## Applications of Group Theory

PD Dr. Andrea Donarini Lectures

Exercises

H33, Mondays, 14:15 H34, Thursdays, 14:15 5.0.21, Wednesdays, 13:15

## Sheet 6

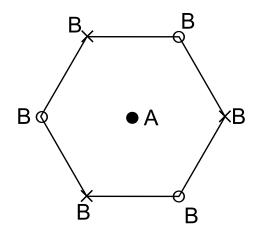
## 1. Basis functions and representations of $D_3$ and $D_{3h}$

Consider the point group  $D_3$  and the basis functions partners to its irreducible representations:

- 1. What are the matrix representations for the bases  $(x^2 y^2, 2xy)$  and  $(R_x, R_y)$  in the point group  $D_3$ ?
- 2. Using the results in 1), find the unitary transformation which transforms the matrices for the representation corresponding to the basis functions  $(x^2 y^2, 2xy)$  into the representation corresponding to the basis functions (x, y)
- 3. Using projection operators, check that xy is a proper basis function of the two dimensional irreducible representation E in the point group  $D_3$ . Using the matrix representation found in 1) and the projection operators find the other partner of the representation.
- 4. Using the basis functions in the character table for  $D_{3h}$ , write a set of  $(2 \times 2)$  matrices for the two two-dimensional representations E' and E''.

## **2.** The molecule $AB_6$

Consider a molecule  $AB_6$  (see Figure) where the atom A lies in the xy plane (z = 0), the three B atoms indicated by a circle lie in the plane z = c > 0 and the B atoms indicated by a cross lie in the plane z = c' < 0. When projected onto the xy plane, all B atoms occupy the corners of a hexagon.



- 1. Find the symmetry elements and classes.
- 2. Construct the character table. To which point group does the molecule correspond? How many irreducible representations are there? How many are one-dimensional and how many are of higher dimensionality?
- 3. Using the basis functions in the character table of this point group, find a set of matrices for each irreducible representation of the group.
- 4. Find the linear combination of the six s-orbitals of the B atoms that transform as each of the irreducible representations of the group.
- 5. What additional symmetry operations result in the limit  $c = c' \neq 0$ ? and in the limit c = c' = 0? Indicate the associated point groups and draw the corresponding stereograms.

Frohes Schaffen!