Photon-mediated drag in double-layer electron gas

A Donarini†, R Ferrari‡, A P Jauho†, L Molinari‡
† Mikroelektronik Centret, Building 345 east, Technical University of Denmark, DK-2800 Kgs.Lyngby, Denmark
‡ Dipartimento di Fisica, Università di Milano, via Celoria 16, 20133 Milano, Italy

Abstract.
A microscopic theoretical description of Coulomb drag can be given based on the Kubo formula, which allows a diagrammatic formulation of the theory. If the interlayer coupling is due to the (screened) Coulomb interaction, or phonon-mediated interactions, the lowest nonvanishing contribution to dc drag is second order in the coupling. In this work we study the first order coupling due to the vector potential. The electromagnetic coupling is known to give rise to small but interesting effects in the bulk electron gas. Our calculations, which are carried out within RPA, show that a first-order correction does exist, even in the dc-limit. We present an approximate analytic evaluation of the photon-exchange contribution to Coulomb drag, and discuss the further steps required in a quantitative calculation.

1. Introduction

The double layer configuration of the electron gas allows one to study unusual transport phenomena which are interesting for the theory and are nowadays accessible to experiments. In 1976, Pogrebrinskii [1] advanced the idea that a current driven in an electron gas should give rise to a potential difference in a spatially separated electron system because of Coulomb scattering with a preferred direction of the exchanged momentum. This Coulomb drag was described in terms of coupled transport equations. Later on the effect, which was originally analyzed for bulk electrons, was experimentally observed in double well heterostructures at low temperature, allowing for a quantitative investigation [2]. Transresistance is the ratio of the induced potential difference and the driving current. It was the main subject of several theoretical works based on transport equations or Kubo formula for transconductivity [3]. Qualitatively, transresistance at $B = 0$ depends on temperature as $T^2$, as a sign of dissipation, and decreases as $d^{-4}$ in the distance between the wells.

The theory was refined by including phonon exchange [4], and provided a good explanation of observed deviations of the $T^2$ behaviour. A “current drag” effect was also proposed [5], and originates from a Van der Waals attraction between wires or layers where there is a relative current. In a strong magnetic field the physics is very rich and depends on the filling factors; the subject of drag in bilayer Hall systems has already a large literature, for review see [6].

2. A legitimate question

The role of Coulomb interaction in processes of momentum exchange between two layers has been extensively considered. A phonon mediated interaction has been also sometimes introduced to refine already qualitatively correct results. In this work we present our analysis concerning the role of an electromagnetic (i.e. photon exchange) interaction. At first sight
it might appear that the effects are negligible, because we are working with non-relativistic systems. Note however:

- in electrostatic interaction and in DC limit first non-vanishing contribution comes from 2\textsuperscript{nd} order interaction:
  \[
  \sigma_{1,2} \propto \left( \frac{\alpha}{\epsilon_r} \right)^2 \approx \left( \frac{1}{1370} \right)^2 \approx 10^{-6}
  \]

  where \( \epsilon_r = 13 \) is the relative permittivity
- the 1\textsuperscript{st} order interaction in electromagnetic interaction is instead:
  \[
  \sigma_{1,2} \propto \left( \frac{v_F}{c} \right)^2 = \frac{2k_BT_F}{m^*c^2} \approx \frac{8 \times 10^{-3}}{5 \times 10^4} \approx 10^{-7}
  \]

Thus it follows, in principle, that the two contributions can be comparable. The e.m. interaction in the electron gas is the source of small though interesting effects, as it modifies the low temperature behaviour of specific heat or the Fermi surface [7]. It has been treated in the RPA scheme, and a main feature of the effective interaction is to be unscreened at zero frequency [8]. We shall employ the same approximation for the polarization [7], which directly yields the current-current correlator required in the Kubo formula [9].

