WS 09/10

Condensed Matter Theory II:

Mesoscopic Physics

Organization matters

- Classes Mondays } 10 – 12 9.02.01
  Thursdays

- Exercises: Wednesdays 15-17 5.01.01 { Miriam del Valle
  Jürgen Wurm

- Exercises will appear on-line every Monday on my personal homepage.

  www.physik.uni-r.de/forschung/grifoni

  → people
  → Andrea Donarini
  → homepage
  → teaching

The written solution should be turned in by the following Monday at 10.00. And the solution will be discussed on Wednesday of the same week.

- Criteria for the Schein: • regular partecipation to lectures and exercises
  • 50% of written exercises.
  • 50% of "oral" exercises.

- Prerequisite:
  - Quantum Mechanics I
  - Quantum Mechanics II
  - Condensed Matter Theory I (we will borrow some of the techniques)

- Literature:
  - S. Datta: Electronic Transport in mesoscopic systems (Cambridge Univ. Press)
  - D. Ferry S. Goodnick: Transport in Nanostructures (Cambridge Univ. Press)
  - H. Bruus K. Flensberg: Many-body Quantum Theory in Condensed Matter Physics (Oxford Graduate Texts)
  - H. Haug A.-P. Jauho: Quantum Kinetics in Transport and Optics of Semiconductors (Springer)
  - K. Blum: Density matrix theory and applications (Plenum Press)

Overview

1. Introduction to Mesoscopic Physics: main concepts and phenomena + relevant length scales

2. Boltzmann equation for transport

3. Quantum systems in reduced dimensions
   - density of states 0-3D
   - Landau-Levels
   - Block-diagonalization Group Theory for small systems

4. Transmission through Nanosystem Landauer Formalism
   - Landauer formula
   - Quantum Point Contact
   - Landauer - Büttiker formula
   - Onsager relations

5. Transmission function, S-matrix and Green's functions

6. Quantum Hall Effect

7. Transport and interference in disordered conductors
   - Linear response
   - Diagrammatic,
   - Impurity scattering Drude revisited
   - Weak localization

8. Single electron effects in weak coupling to the leads
   - Coulomb blockade - Quantum dot physics - FC blockade
   - Master Equation → Generalized Master Equation
   - Interference Blockade, Spin Accumulation.

9. Non - equilibrium Green's function formalism. Current formula (Wingreen - Meir - Lee)

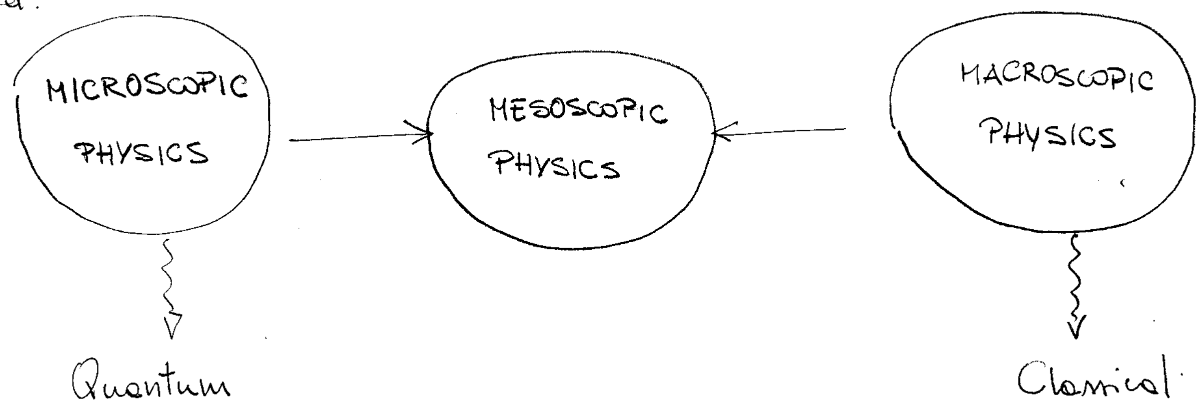
Mesoscopic Physics (1976, van Kampen)

μέσος : in the middle

σκοπέω : to look at, observe

φύσις : nature

It represents the ensemble of natural phenomena that can be observed in the MIDDLE between macroscopic and microscopic world.



