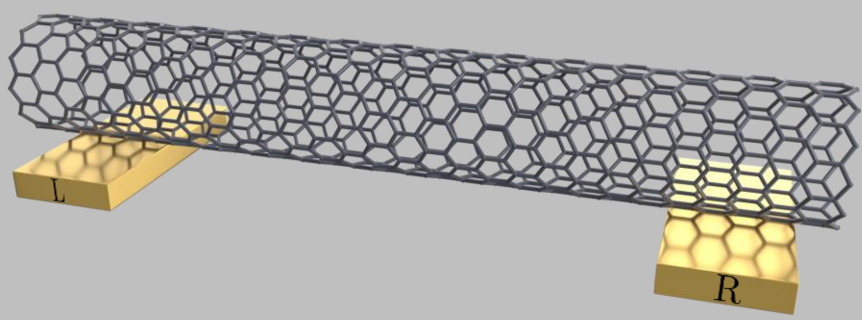
Dark states in carbon nanotubes and molecular quantum dots



arXiv:1804.02234 (2018)

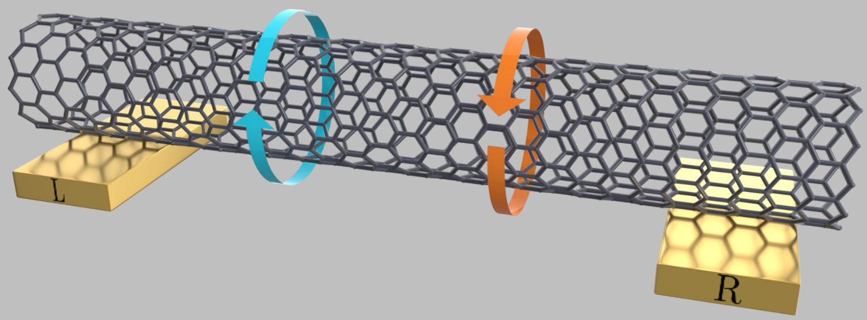
Phys. Rev. B **95**, 115133 (2017)

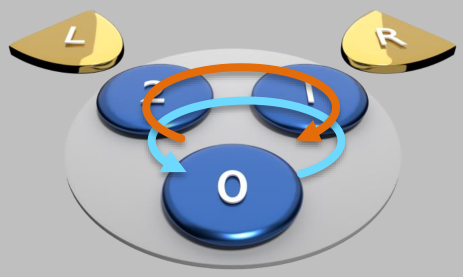
Andrea Donarini

San Sebastian
17.07.2018

University of Regensburg, Germany

Dark states in carbon nanotubes and molecular quantum dots





arXiv:1804.02234 (2018)

Phys. Rev. B **95**, 115133 (2017)

Andrea Donarini

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San Sebastian
17.07.2018

Dark states in optics

G. Alzetta et al. *Nuovo Cimento*. **36**, 5 (1976),
E. Arimondo and G. Orriolis, *Lettere al Nuovo Cimento*, **17**, 33, (1976)

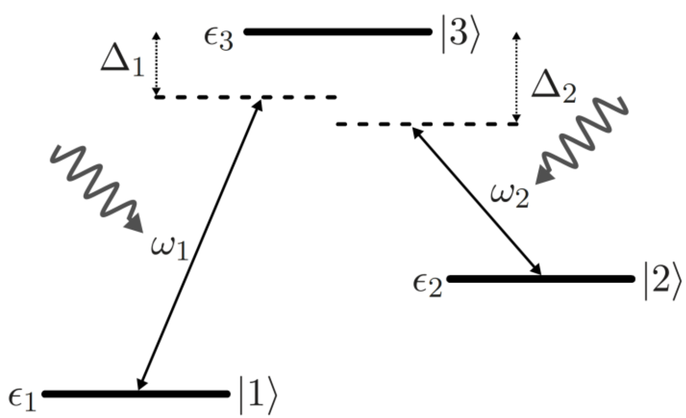
Nonabsorbing Atomic Coherences by Coherent Two-Photon Transitions in a Three-Level Optical Pumping.

E. Arimondo

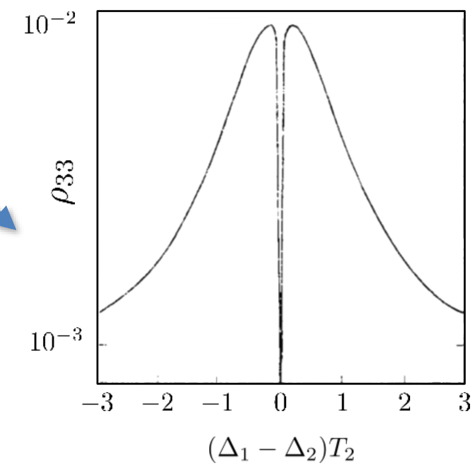
Istituto di Fisica dell'Università - Pisa

G. Orriols

Laboratorio di Fisica Atomica e Molecolare del C.N.R. - Pisa
Departament d'Optica, Universitat de Barcelona - Barcelona