3. Model and framework

We study a system of two 2DEG’s realized using semiconductor heterostructures [2] and model the system as two infinite parallel layers of electron gas, confined in narrow potential wells centered in \( z_1 = 0 \) and \( z_2 = d \) with negligible overlap. It is useful to introduce fermion operators that create or remove a particle with spin \( \sigma \) and position \( \mathbf{x} \) in layer \( \ell = 1, 2 \). Relevant operators such as charge or current density are sums of commuting one-layer parts:

\[
\rho(\mathbf{x}) = \sum_\ell \rho_\ell(\mathbf{x}), \quad \rho_\ell(\mathbf{x}) = -e \sum_\sigma \psi_\ell\dagger(\mathbf{x})\psi_\ell(\mathbf{x})
\]

\[
J^i(\mathbf{x}) = \sum_\ell \left[ j^i_\ell(\mathbf{x}) + \frac{e}{2mc^2} \rho_\ell(\mathbf{x}) A^i(\mathbf{x}) \right]
\]

\[
j^i_\ell(\mathbf{x}) = \frac{i\hbar e}{2mc} \sum_\sigma \psi_\ell\dagger(\mathbf{x}) \partial_i \psi_\ell(\mathbf{x}) - (\partial_i \psi_\ell\dagger(\mathbf{x})) \psi_\ell(\mathbf{x})
\]

In the limit of thin layers, the layer operators only depend on \( \mathbf{r} = (x, y) \). The Hamiltonian is the sum of decoupled terms \( H_\ell \) which take into account the kinetic and the Coulomb energy contributions from the electrons in each layer, the free photon Hamiltonian \( H_{em} \), the interlayer Coulomb interaction \( U_{es} \), and the e.m. coupling \( H_{int} \):

\[
H = H_1 + H_2 + H_{em} + U_{es} - \frac{1}{c} \int d^3x J^i(\mathbf{x}) A^i(\mathbf{x})
\]

The absence of tunneling ensures charge conservation in each layer, given by the operator identity

\[
\frac{1}{i\hbar} [H, \rho_\ell(\mathbf{x})] = \text{div} J_\ell(\mathbf{x})
\]
To derive the Kubo formula for conductivity, one perturbs the Hamiltonian with a term
\[ \delta H = \frac{1}{e} \int d^3x A_i^\text{ext}(x,t) J^i(x) \]
(6)
describing the linear coupling of the total current to a weak external electric field. Linear response gives the conductivity tensor
\[ \sigma_{ij}(x,x',\omega) = \frac{i}{\hbar \omega} \pi^{\text{ret}}_{ij}(x,x',\omega) - \frac{i e}{m \omega} \langle \rho(x) \rangle \delta_{ij} \delta_3(x-x') \]
(7)
i, j are space directions. \( \pi^{\text{ret}}_{ij} \) is the retarded current-current correlator. In particular, transconductivity is obtained when \( z \) and \( z' \) belong to different layers. Because of translational invariance in the \( (x,y) \) directions, we Fourier transform with a two-dimensional wave-vector \( \mathbf{q} \). The real part of conductivity in the static homogeneous limit is:
\[ \text{Re} \sigma_{ij}(z,z') = - \lim_{\omega \to 0} \lim_{\mathbf{q} \to 0} \frac{1}{\hbar \omega} \text{Im} \pi^{\text{ret}}_{ij}(\mathbf{q},\omega,z,z') \]
(8)
It is convenient to use a four-component notation, and define \( J^0(\mathbf{x}) = c \rho(\mathbf{x}) \). Since we shall use the Matsubara, or thermal formalism, we employ Euclidean metric. Therefore \( x^\mu = x_\mu \). The retarded correlator is obtained from the thermal time-ordered correlator
\[ \pi^{\mu\nu}(x,\tau,x',\tau') = - \langle T A^\mu(x,\tau) J^\nu(x',\tau') \rangle \]
\[ = \frac{1}{\hbar \beta} \sum_n \int \frac{d^2q}{(2\pi)^2} e^{i \mathbf{q} \cdot (\mathbf{r}-\mathbf{r}')} e^{-i \omega_n (\tau-\tau')} \pi^{\mu\nu}(\mathbf{q},\omega_n,z,z') \]
(9)
with the analytic continuation \( i \omega_n \to \omega + i \eta \). The thermal current-current correlator can be related to the polarization tensor of the e.m. field, to be discussed in the next section.