In general we can say that objects wich are of atomic or molecular size ($1 Å \to 100\,nm$) are described by quantum mechanics, while visible objects (by naked eye) are described by classical physics.

Nevertheless:

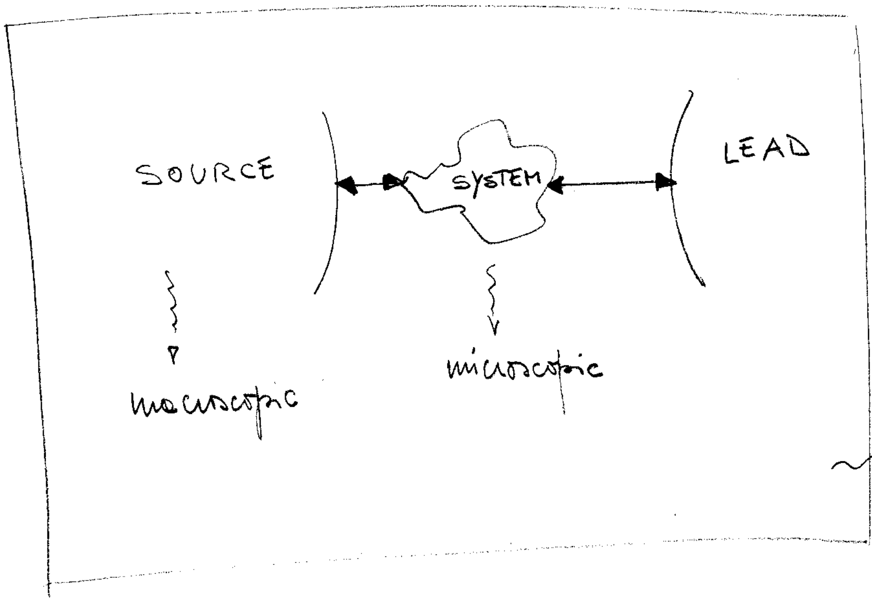
1- the measurement of nanoscale objects always pass through macroscopic apparatus

2- we would like to have a length, time, energy scale to compare with

3- What do we consider fundamentally quantum?

- Energy discretization ⟵ Low dimensionality
- Interference / Coherence ⟵ particle-wave duality, superconductivity
- Tunnelling

We will analyze in this course mesoscopic systems in the sense of SMALL OPEN SYSTEM and try to find the quantum vs. classical behavior in their transport characteristics.



$\rightsquigarrow$ mesoscopic

* Examples of mesoscopic effects:

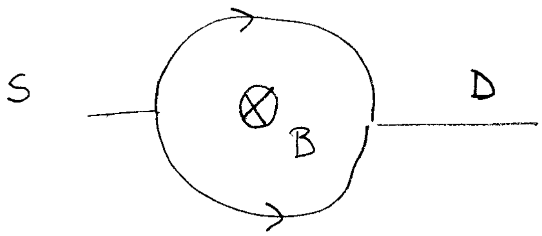
- Aharonov-Bohm effect
- Quantum Hall effect
- Universal conductance fluctuation
- Weak localization
- Conductance quantization
- Coulomb blockade
- Franck Condon Blockade
- Interference blockade
- Spin blockade
- Kondo effect (in QD)

etc...

* Examples of mesoscopic systems

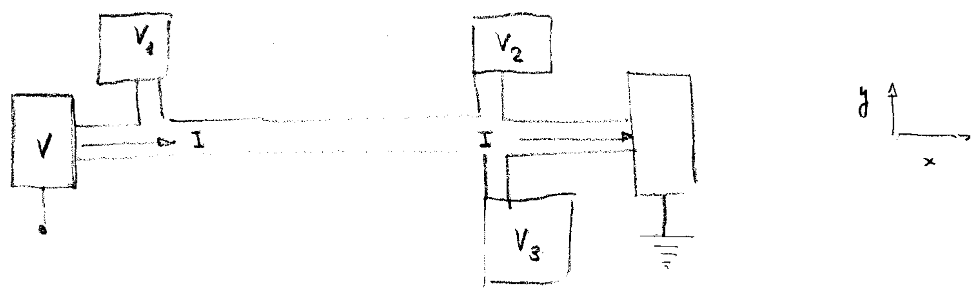
- Quantum rings
- Mesoscopic Hall bars
- Quantum point contact
- Quantum dot
- 2 Dimensional electron gas
- Single electron transistor — Semiconductor, Metal, Nanotube, Single molecule

- <u>Aharonov-Bohm effect</u> : Periodic oscillations of the conductance as a function of the strength of the magnetic field.



