Quantum Theory of Condensed Matter

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Mo 10:15, 5.0.21
We 16:15, H33

Sheet 10

1. Electronic lifetime

Consider a system made of an atom on a metal surface. To first approximation let us assume only one state for the atom and describe the system with the Hamiltonian:

$$H = \varepsilon_0 d^\dagger d + \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}} c_{\mathbf{k}} + t c^\dagger_{\mathbf{k}} d + t^* d^\dagger c_{\mathbf{k}}$$

where $d^\dagger$ creates an electron in the atomic state while $c^\dagger_k$ creates an electron with momentum $\hbar\mathbf{k}$ in the metal. As far as double occupancy of the metal can be excluded it is harmless to neglect the spin degree of freedom.

a) Making use of the equation of motion technique, calculate the retarded Green's function for the system in the frequency domain.

b) Prove the validity of the relation between distributions:

$$\frac{1}{x + i0^+} = -i\pi\delta(x) + \mathcal{P}\left(\frac{1}{x}\right)$$

where $\mathcal{P}$ indicates the Cauchy principal value of the otherwise ill defined integral associated with the distribution.

c) Assuming that the atom is only exchanging electrons with electrons on the surface of the metal calculate the retarded Green's function for the atom also in the time domain. Assume the dispersion relation of the electrons on the metal surface to be quadratic. Can you recognize which is the life time of the electron on the metal?

d) Calculate the average probability of finding an electron on the metal as a function of the on-site energy of the atomic orbital ε_0 at zero temperature. Compare the result with the one calculated in the second exercise of Sheet 6. Can you distinguish the occupation broadening due to temperature or to the tunnelling coupling to the metal?

Frohes Schaffen!