4. The polarization tensor

The thermal Green functions for the photon field interacting with matter are [10]
\[ D_{ij}(x,\tau,x',\tau') = - \frac{1}{\hbar} \langle T A^i(x,\tau) A^j(x',\tau') \rangle \]
(10)

We include the prefactor \( 1/\hbar \) to obtain the same dimension as the effective Coulomb interaction \( D_{00} \). The propagators for free photons and the bare Coulomb interaction are best written in momentum space:
\[ D_{0ij}(k,\omega_n) = - \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \frac{4 \pi e^2}{\omega_n^2 + c^2 k^2} \]  
\[ D_{00}(k,i \omega_n) = \frac{4 \pi}{k^2} \]
(11)
They are the components of a tensor \( D_{0\mu\nu}, \mu, \nu = 0, 1, 2, 3 \) with \( D_{0i} = D_{00} = 0 \). With interaction included, the dressed photon propagator and the effective Coulomb interaction are components of a tensor \( D_{\mu\nu} \) which differs from the bare one by polarization insertions:
\[ D_{\mu\nu}(x,x',i \omega_n) - D_{0\mu}(x,x',i \omega_n) = D_{\mu\rho}(x,x_1,i \omega_n) P^{\rho\sigma}(x_1,x_2,i \omega_n) D_{0\sigma\nu}(x_2,x',i \omega_n) \]
(12)
where summation and integration of repeated variables are understood. The identification of the polarization insertion of the Coulomb interaction ($\mu = \nu = 0$) with the connected density-density correlator is well known from textbooks [10]:

$$P^{00}(\vec{x}, \vec{x}', i\omega_n) = \frac{1}{\hbar c^2} \pi^{00}(\vec{x}, \vec{x}', i\omega_n)$$  \hspace{1cm} (13)

A perturbative analysis shows that the polarization insertion of the photon propagator has the following structure:

$$P^{ij}(\vec{x}, \vec{x}', i\omega_n) = -\frac{e}{mc^2} \langle \rho(\vec{x}) \rangle \delta^{ij} \delta(\vec{x} - \vec{x}') + \frac{1}{\hbar c^2} \pi^{ij}(\vec{x}, \vec{x}', i\omega_n)$$  \hspace{1cm} (14)

and for the other components of the polarization:

$$P^{0i} = \frac{1}{\hbar c^2} \pi^{0i}, \quad P^{i0} = \frac{1}{\hbar c^2} \pi^{i0}$$  \hspace{1cm} (15)

The conductivity tensor turns out to be proportional to the retarded photon polarization

$$\sigma_{ij}(\vec{x}, \vec{x}', \omega) = \frac{ie^2}{\omega} P^{ij}(\vec{x}, \vec{x}', \omega + i\eta)$$  \hspace{1cm} (16)

We thus face the problem of evaluating the polarization tensor. We first write a Dyson equation for it.

### 5. The Dyson equation

Since the model is not translation-invariant in the $z$ direction, we put $\vec{k} = (\vec{q}, k_3)$ in the bare e.m. propagators and Fourier transform to coordinates $z - z'$:

$$D_{\mu\nu}^0(q, z - z', i\omega_n) = \int \frac{dk_3}{2\pi} D_{\mu\nu}^0(q, k_3, i\omega_n) e^{ik_3(z - z')}$$  \hspace{1cm} (17)

With the definition of the auxiliary function

$$\hat{D}(q, z - z', i\omega_n) = 2\pi c \frac{e^{-\frac{1}{c} |z - z'| \sqrt{\omega_n^2 + q^2 c^2}}}{\sqrt{\omega_n^2 + q^2 c^2}}$$  \hspace{1cm} (18)

we evaluate

$$D_{00}^0 = 2\pi \frac{e^{-q |z - z'|}}{q}, \quad D_{ab}^0 = -\delta_{ab} \hat{D} - \frac{q_a q_b c^2}{\omega_n^2} (\hat{D} - D_{00}^0), \quad D_{33}^0 = \frac{q^2 c^2}{\omega_n^2} (\hat{D} - D_{00}^0)$$  \hspace{1cm} (19)

$$D_{3a}^0 = D_{3a}^0 = 2\pi i \text{sign}(z - z') \frac{q^a c^2}{\omega_n^2} \left( e^{-\frac{1}{c} |z - z'| \sqrt{\omega_n^2 + q^2 c^2}} - e^{-q |z - z'|} \right)$$  \hspace{1cm} (20)