They are explained in terms of interference between the 2 paths which are <u>coherently</u> undertaken by the incoming electrons. The phase acquired by the electronic wave function is different in the two paths and the phase difference is proportional to the magnetic flux.

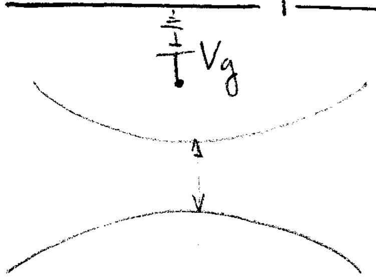
- <u>Quantum Hall effect</u> :



Constant current flow

Minima in ρ_{xx} (V_x) accompanied by plateaus in ρ_{xy} (V_H) as a function of the magnetic field or, at high magnetic fields, as a function of carrier concentration.

- Conductance quantization:



The conductance through a narrow constriction of a 2DEG (a Quantum point contact) is quantized in units of $\frac{2e^2}{h}$ where 2 stands for the spin degeneracy and the number of quanta depends on the width of the constriction. By modulating V_g the conductance moves in steps of $\frac{2e^2}{h}$.

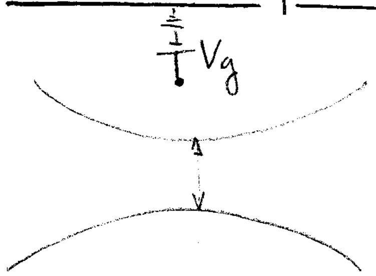
- Universal conductance fluctuations: The conductance of a mesoscopic sample exhibiting weak localization fluctuates as a function of

 - therically: POSITION OF THE IMPURITIES
 - experimentally: magnetic field, gate voltage → carrier density

the amplitude of the fluctuations is $\frac{2e^2}{h}$, independent on the value of the conductance

- Weak localization: In mesoscopic samples the conductance is reduced with respect to its classical value due to interference between time reversed path over the same scatterers. This reduction can be removed by application of magnetic field.

- Conductance quantization:



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- Week localization: In mesoscopic samples the conductance is reduced with respect to its classical value due to interference between time reversed path over the same scatterers. This reduction can be removed by application of magnetic field.

- <u>Coulomb blockade</u>: Transport through weakly coupled QD system is hindered due to the discreteness of the charge and the dimension of the QD. Charging energy.
  Result: Strong conductance oscillations. (Coulomb oscillations)

- <u>Franck Condon blockade</u>: In molecular SET: current blocking due to the interplay between electrical and mechanical degrees of freedom. Result: suppression of the conductance in molecular SET. (opening of the stability diagrams).

- <u>Interference blockade</u>: Current blocking and selective suppression due to interference between orbitally degenerate states.

In all these examples, for different reason, one notices a highly non Ohmic behaviour of the TRANSPORT CHARACTERISTICS.
In particular, the conductance, which classically is given for a rectangular 2 dimensional conductor by $G = \frac{\sigma W}{L}$ and σ is just a constant characteristic of the material, assumes very different forms as a function of T, B, V_g, $\underset{\uparrow}{V_b}$ ...

in this case one should speak of differential conductance.

Preliminary concepts

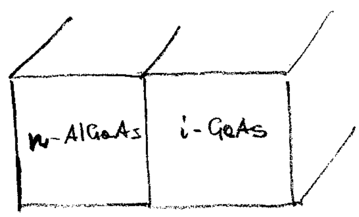
2-DEG

Almost all mesoscopic devices contain an electronic system confined to low dimensions (≤ 2). Historically the first low dim electron system that has been used is 2 Dimensional Electron Gas

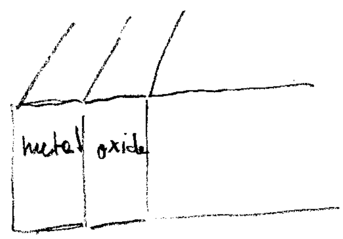
- Metal Oxide Semiconductor Field Effect Transistors
- GaAs / AlGaAs Heterostructures.

