_s = s'\frac{1}{\sqrt{L}}\chi^-(y)\exp[ik^-x]$$

$$I_i = \frac{e}{mL}\int[\chi^+(\hbar k^+ - eA_x)\chi^+]\,dy \qquad \chi \text{ real}$$

$$I_s = \frac{e}{me}|s'|^2\int[\chi^-(\hbar k^- - eA_x)\chi^-]\,dy$$

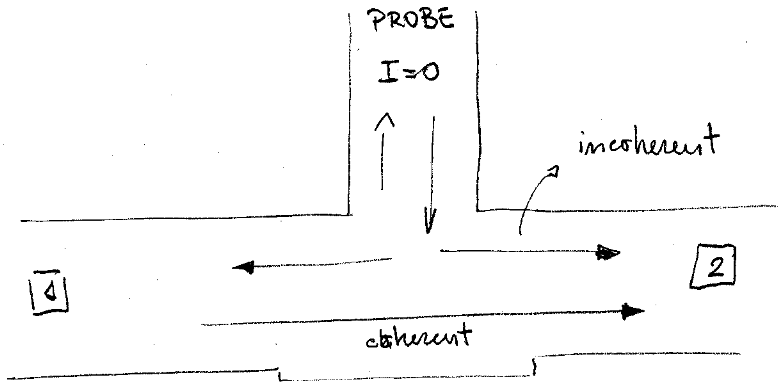
And what about the cross terms

$$\frac{e}{mL}\int[\chi^+\left(\frac{\hbar(k^+ + k^-)}{2} - eA_x\right)\chi^-]\,dy \qquad ?$$

They vanish due to the orthogonality relation

$$\int\left[\chi_{m,k}\left(\frac{\hbar(k+k')}{2} - eA_x\right)\chi_{n,k'}\right]dy = \delta_{kk'} \qquad \text{(exercise)}$$

A voltage probe is a source of decoherence



we call φ the voltage probe

$$i_p(E) = \frac{2e}{h}\sum_{q\neq\varphi} \bar{T}_{pq}(E)\left[f_p(E) - f_q(E)\right] + \frac{2e}{h}\bar{T}_{p\varphi}\left[f_p(E) - f_\varphi(E)\right]$$

for what concerns the voltage probe we do not know the distribution function. We just know that $I_\varphi = 0$

$$i_\varphi(E) = \frac{2e}{h}\sum_q \bar{T}_{\varphi q}(E)\left[f_\varphi(E) - f_q(E)\right] \qquad \bar{R}_\varphi \equiv \sum_q \bar{T}_{\varphi q}$$

$$f_\varphi(E) = \frac{h}{2e}\frac{1}{\bar{R}_\varphi}\left[i_\varphi + \frac{2e}{h}\sum_q \bar{T}_{\varphi q} f_q\right] = \frac{1}{\bar{R}_\varphi}\left[\frac{h}{2e} i_\varphi + \sum_q \bar{T}_{\varphi q} f_q\right]$$

$$\Rightarrow \quad i_p(E) = \frac{2e}{h}\sum_{q\neq\varphi}\bar{T}_{pq}(E)\left[f_p(E) - f_q(E)\right] + \frac{2e}{h}\bar{T}_{p\varphi}\left[f_p(E) - \frac{1}{\bar{R}_\varphi}\left(\frac{h}{2e} i_\varphi + \sum_{q\neq\varphi}\bar{T}_{\varphi q} f_q\right)\right]$$

$$= \frac{2e}{h}\sum_{q\neq\varphi}\bar{T}_{pq}(E)\left[f_p(E) - f_q(E)\right] + \frac{2e}{h}\bar{T}_{p\varphi} f_p(E) - \frac{2e}{h}\sum_{q\neq\varphi}\frac{\bar{T}_{p\varphi}\bar{T}_{\varphi q}}{\bar{R}_\varphi} f_q$$

$$- \bar{T}_{p\varphi}/\bar{R}_\varphi \; i_\varphi$$

$$\bar{T}_{p\varphi} = \bar{T}_{p\varphi}\frac{\bar{R}_\varphi}{\bar{R}_\varphi} = \bar{T}_{p\varphi}\frac{1}{\bar{R}_\varphi}\sum_q \bar{T}_{\varphi q} = \sum_q \frac{\bar{T}_{p\varphi}\bar{T}_{\varphi q}}{\bar{R}_\varphi}$$

$$i_p(E) = \frac{2e}{h} \sum_{q \neq \varphi} \left(\bar{T}_{pq}(E) + \frac{\bar{T}_{p\varphi} \bar{T}_{\varphi q}}{\bar{R}_\varphi} \right) \left[f_p(E) - f_q(E) \right] - \frac{T_{p\varphi}}{\bar{R}_\varphi} i_\varphi$$

The condition that we have is that $\int dE \, i_\varphi(E) = 0$. This condition tells us that electrons are entering the voltage probe with a certain energy and leaving it at a different one. => implying a so called VERTICAL FLOW in the probe. Vertical in the sense of flow between different energies. In reality every probe is having this effect. The difference is just that a voltage probe is without current and in this sense can be considered as "mute" source of dephasing.

In principle one should provide a model for $i_\varphi(E)$. In most cases a good approximation (even if crude) is $i_\varphi(E) = 0$.

$$i_p(E) = \frac{2e}{h} \sum_{q \neq \varphi} \left(\underset{\text{coherent}}{\underset{\uparrow}{\bar{T}_{pq}(E)}} + \underset{\text{incoherent}}{\underset{\uparrow}{\frac{\bar{T}_{p\varphi} \bar{T}_{\varphi q}}{\bar{R}_\varphi}}} \right) \left[f_p(E) - f_q(E) \right]$$

This approach can be called <u>non-coherent elastic transport</u> since there is a source of incoherence $\left(\frac{\bar{T}_{p\varphi} \bar{T}_{\varphi q}}{\bar{R}_\varphi} \right)$ but the net energy flow due to this source of decoherence. In this approximation the current is NOT modified is

* $\bar{T}(E)$ is constant in the $\mu_1 + \text{few} \cdot k_B T > E > \mu_2 - \text{few} \cdot k_B T$ since in the $\int dE \, i_\varphi(E)$ the contribution of $i_\varphi(E)$ vanish identically.
* in other cases LB must be used consciously...