The Coulomb gauge is expressed in terms of photon propagator as $q_a D_{0i}^0 - i \partial_z D_{ai}^0 = 0$. If we denote $P^{\mu\nu*}$ the polarization insertions that cannot be disconnected by cutting one photon or Coulomb line, we can write a Dyson equation

$$P^{\mu\nu}(q, i\omega_n, z, z') = P^{\mu\nu*}(q, i\omega_n, z, z')$$

$$+ P^{\mu\nu}(q, i\omega_n, z, z_1) D_{\rho\sigma}^0(q, i\omega_n, z_1 - z_2) P^{\sigma\nu}(q, i\omega_n, z_2, z')$$  \hspace{1cm} (21)
where, as usual, repeated variables are summed and integrated. To proceed further we consider the limit of thin layers at \( z_1 = 0 \) and \( z_2 = d \). The Dyson equations become algebraic, due to the delta function in \( z \) coordinate:

\[
P^{\mu\nu}(\ell \ell') - P^{\mu\nu*}(\ell \ell') = P^{\mu\nu*}(\ell_1 \ell_1)D^0_{\rho\sigma}(\ell_1,\ell_2)P^{\rho\sigma}(\ell_2 \ell') \tag{22}
\]

where repeated indices are summed and we suppressed the variables \( \mathbf{q}, \omega_n \) for brevity. In Eq. (22)

\[
D^0_{\mu\nu}(\ell \ell') = D^0_{\mu\nu}(\mathbf{q}, z_\ell - z_{\ell'}, \omega_n), \quad P^{\mu\nu}(\ell, \ell') = P^{\mu\nu}(\mathbf{q}, \omega_n, z_\ell, z_{\ell'}) \tag{23}
\]

are respectively the elements of two \( 4 \times 4 \) matrices \( D^0(\ell \ell') \) and \( P(\ell \ell') \). The Dyson equations then correspond to 4 matrix equations \((\ell, \ell' = 1, 2)\):

\[
P(\ell \ell') = P^*(\ell \ell') + P^*(\ell_1 \ell_1)D^0(\ell_1 \ell_2)P(\ell_2 \ell') \tag{24}
\]

The structure of the polarization tensor is greatly constrained by covariance and charge conservation. The latter implies the following exact relations:

\[
i\frac{\omega_n}{c} P^{\mu\nu}(\mathbf{q}, \omega_n, \ell, \ell') = q_a P^{\mu\nu}(\mathbf{q}, \omega_n, \ell, \ell'), \quad \nu = 0, 1, 2, 3 \tag{25}
\]

The same holds when the sums are on the second index. These relations correspond to the Ward identity relating the vertex functions for Coulomb and e.m. coupling to the electron field. If we set

\[
P^{ab}(\mathbf{q}, \omega_n, \ell, \ell') = \delta^{ab} A(\mathbf{q}, \omega_n, \ell, \ell') + \frac{q_a q_b}{q^2} B(\mathbf{q}, \omega_n, \ell, \ell') \tag{26}
\]

we find:

\[
P^{0\alpha} = P^{\alpha 0} = i\frac{\omega_n}{cq^2}q^a P^{a0}, \quad A + B = -\frac{\omega_n^2}{c^2 q^2} P^{00} \tag{27}
\]

Similar relations link \( P^{03} \) or \( P^{30} \) to \( P^{03} \) or \( P^{30} \).

6. The Random Phase approximation

We shall solve Dyson’s equation in RPA and for identical layers. In this approximation one has

\[
P^*_{\text{RPA}}(\ell \ell') = \delta_{\ell \ell'} P_0 \tag{28}
\]

where \( P_0 \) is the interlayer irreducible polarization matrix in the RPA. With this simplification Dyson’s equations are:

\[
P^{(12)} = P_0 D P^{(12)} + P_0 D^0(12) P^{(22)}
\]

\[
P^{(22)} = P_0 + P_0 D P^{(22)} + P_0 D^0(21) P^{(12)} \tag{29}
\]

where \( D \equiv D^{(11)} = D^{(22)} \) is the matrix for bare intralayer e.m. interaction. If we set the interlayer interaction \( D^{(12)} \) and \( D^{(21)} \) to zero we obtain the polarization matrix for the single isolated layer

\[
P = (I - P_0 D)^{-1} P_0 = P_0 (I - D P_0)^{-1} \tag{30}
\]