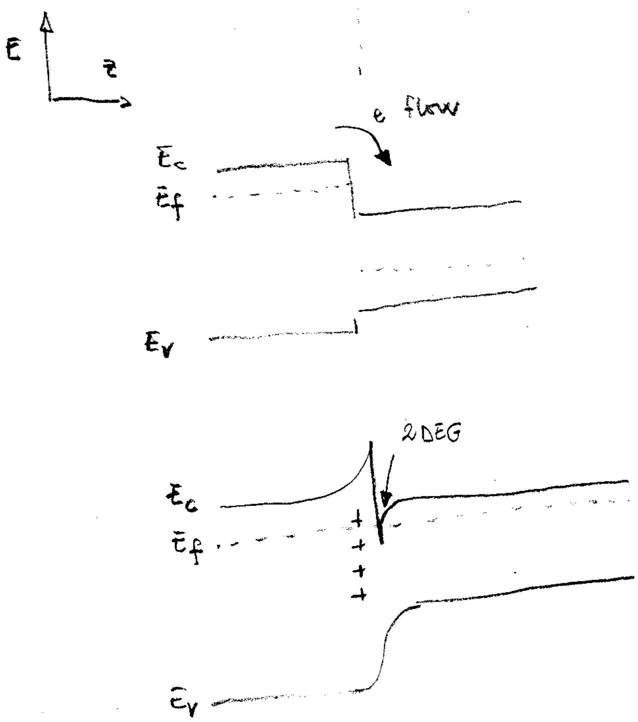
The main idea is the same

Heterostructures



MOSFET





in analogy at the oxide metal interface. The electrons density can be modulated by the metal gate.

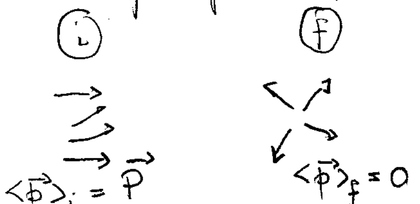
Advantages :
- very high carrier density : the 2 DEG is "very" thin
- very high mobility : the donor impurities are "far" inside n-AlGaAs.

Other examples of low dimensional electron gases are the carbon based structures graphene, carbon nanotube, fullerene and they are also widely used in mesoscopic physics.

The concept of $\boxed{\text{MOBILITY}}$

$$\left[\frac{d\vec{p}}{dt}\right]_{\text{scattering}} = \left[\frac{d\vec{p}}{dt}\right]_{\text{field}}$$

In a classical sense: it take τ_m to get the momentum completely randomized. Say given a group of electrons with average momentum $\vec{P}$ goes into a group of electron with average momentum 0.



$$\Rightarrow \left[\frac{d\vec{p}}{dt}\right]_{\text{scattering}} = \frac{\vec{P}}{\tau_m}$$

With electric field we know that in some sense we do the opposite by applying an external force. The result is the average momentum

$$\langle\vec{p}_f\rangle = m\vec{v}_d$$

$$\frac{m\vec{v}_d}{\tau_m} = e\vec{E} \qquad \mu = \left|\frac{v_d}{E}\right| = \frac{|e|\tau_m}{m} \qquad \frac{C\,s}{g}\cdot\frac{s}{s}\frac{cm^2}{cm^2} =$$

$$= \frac{C\,s^2}{g\,cm^2}\cdot\frac{cm^2}{s} = \frac{cm^2}{Vs}$$

τ_m is given by: impurity scattering, phonon scattering

N.B. e-e does NOT give any contribution to τ_m

At low temperature τ_m is dominated by impurity scattering $\Rightarrow$ high carrier concentration $\Rightarrow$ low τ_m and low mobility:

- n-AlGaAs $\quad n = 10^{17}\,cm^{-3} \quad \mu \lesssim 10^4\,cm^2/(V\,s)$
- 2-DEG $\quad n = 10^{12}/cm^2 \quad \mu \gtrsim 10^6\,\frac{cm^2}{Vs}$ (improved by buffer layers)
  thickness ~ 100 Å
  equiv. bulk $10^{18}\,cm^{-3}$

Effective mass, density of states

Electron in conduction band:

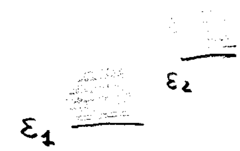
$$\left[E_c + \frac{(i\hbar\vec{\nabla} + e\vec{A})^2}{2m} + U(\vec{r}) \right] \psi(\vec{r}) = E\,\psi(\vec{r})$$

* $U(\vec{r})$ describes space distributed charge (impurities)
* $\vec{A}$ vector potential due to external em fields
* m effective mass: $\Leftarrow$ Bloch theorem already implied and far from the borders of BZ.

