Applications of Group Theory

Lectures	Tue	10:15 - 11:45	PHY 9.1.09
	Thu	10:15 - 11:45	PHY 9.1.09
Exercises	Fri	10:15 - 11:45	PHY 5.0.21

Sheet 2

1. Groups of prime-number order

Prove that all groups with order equal to a prime number n are isomorphic to the cyclic group C_n.

2. Matrix representations

In the lecture we have introduced the homomorphism connecting point symmetry groups to groups of 3x3 matrices representing linear mappings of $\mathbb{R}^3$ into itself. Moreover we related the latter to a group of functionals which can eventually be mapped into a matrix group once a vectorial space invariant under the functionals group is introduced. Let us now consider concrete examples:

1. Construct the matrix representative of the point symmetry operation C_4^+, *i.e.* the anticlockwise rotation of $\pi/2$ with respect of the z axis, within $\mathbb{R}^3$.
2. Consider the associated function operator $\hat{C}_4^+$ and find the transformed function for each of the 5 atomic orbitals of the 3d subshell. Find the associated matrix representation of the point symmetry operation in the Hilbert space generated by these orbitals.
3. Repeat the first two steps for all the elements of the cyclic group C_4. Are the corresponding representations reducible or irreducible?
4. Find the matrix representation of the dihedral group D_4 in the Hilbert space of the 3d subshell and calculate the size of the irreducible representations.

3. Group of the Hamiltonian

Consider the linear hermitian operator $\hat{H}$ that maps a given Hilbert space $\mathcal{H}$ into itself. Prove that the set of all linear, regular operators $\hat{R}$ defined on the same Hilbert space and with the property $[\hat{R}, \hat{H}] = 0$ form a group. Take as binary composition the usual multiplication between operators.

Frohes Schaffen!