Its components \( P^{ab} \) provide the conductivity of a single layer. In terms of \( P \), Eq. (29) becomes

\[
P^{(12)} = P D^0(12) P^{(22)}
\]

\[
P^{(22)} = P + P D^0(21) P^{(12)} \tag{31}
\]
The sub-matrix $P(11)^{ab}$ is proportional to the conductivity tensor of layer 1 in presence of layer 2, while $P(12)^{ab}$ is proportional to transconductivity among layers 1 and 2. Some matrix elements of $P_0$ are zero in the RPA. The matrix describes the polarization for the single isolated layer for free electrons (with no e.m. interactions).

$$
P_0 = \begin{pmatrix}
P_{00} & P_{01}^0 & P_{02}^0 & 0 \\
P_{10}^0 & P_{11} & P_{12}^0 & 0 \\
P_{20}^0 & P_{21}^0 & P_{22}^0 & 0 \\
0 & 0 & 0 & P_{33}^0
\end{pmatrix}$$  (32)

The various components fulfill the relations for charge conservation that characterize the exact polarization matrix $\Pi$. Therefore, if we set

$$
P^{ab} = A_0 \delta^{ab} + B_0 \frac{q^a q^b}{q^2}, \quad P_{33}^0 = \frac{e^2 n}{m c^2}$$  (33)

where the latter corresponds to the bubble diagram, we have:

$$
P_0^{0a} = P_0^{0b} = i q^a \frac{\omega_n}{q^2 c} P_{00}, \quad A_0 + B_0 = \frac{q^a q^b}{q^2} P_0^{ab} = -\frac{\omega_n^2}{c^2 q^2} P_{00}^{0a}$$  (34)

7. Single layer polarization

We begin by evaluating the polarization matrix $\Pi$ for a single isolated layer, which is the solution of the equation

$$
\Pi = \Pi_0 + \Pi_0 D \Pi
$$  (35)

Given the matrix structure of $D$, it turns out that the matrix $\Pi$ has the same structure as $\Pi_0$:

$$
\Pi = \begin{pmatrix}
P_{00} & i \frac{q^a \omega_n}{q^2 c} P_{00}^0 & i \frac{q^a \omega_n}{q^2 c} P_{00}^0 & 0 \\
i \frac{q^a \omega_n}{q^2 c} P_{00}^0 & A_0 + B_0 q^2 & \frac{q^a q^b}{q^2} B & 0 \\
i \frac{q^a \omega_n}{q^2 c} P_{00}^0 & \frac{q^a q^b}{q^2} B & A_0 + B_0 q^2 & 0 \\
0 & 0 & 0 & P_{33}^0
\end{pmatrix}
$$  (36)

From the equation (35) one easily derives the exact relations:

$$
P_{00} = \frac{q^2 c P_{00}^{00}}{q^2 c - 2 \pi P_{00}^{00} \sqrt{\omega_n^2 + q^2 c^2}}, \quad P_{33} = \frac{P_{33}^0}{1 - 2 \pi P_{00}^{00} \frac{q^2 c^2}{\sqrt{\omega_n^2 + q^2 c^2}} \left(1 - \frac{\omega_n^2}{q^2 c^2}ight)}$$  (37)

$$
(A + B) = -\frac{\omega_n^2}{q^2 c^2} P_{00}, \quad A = \frac{A_0}{1 + \frac{2 \pi c}{\sqrt{\omega_n^2 + q^2 c^2}} A_0}
$$  (38)

For $q = 0$ and in the limit of small frequency one obtains $A + B = |\omega_n|/2 \pi c$, which yields a finite d.c. conductivity $\sigma = c/2 \pi$. In the standard treatment of intra-layer conductivity [10] one considers independent electrons interacting with impurities. With low density of impurities the single bubble diagram with dressed propagators and vertex is considered, and it gives Drude’s formula $\sigma = e^2 n \tau / m$. 