The z direction is special (growth direction)

$$\psi(\vec{r}) = \phi_n(z)\, e^{ik_x x}\, e^{ik_y y}$$

$$E = E_c + \varepsilon_n + \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2\right)$$



For low T and low carrier density we can stick to the lower z-band. At this point we are saying the our system is effectively 2-Dimensional.

$$\left[E_s + \frac{(i\hbar\vec{\nabla} + e\vec{A})^2}{2m} + U(x,y) \right] \psi(x,y) = E\,\psi(x,y)$$

A usefull concept and exercise with low dimensional e gases is the determination of the density of states:

$D(E)$ is the number of states per unit energy per unit "Volume".

$$\rho(E) = \frac{1}{S}\sum_{\sigma k} \delta(E - \varepsilon_k) = \frac{2}{S}\sum_k \delta\left(E - E_s - \frac{\hbar^2 k^2}{2m}\right) =$$

$$= 2\int_0^{2\pi} d\varphi_k \int_0^{\infty} \frac{dk\,k}{(2\pi)^2}\, \delta\left(E - E_s - \frac{\hbar^2 k^2}{2m}\right) =$$

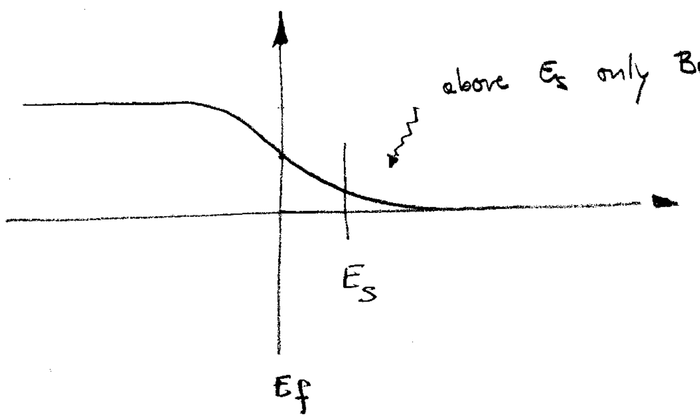
$$= \frac{1}{\pi}\int_0^{\infty} dk\, k \cdot \frac{1}{\frac{\cancel{2}\hbar^2 k}{\cancel{2}m}}\, \delta(k - k(E))$$

$$k(E) = \frac{\sqrt{2m(E - E_s)}}{\hbar}$$

$$\boxed{\rho(E) = \frac{m}{\pi\hbar^2}\theta(E - E_s)} \longleftarrow$$ number of states per unit energy and surface

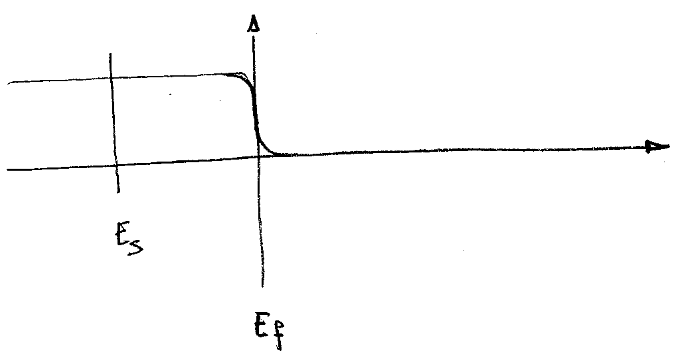
Degenerate and non-degenerate conductors

above E_s only Boltzmann distribution



non-degenerate

$$\frac{E_s - E_f}{k_B T} \gg 1$$



$$\frac{E_s - E_f}{k_B T} \ll -1$$

the distribution for the 2DEG $f_0(E) \approx \theta(E_f - E)$

$$N = \int_S d\vec{r} \int \frac{d\vec{k}}{(2\pi)^2} f_0(\varepsilon(\vec{k})) = S \int dE\, N(E) f_0(E) = S (E_f - E_s) \frac{m}{\pi\hbar^2}$$

