## Applications of Group Theory

| Lectures | Tue $10: 00-11: 30$ | PHY 9.1.09 |
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|  | Thu $10: 00-11: 30$ | PHY 9.1.09 |
| Exercises | Fri $10: 00-11: 30$ | PHY 5.0.21 |

## Sheet 9

## 1. Character tables of double groups

1. Consider the group $\mathrm{D}_{4}$ and find the classes of the associated double group $\overline{\mathrm{D}}_{4}$ (Hint: make use of the Opechowski's rules)
2. Which is the number of irreducible spinor representations in $\overline{\mathrm{D}}_{4}$ ?
3. Complete now the character table of the double group $\overline{\mathrm{D}}_{4}$ by adding to it the characters associated to the spinorial representations.

## 2. Geometry of rotations

1. Prove that all the $C_{3}$ operations of the octahedral group O (group of the cube) are conjugate, but that this is not so for the T group (group of the tetrahedron). Discuss the corresponding classes in these groups. Hint: make use of the concept of pole.
2. Prove that

$$
R(\lambda ; \boldsymbol{\Lambda}) \mathbf{r}=\left(1-2 \Lambda^{2}\right) \mathbf{r}+2 \lambda(\boldsymbol{\Lambda} \times \mathbf{r})+2(\boldsymbol{\Lambda} \cdot \mathbf{r}) \boldsymbol{\Lambda}
$$

where $R(\lambda ; \boldsymbol{\Lambda})$ is a rotation in terms of the Euler-Rodrigues parameters and $\mathbf{r}$ is a generic space vector.

## Frohes Schaffen!