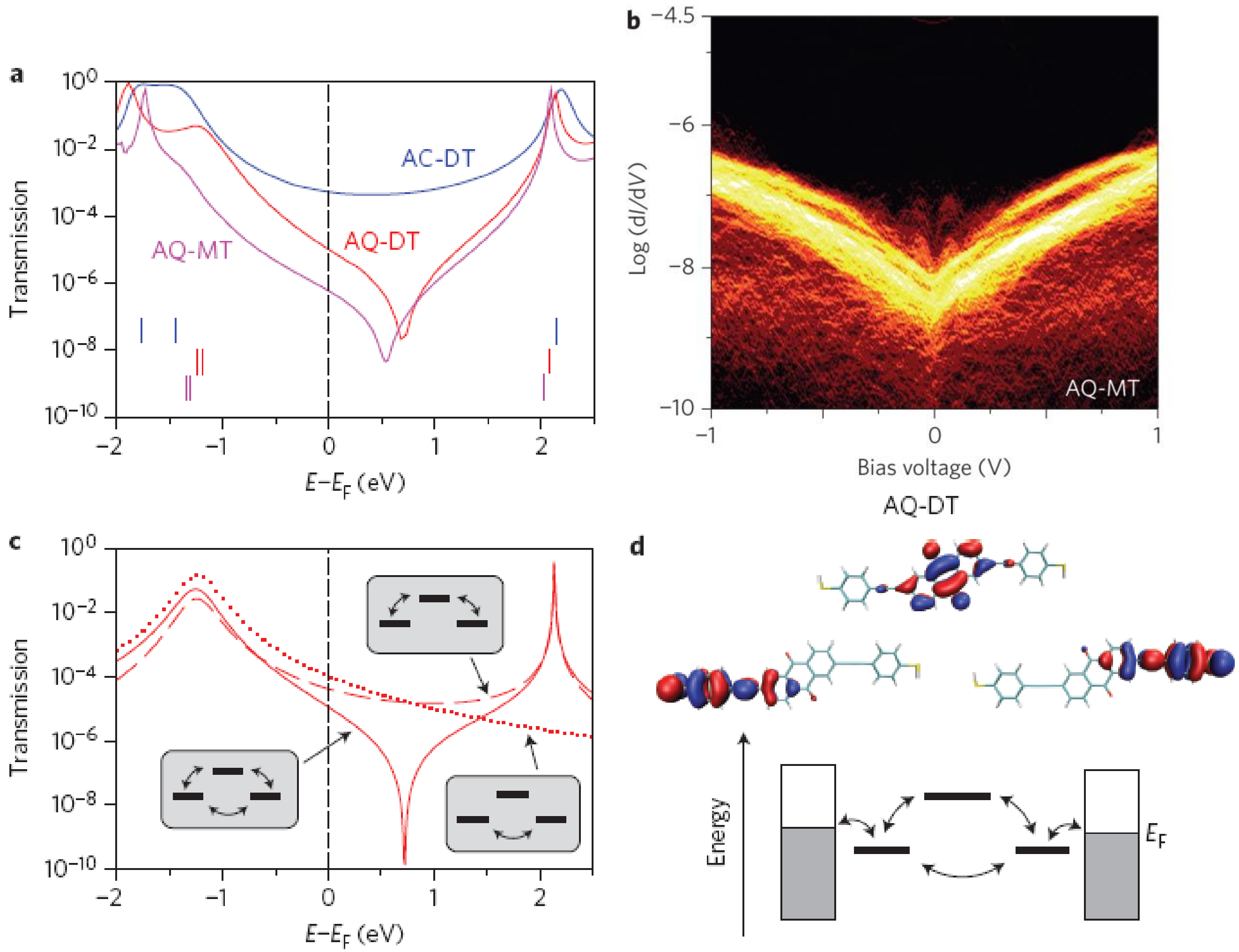


$|\mathrm{DS}\rangle$



Molecular interference

C. M. Guédon, H. Valkenier et al. *Nature Nanotech*. **7**, 305 (2012)



- Zero bias conductance
- Strong coupling to the leads → coherent (single-particle) transport

PRL **104,** 186804 (2010) PHYSICAL REVIEW LETTERS week ending 7 MAY 2010

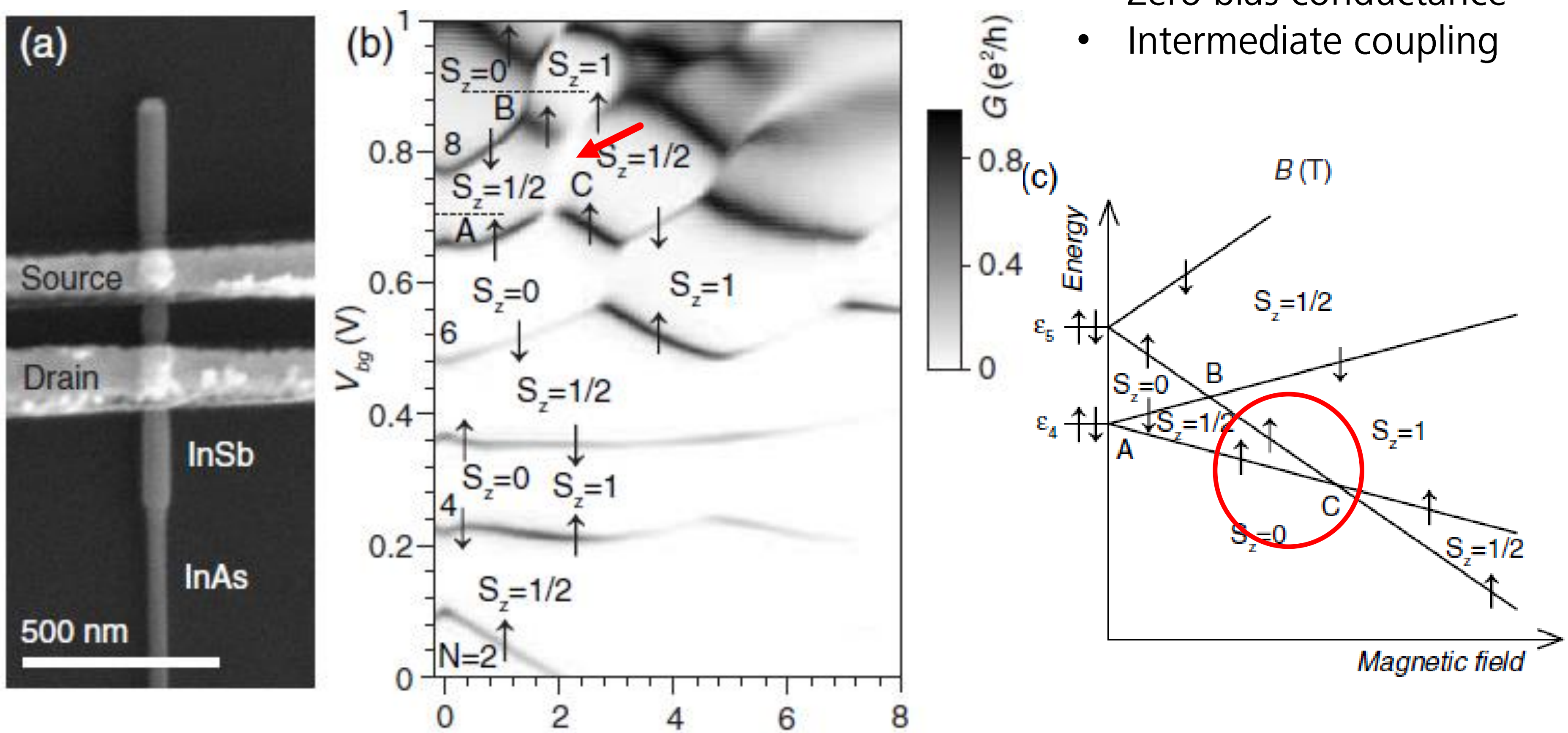
Correlation-Induced Conductance Suppression at Level Degeneracy in a Quantum Dot