$m \cong 0.063 m_0$

$$n_s = (E_f - E_s) \frac{m\pi}{\hbar^2} \qquad E_f - E_s = \frac{\hbar^2 k_f^2}{2m} \qquad n_s = \frac{\cancel{\hbar^2} k_F^2}{2\cancel{m}} \cdot \frac{\cancel{m\pi}}{\cancel{\hbar^2}} = \frac{k_F^2}{2\pi}$$

$$k_F = \sqrt{2\pi n_s}$$

Characteristic Lengths

A conductor shows Ohmic behaviour if L_x, L_y, L_z < Characteristic lengths

1• λ = de Broglie wavelength

2• L_m = mean free path

3• L_φ = phase-relaxation length

[1]

$$\lambda_f = \frac{2\pi}{k_f} = \frac{2\pi\hbar}{\hbar k_f} = \boxed{\frac{h}{m v_f}}$$

2DEG

$$= \sqrt{\frac{2\pi}{n_s}}$$

this is the common definition of the de Broglie wavelength ($v_f \to v$) which tells us why we do not normally observe particle wave duality in macroscopic objects.

$$n_s = 5 \cdot 10^{11}\ cm^{-2} \Rightarrow \boxed{\lambda_f \approx 35\ nm} = 35 \cdot 10^{-9} m$$

$$= 5 \cdot 10^{15}\ m^{-2}$$

One takes the Fermi wave number because effectively only electrons close to the Fermi surface participate to transport.

[2] An electron in a perfect crystal moves as if it were in vacuum but with a different mass. But

– impurities, phonons, e-e interaction

$$\frac{1}{\tau_m} = \sum_{i} \frac{\alpha_m^i}{\tau_c}$$

i process

α_m^i is measuring the effectiveness in changing momentum for the process i

$\alpha_m^{(i)} \in [0, 1]$

$$L_m = v_f \tau_m$$

$$v_f = \frac{\hbar k_f}{m} = \frac{\hbar}{m}\sqrt{2\pi n_s} = 3\times10^7\ \mathrm{cm/s}$$

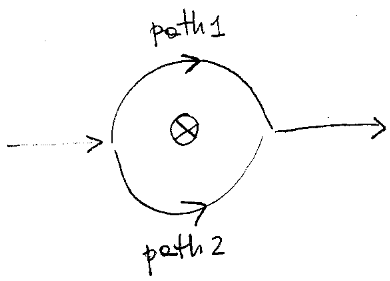
$$L_m = 30\,\mu m \qquad \tau_m \cong 100\ ps$$

N.B. e-e is not influencing L_m since the momentum is conserved in the interaction

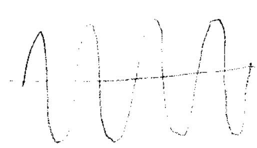
$\boxed{3}$ The first point is to discuss the phase-relaxation time

$$\frac{1}{\tau_\varphi} = \sum_i \frac{\alpha_\varphi^i}{\tau_c}$$

AB effect experiment



in a situation without dephasing



G(B)

the amplitude of the fringes should be depressed $\exp[-\tau_t/\tau_\varphi]$

in case of dephasing process take place.

1- Static scatterers do not contribute to τ_φ NO

2- Phonons
3- Electron-electron interaction
4- Magnetic impurities

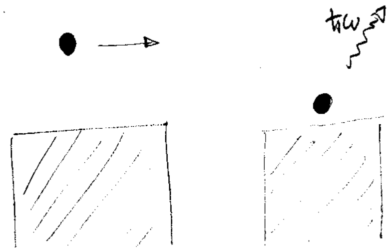
} inelastic scattering YES

Altshuler, Aronov, Khmelnitsky (1982)
J. Phys. C. 15 7367

- phonon analysis

$$(\Delta\varepsilon)^2 = (\hbar\omega)^2 \underbrace{\left(\tau_\varphi/\tau_c\right)}_{\text{\# collisions in the dephasing time}}$$