8. Interlayer polarization: approximate solutions and discussion

The interlayer polarization is the solution of the equation

\[ \mathbf{P}(12) = \mathbf{Q} + \mathbf{QD}^0(21)\mathbf{P}(12) \]  

(39)

where the matrix \( \mathbf{Q} = \mathbf{PD}^0(12)\mathbf{P} \) is needed. Before solving Eq. (39), let us evaluate \( Q^{ab} \), which would give the lowest order approximation in the interlayer interaction to transconductivity.

\[ Q^{ab} = P(1)^{a0} D^0(12)_{00} P(2)^{0b} + P(1)^{ac} D^0(12)_{cd} P(2)^{db} \]  

(40)

where \( a, b = 1, 2 \). A way of improving this first order approximation is to include intralayer interaction at a higher level. We assume that \( D^0(\ell) \) are the exact polarization matrices for isolated layers, averaged over independent impurity distributions and, by using charge conservation for the polarizations we obtain

\[ Q^{ab} = -\tilde{D}(12)[A(1)A(2)(\delta^{ab} - \frac{q^aq^b}{q^2}) + \frac{q^aq^b}{q^2} P^{00}(1)P^{00}(2) \frac{\omega^2}{c^2q^4}(\omega^2_n + c^2q^2)] \]  

(41)

Here \( A(\ell) \) is the isotropic part of the polarization tensor \( P(\ell) \). The exact value of \( A(\ell) \) would provide the d.c. conductivity \( \sigma_\ell \) of the isolated layer. By taking the isotropic part of \( Q^{ab} \) we obtain

\[ \sigma_{12} = \frac{2\pi}{c} \frac{x}{\sigma_1 \sigma_2} \]  

(42)

One cannot perform a full treatment of disorder when higher orders of RPA (in the interlayer interaction) are considered. Dressing all bubbles with interactions poses the problem of cross-averaging. Moreover, before the average, one deals with a theory which is not invariant under space translations. The Eq. (39) can anyway be solved for the theory with no disorder. For \( q = 0 \), with the systematic use of charge conservation, we obtain

\[ A(12) + B(12) = \frac{1}{2\omega_n} \]

Some conclusions can also be driven in presence of disorder. Full RPA in the interlayer interaction can be calculated assuming disorder only inside the single layer polarization. The correction to the conductivity due to photon exchange between the two layers can be expressed in terms of the single layer conductivity:

\[ \sigma_{12} = \frac{c}{2\pi} \frac{x}{1 + x} \]  

(43)

where

\[ x = \sigma_1 \sigma_2 \frac{4\pi^2}{c^2} \]  

(44)

Finally the correction to transresistivity due to electromagnetic interaction is:

\[ \rho_{D,em} = \frac{c}{2\pi\sigma_1 \sigma_2} \]  

(45)

This preliminary result suggests that electromagnetic interaction is not an “irrelevant relativistic” correction: first order correction to trans-conductivity is different from zero in DC limit at zero temperature, though present level of approximation is not sufficient. The magnitude and dependence on parameters (like interwell distance for example) are not captured by the present calculation. A possible improvement can be probably achieved by treating impurities at a mesoscopic level without self-averaging [11]. This procedure
on the other hand breaks the translation-invariance and will result in a very difficult RPA summation. A non-vanishing first order contribution to transconductivity can not be straightforwardly understood in terms of semiclassical Boltzmann equation. Finally it is crucial for the finiteness of the transconductivity that the photon propagator diverges like $1/\omega$ for small frequencies. It is still unclear if this peculiar divergence survives in a “better” approximation. The key feature of this effect is the non-vanishing contribution to drag resistivity at zero temperature. Some evidence of this low temperature behaviour are also present in literature [12]. Even if it is still hard to give a precise indication on the magnitude of the effect, this seems to be its characteristic signature to be investigated also from the experimental point of view. Deviation from the quadratic temperature dependence at low temperatures are expected, especially for devices with a high purity and a geometry that is as close as possible to the infinite layers approximation (distance between the 2DEGS’s much smaller than lateral dimension of the sample).

References

[10] G D Mahan, Many-Particle Physics Plenum Press, New York (1990), cited respectively, Sec. 2.10, Sec.5.5, Chap.7.