H. A. Nilsson, O. Karlström, M. Larsson, P. Caroff, J. N. Pedersen, L. Samuelson, A. Wacker, L.-E. Wernersson, and H. Q. Xu*

Nanometer Structure Consortium, Lund University, Box 118, 221 00 Lund, Sweden

(Received 10 November 2009; published 4 May 2010)

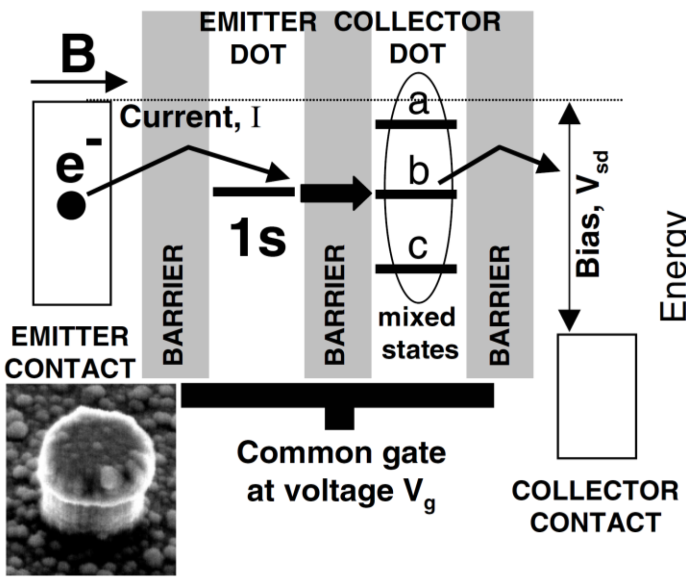
- Zero bias conductance
- Intermediate coupling

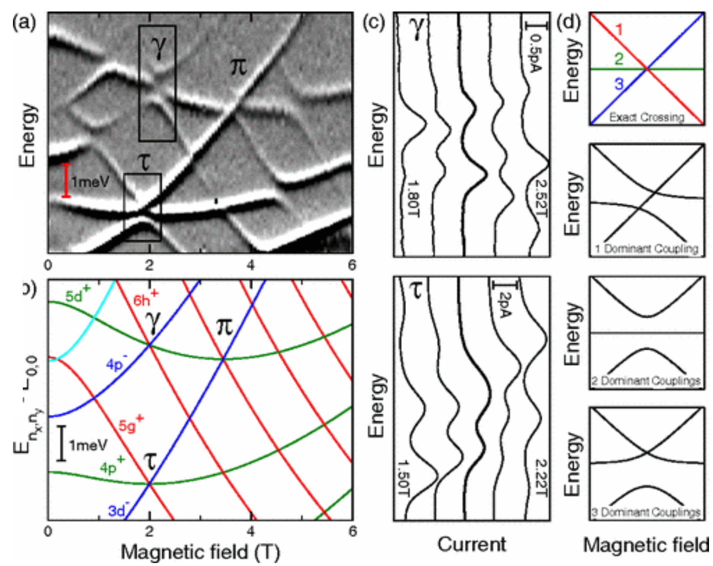


PRL 102, 026808 (2009) PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2009

Coherent Three-Level Mixing in an Electronic Quantum Dot

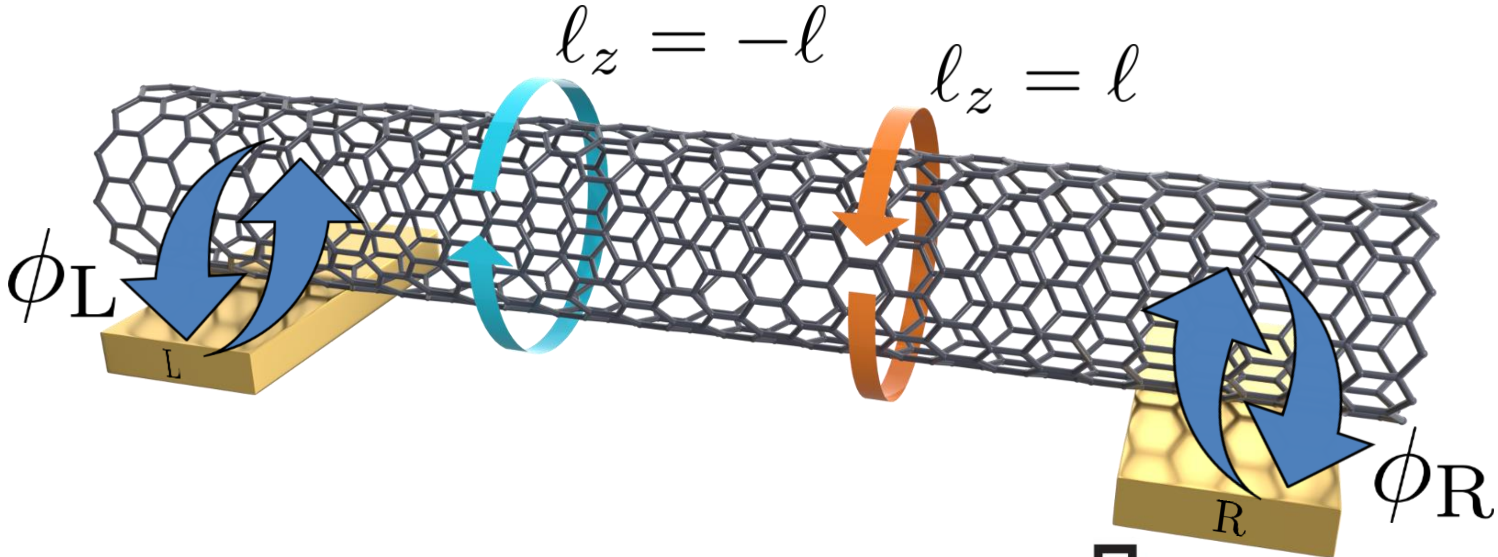
C. Payette,[1,2] G. Yu,[1] J. A. Gupta,[1] D. G. Austing,[1,2,*] S. V. Nair,[3] B. Partoens,[4] S. Amaha,[5] and S. Tarucha[5,6]
[1]Institute for Microstructural Sciences M50, NRC, Montreal Road, Ottawa, Ontario, K1A 0R6, Canada
[2]Department of Physics, McGill University, 3600 rue University, Montréal, Quebec, H3A 2T8, Canada
[3]Center for Advanced Nanotechnology, University of Toronto, 170 College Street, Toronto, Ontario, M5S 3E3, Canada
[4]Departement Fysica, Universiteit Antwerpen, Groenenborgerlaan 171, B-2020 Antwerpen, Belgium
[5]Quantum Spin Information Project, ICORP, JST, Atsugi-shi, Kanagawa 243-0198, Japan
[6]Department of Applied Physics, University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-8656, Japan
(Received 15 August 2008; published 16 January 2009)