$(\Delta\varepsilon)^2$ is the mean squared energy spread due to e-ph collision

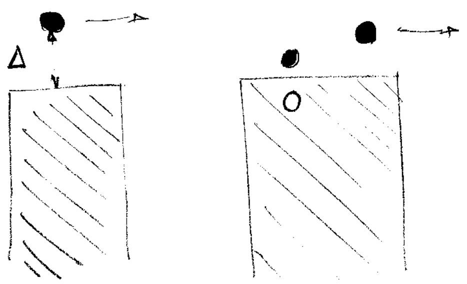


The phase relaxation time $\Delta\varphi \sim \frac{\Delta\varepsilon\, \tau_\varphi}{\hbar} \sim 1 \quad = \quad \Delta\varepsilon = \frac{\hbar}{\tau_\varphi}$

$$\frac{\cancel{\hbar^2}}{\tau_\varphi^{\,2}} = \frac{\cancel{\hbar^2}\,\omega^2}{\tau_c}\,\tau_\varphi \quad \Rightarrow \quad \tau_\varphi \cong \left(\tau_c/\omega^2\right)^{1/3}$$

This relation seems reasonable and, for example, takes into account the fact that low frequency phonons will scatter all electrons simultaneously giving rise to no dephasing.

- e-e analysis



the current is carried afterwards by electrons carrying a different phase.

$$\frac{\hbar}{\tau_\varphi} \sim \varepsilon_F \left|\frac{\Delta}{\varepsilon_F}\right|^2 \left[\ln\left(\frac{\varepsilon_F}{\Delta}\right) + \text{const}\right]$$

the higher Δ, the shorter is the dephasing time τ_φ.

But we would like to obtain a dephasing length

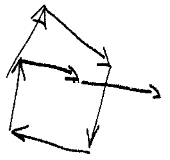
idea $\boxed{L_\varphi = v_f \tau_\varphi}$

ballistic limit

$\tau_\varphi \ll \tau_m$

this is the case only if $\tau_c \sim \tau_\varphi$ which means that el low temperature no scattering from fixed impurity is left => for ballistic conductors.

If $\tau_\varphi \gg \tau_m$



diffusive motion over a phase coherent region. Which is the size of the phase coherent region?

$$L_\varphi^2 = \frac{\tau_\varphi}{\tau_m}(v_f \tau_m)^2 \langle \cos^2\theta \rangle \qquad \langle \cos^2\theta \rangle = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} \cos^2\theta = \frac{1}{2}$$

$$L_\varphi^2 = \frac{v_f^2 \tau_m}{2} \tau_\varphi \equiv D \tau_\varphi$$

D = diffusion constant in 2 dimension

Transport classification (L = system size)

$$\left.\begin{array}{lll} L \gg L_m & \text{diffusive} \\ L \gg L_\varphi & \text{incoherent} \\ L \gg \lambda & \text{semiclassical} \end{array}\right\} \text{conventional}$$

$$\left.\begin{array}{lll} L \ll L_m & \text{ballistic} \\ L \ll L_\varphi & \text{coherent} \\ L \ll \lambda & \text{quantum mechanical} \end{array}\right| \Rightarrow \text{MESOSCOPIC PHYSICS}$$

Other relevant lengths

MAGNETIC LENGTH

$$\ell_B = \sqrt{\frac{\hbar}{eB}}$$ spacial extent of a wave function in a " etic field

$$\underbrace{r_c}_{\text{cyclotron radius}} = k_F \ell_B^2 = \frac{\hbar k_F}{eB} = \frac{m v_F}{eB} = \frac{v_F}{\omega_c} \qquad \underbrace{\omega_c = \frac{eB}{m}}_{\text{cyclotron frequency}}$$

$$B = 1T \,, \quad \ell_B \sim \left(\frac{10^{-34}}{10^{-19}} \right)^{1/2} \sim 0.3 \times 10^{-7}\,m \sim 30\,nm$$

$$r_c \sim \ell_B^2 / \lambda_F \sim 10^{-15}$$

THERMAL LENGTH

length over which thermal smearing takes place

$$\tau_{th} \approx \frac{\hbar}{k_B T} \,, \quad \ell_{th} = \sqrt{D \tau_{th}} \;; \quad D: \text{diffusion constant } \frac{v_f^2 \tau_m}{d}$$

d is the dimension of the conductor