Quantentheorie II

Prof. Milena Grifoni Dr. Andrea Donarini

Blatt 5

1. Electron in rotating magnetic field

Consider an electron in the rotating magnetic field $\vec{B}(t) = [B_0 \cos(\omega t), B_0 \sin(\omega t), B_1]$.

a) • Prove that, if the electron is an orbital state with zero angular momentum, the Hamiltonian for the system can be written in the form:

$$H(t) = -\frac{\delta E}{2}\sigma_z + V\cos(\omega t)\sigma_x + V\sin(\omega t)\sigma_y$$

.

with $\delta E = g\mu_{\rm B}B_1$ and $V = -g\mu_{\rm B}\frac{B_0}{2}$. (**2 Points**)

b) • Solve the time dependent Schrödinger equation associated to the Hamiltonian introduced in point a) and calculate the evolution of the occupation probability $P_0(t)$ for the ground state assuming the ground state initial condition: $P_0(0) = 1$. Make a sketch of the occupation probabilities for the two spin states as a function of time.

(**3 Points**)

c) Prove that this problem can be mapped into the Rabi oscillation problem described by the Hamiltonian

$$H(t) = -\frac{\delta E}{2}\sigma_z + V\cos(\omega t)\sigma_x$$

assuming for the latter the rotating wave approximation.

2. Floquet states of a three-level system

Consider an electron in a state with total angular momentum $F = 1$. We assume that a static magnetic field of strength B is applied in the z-direction together with a static field B_0 and an oscillating field $B_1\cos(\omega t)$ in the x-direction. The Hamiltonian restricted to the $F = 1$ subspace can be written as:

$$H(t) = g_1\mu_{\rm B}B\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} + g_1\mu_{\rm B}[B_0 + B_1\cos(\omega t)]\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \equiv H^{\rm B} + H_0(t).$$

a) • Evaluate the Floquet functions $\phi_\alpha(t)$ and the quasi energies ϵ_α for the case in which $B = 0$. *Hint*: It is convenient to perform the unitary transformation which diagonalizes $H_0(t)$ at every time. (**3 Points**)

Let us now consider the situation $B \neq 0$ but still $B \ll B_0 \approx B_1$, such that the Hamiltonian H^B can be treated as a perturbation.

b) • Restrict first to the case $g_1\mu_{\rm B}B_0 \neq n\frac{\hbar\omega}{2}$, , $\forall n \in \mathbb{Z}$ and calculate the first order correction to the quasi-energies. (**2 Points**)

c) Consider now the resonant case $g_1\mu_\mathrm{B}B_0 = \tilde{n}\frac{\hbar\omega}{2}$ for a particular $\tilde{n} \in \mathbb{Z}$. Use degenerate perturbation theory to find the quasi energies to first order in B. *Hint*: Notice that the system has different degenerations for $\tilde{n}$ even or odd.

d) Sketch the Floquet eigenenergies as a function of B_0 first for the case $B = 0$ and then also for the case $B \neq 0$ in the first order perturbation in B. Do you expect any degenerate quasi-energy? *Hint*: Remember that the quasi-energies ϵ_α are defined $\mod (\hbar\omega)$

Frohes Schaffen!

Return the solution of the exercises marked with • by Monday 17th of November at 10:00.