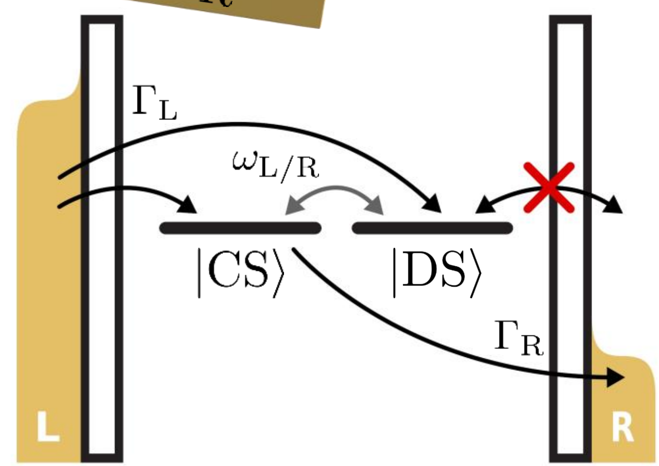




- Symmetry breaking perturbation **within** the system → Dark States

All electrical dark states



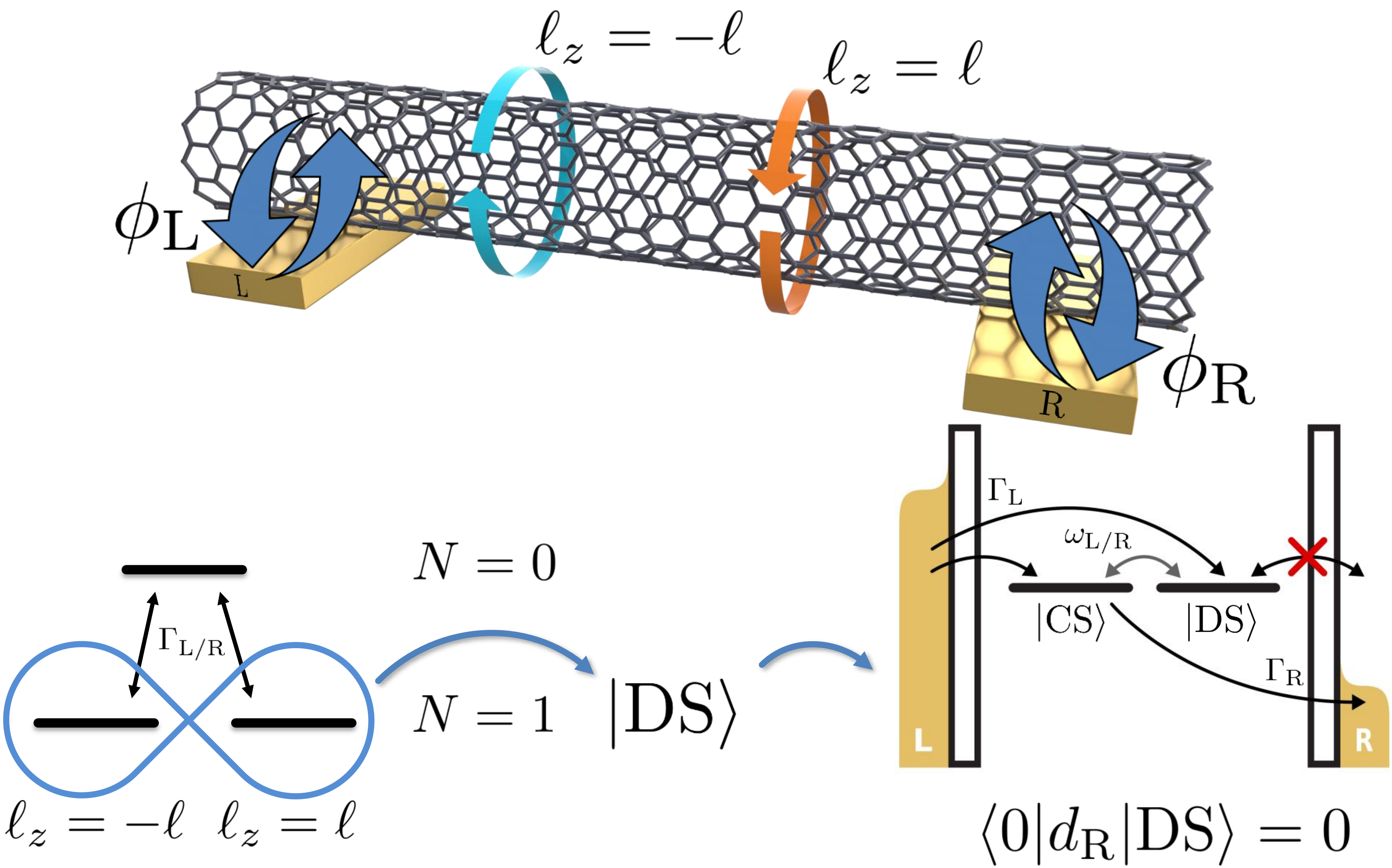


$$\langle 0|d_R|DS\rangle = 0$$

Ingredients:

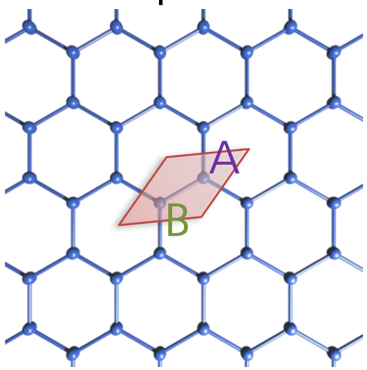
- Orbital (quasi)-degeneracy
- Weak coupling to the leads → charging effects
- Phase coherent tunnelling dynamics
- Finite bias → directed transport

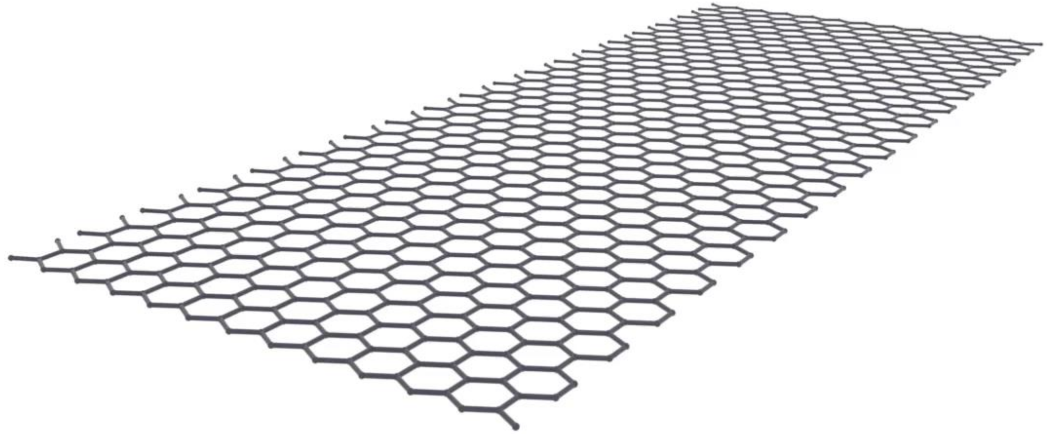
All electrical dark states

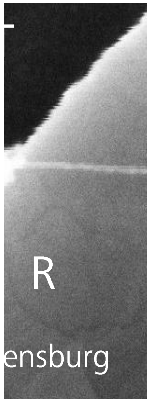


