Applications of Group Theory

Lectures	Tue	10:15 - 11:45	PHY 9.1.09
	Thu	10:15 - 11:45	PHY 9.1.09
Exercises	Fri	10:15 - 11:45	PHY 5.0.21

Sheet 1

---

1. Cyclic groups

A group is generally defined as a set of elements $\mathcal{G}$ and a binary composition $(\cdot)$ satisfying the four properties: i) Closure of $\mathcal{G}$ with respect to the binary composition; ii) Existence of the identity element; iii) Validity of the associative law; iv) Existence of the inverse of each element of $\mathcal{G}$ inside the set $\mathcal{G}$ itself. Consider an element (e.g. a symmetry operation) g and a binary composition $(\cdot)$ that allows you to construct the sequence $g_1 \equiv g$, $g_2 \equiv g \cdot g$, $g_3 \equiv g \cdot g \cdot g$, ... Further assume that $g_n \cdot g = g$ with a particular $n \in \mathbb{N}$.

1. Prove that the set $\{g_1, g_2, \ldots, g_n\}$ with the operation $(\cdot)$ is a group by explicitly verifying the four defining axioms listed above.
2. Prove that for any group of finite order the existence of an inverse is a consequence of the other three axioms defining a group.

2. Groups of order 4

1. List all possible point groups of order 4, write their projection diagrams and multiplication tables. Find the isomorphisms.
2. Prove that, in general, there are only 2 different groups of order 4 which are not isomorph. Construct the multiplication tables of these two groups, $\mathcal{G}_4^{(1)}$ and $\mathcal{G}_4^{(2)}$. Hints: Start with the multiplication table of C_4 i.e. the cyclic group of order 4. How many elements of C_4 are equal to their inverse? Now try to construct further groups in which a different number of elements are equal to their own inverse. Remember to fulfil the rearrangement theorem. It is a bit like sudoku!

Frohes Schaffen!