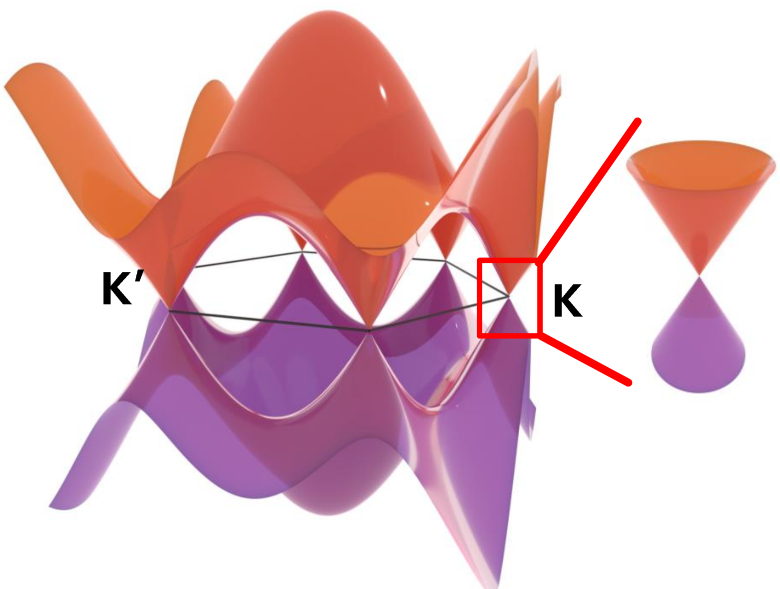
CNT quantum dots

Graphene



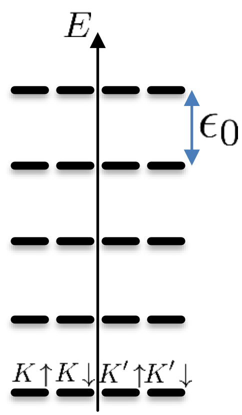






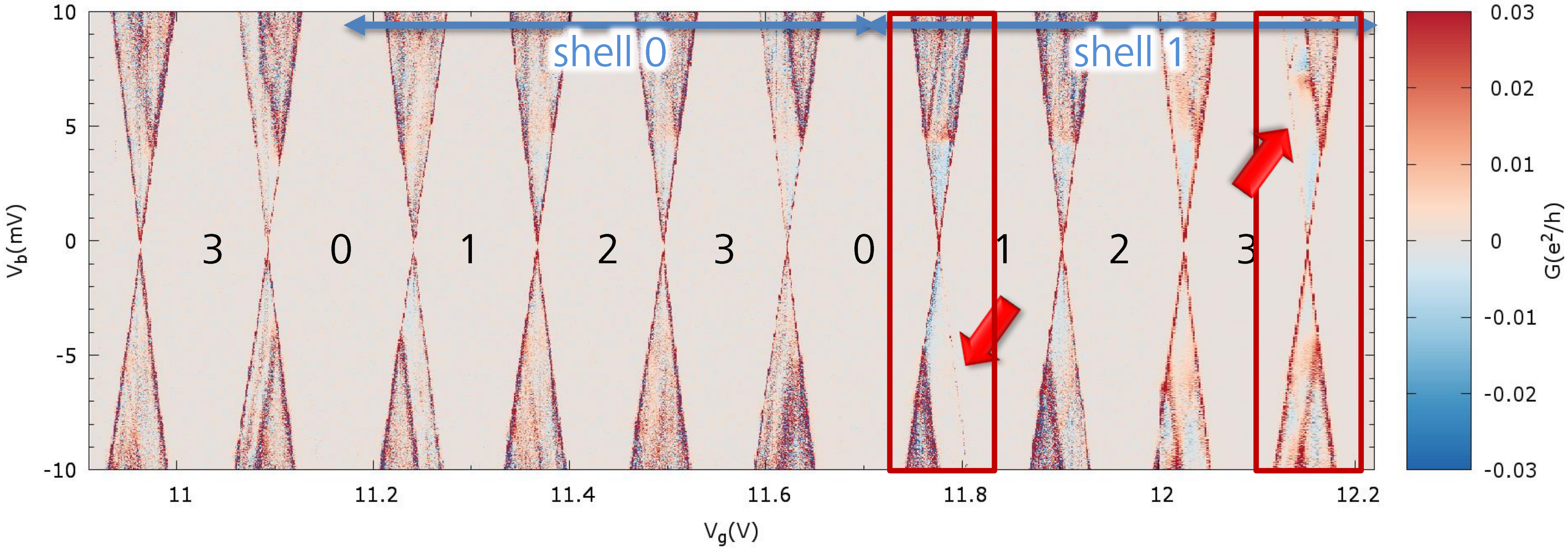
Transverse + Longitudinal quantization

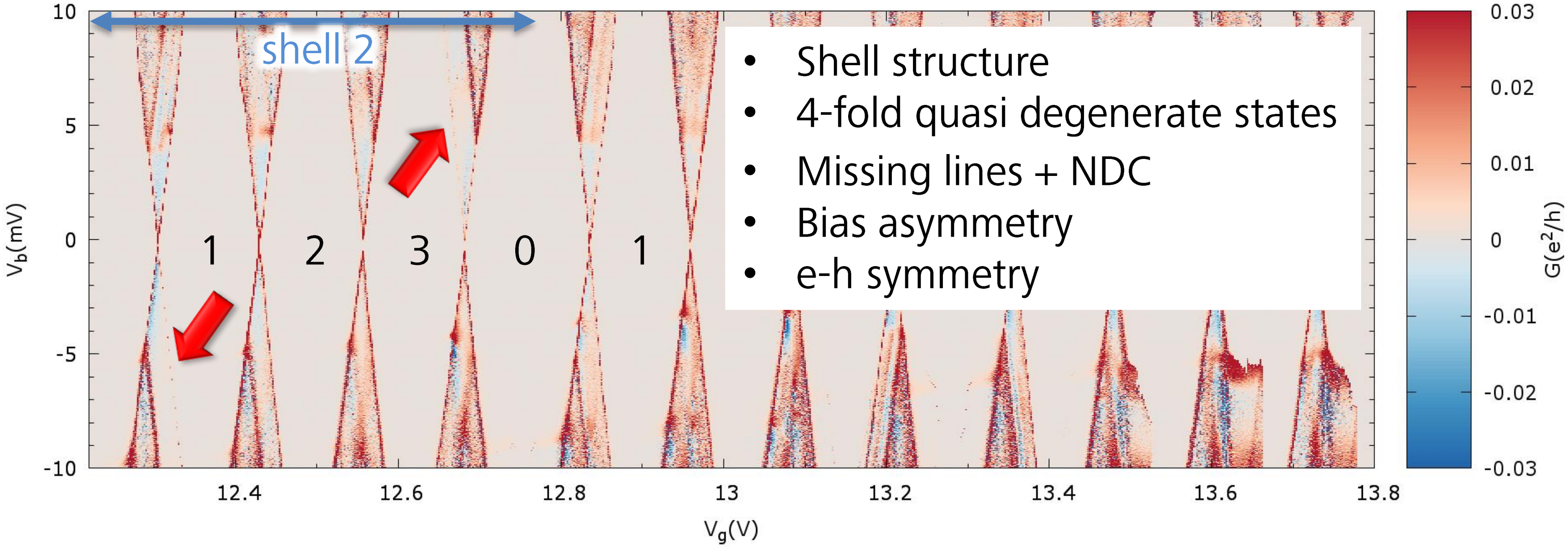
Quantum dot:



- Shell structure
- 4-fold degenerate:
  - 2x spin
  - 2x angular momentum (valley)

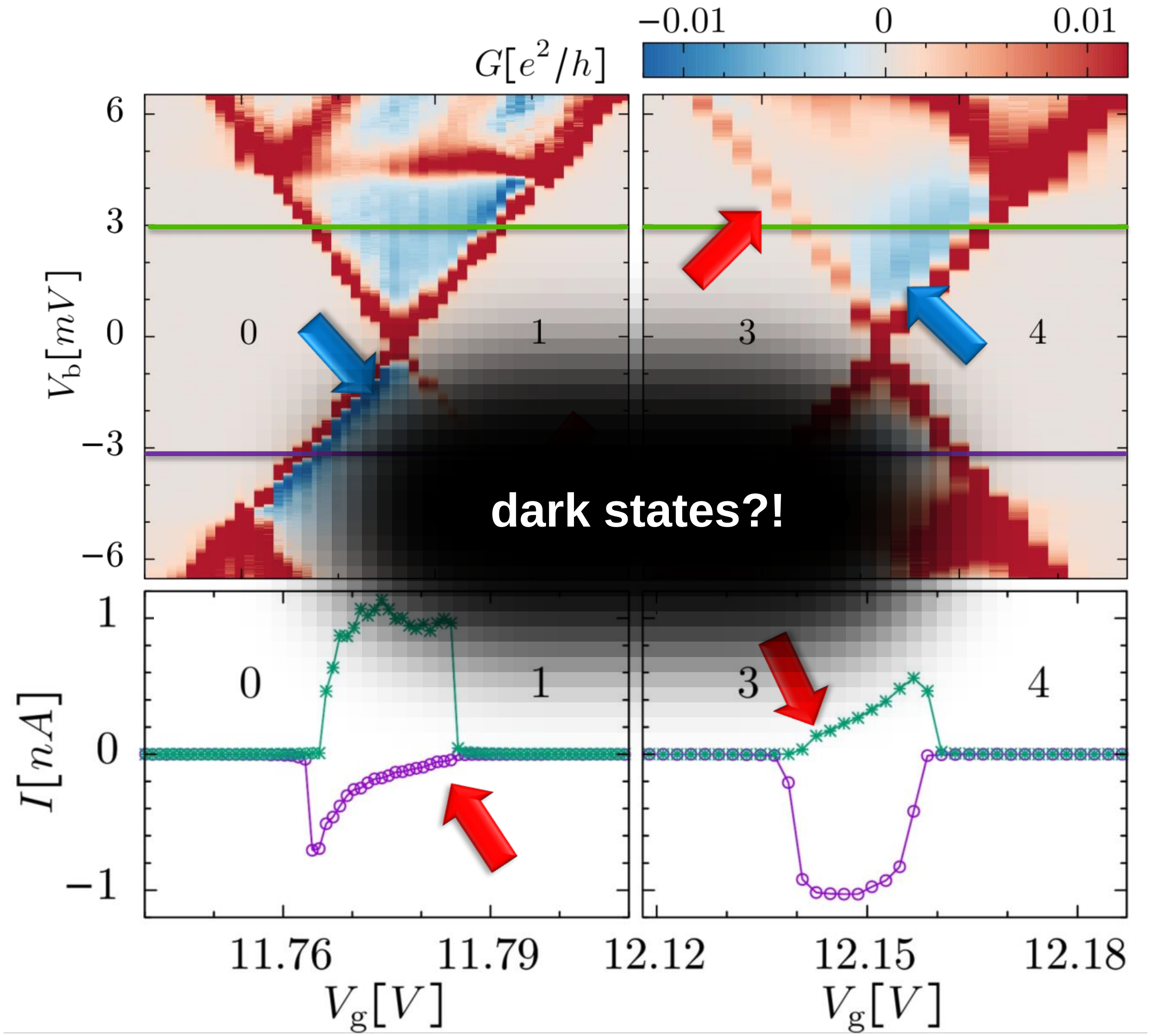
Stability diagram



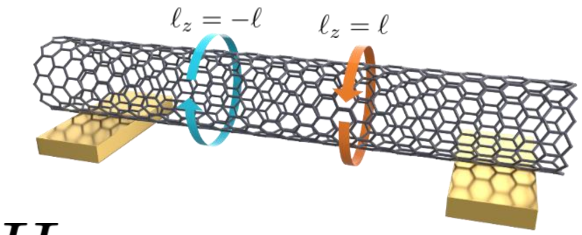


- Shell structure
- 4-fold quasi degenerate states
- Missing lines + NDC
- Bias asymmetry
- e-h symmetry

Dark states in transport



Total Hamiltonian



$$H = H_{\mathrm{CNT}} + H_{\mathrm{leads}} + H_{\mathrm{tun}}$$

$$H_{\mathrm{CNT}} = \sum_{m\ell_z} (m\epsilon_0 - \xi)\, \hat{n}_{m\ell_z} + \frac{U}{2}\hat{N}^2 + J \sum_m \left(\hat{\mathbf{S}}_{m\ell} \cdot \hat{\mathbf{S}}_{m-\ell} + \frac{1}{4}\hat{n}_{m\ell}\hat{n}_{m-\ell} \right)$$

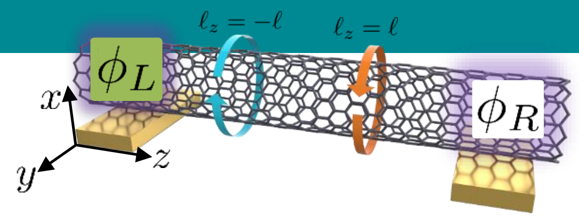
- $m\epsilon_0$: Level spacing
- ξ: V_{g}
- $\frac{U}{2}\hat{N}^2$: Constant interaction
- J term: Exchange interaction for zig-zag class CNTs

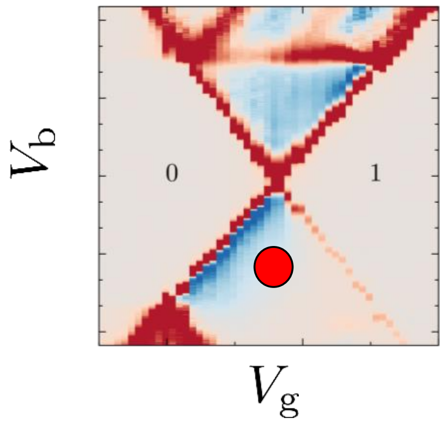
$$H_{\mathrm{leads}} = \sum_{\alpha\mathbf{k}\sigma} \varepsilon_{\mathbf{k}} c^{\dagger}_{\alpha\mathbf{k}\sigma} c_{\alpha\mathbf{k}\sigma}$$

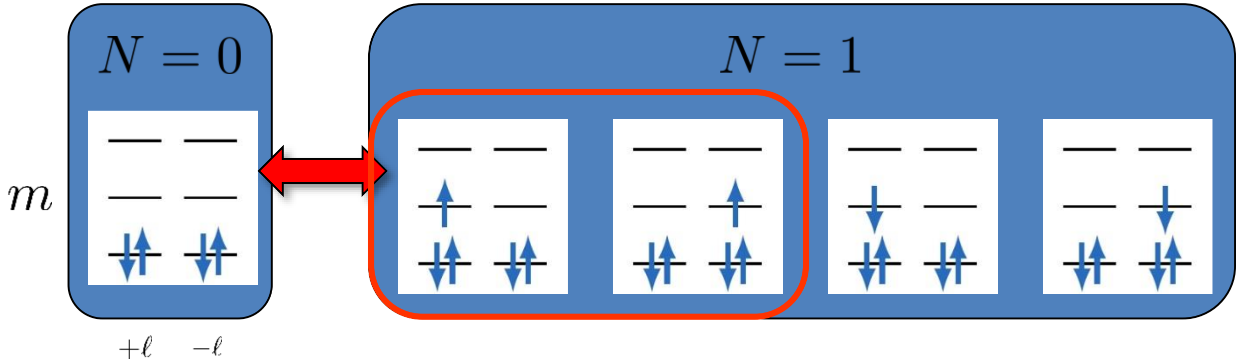
$$H_{\mathrm{tun}} = \sum_{\alpha\mathbf{k}m\ell_z\sigma} t_{\alpha\mathbf{k}m\ell_z} d^{\dagger}_{m\ell_z\sigma} c_{\alpha\mathbf{k}\sigma} + \mathrm{h.c.}$$

Includes the geometry of the contacts.
Is treated perturbatively

One particle dark state







$$\Gamma^{\alpha}_{\ell_z \ell'_z}(E_1 - E_0) = \frac{2\pi}{\hbar} \sum_{\mathbf{k}} (t_{\alpha \mathbf{k} \ell_z})^* t_{\alpha \mathbf{k} \ell'_z} \delta(\epsilon_{\mathbf{k}} - E_1 - E_0)$$

$$\mathbf{\Gamma}^{\alpha} = \Gamma^{\alpha} \underbrace{\begin{pmatrix} 1 & a e^{2i\ell\phi_\alpha} \\ a e^{-2i\ell\phi_\alpha} & 1 \end{pmatrix}}$$

$\mathcal{R}_l$: coherence matrix

Atomically localized tunneling
or
Surface Γ-point approximation
(i.e. $k_x \approx k_F \quad k_y, \, k_z = 0$)

$$a = 1$$

Tunneling rate matrix

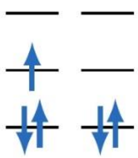
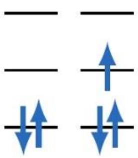
$$\mathcal{R}_{\mathrm{R}} = \begin{pmatrix} 1 & e^{i2\phi_{\mathrm{R}}} \\ e^{-i2\phi_{\mathrm{R}}} & 1 \end{pmatrix} \quad \xrightarrow{\text{diagonalize}} \quad \mathcal{R}_{\mathrm{R}} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$$

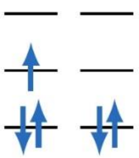
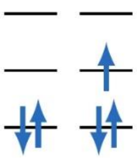
$$\mathcal{R}_{\mathrm{L}} = \begin{pmatrix} 1 & e^{i2\phi_{\mathrm{L}}} \\ e^{-i2\phi_{\mathrm{L}}} & 1 \end{pmatrix} \quad \longrightarrow \quad \mathcal{R}_{\mathrm{L}} = \begin{pmatrix} 1-\cos\Delta\phi & -i\sin\Delta\phi \\ i\sin\Delta\phi & 1+\cos\Delta\phi \end{pmatrix}$$

$$\Delta\phi = \phi_{\mathrm{L}} - \phi_{\mathrm{R}}$$

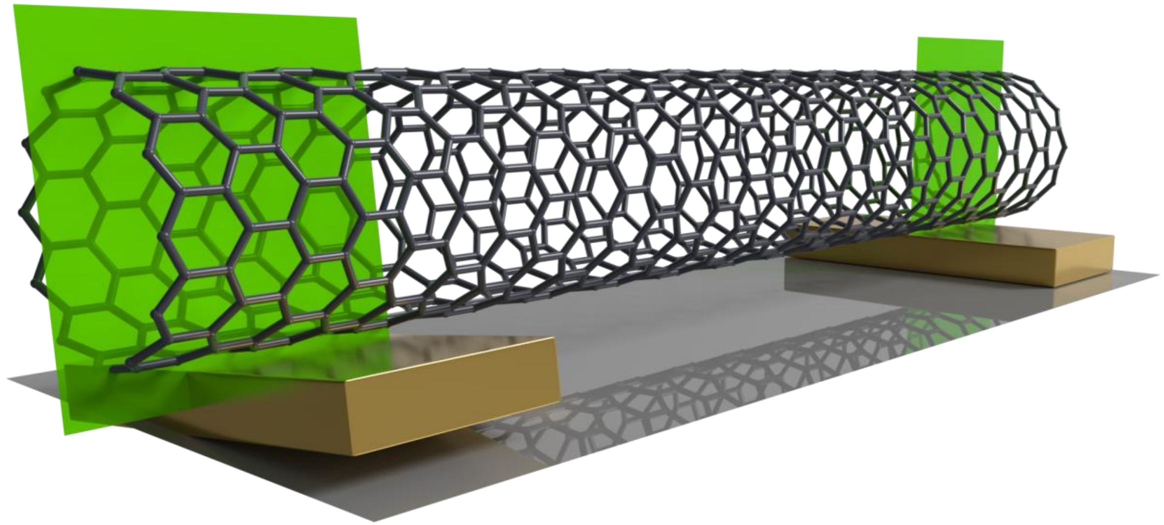
Cannot be diagonalized at the same time if $\Delta\phi \neq n\pi$

The dark state is defined with respect to a **specific** lead

$$|DS,\uparrow\alpha\rangle = \frac{1}{\sqrt{2}}\left(e^{i\ell\phi_\alpha}\right.$$

$$- \, e^{-i\ell\phi_\alpha}$$

$$\left.\right)$$

$$|CS,\uparrow\alpha\rangle = \frac{1}{\sqrt{2}}\left(e^{i\ell\phi_\alpha}\right.$$

$$+ \, e^{-i\ell\phi_\alpha}$$

$$\left.\right)$$

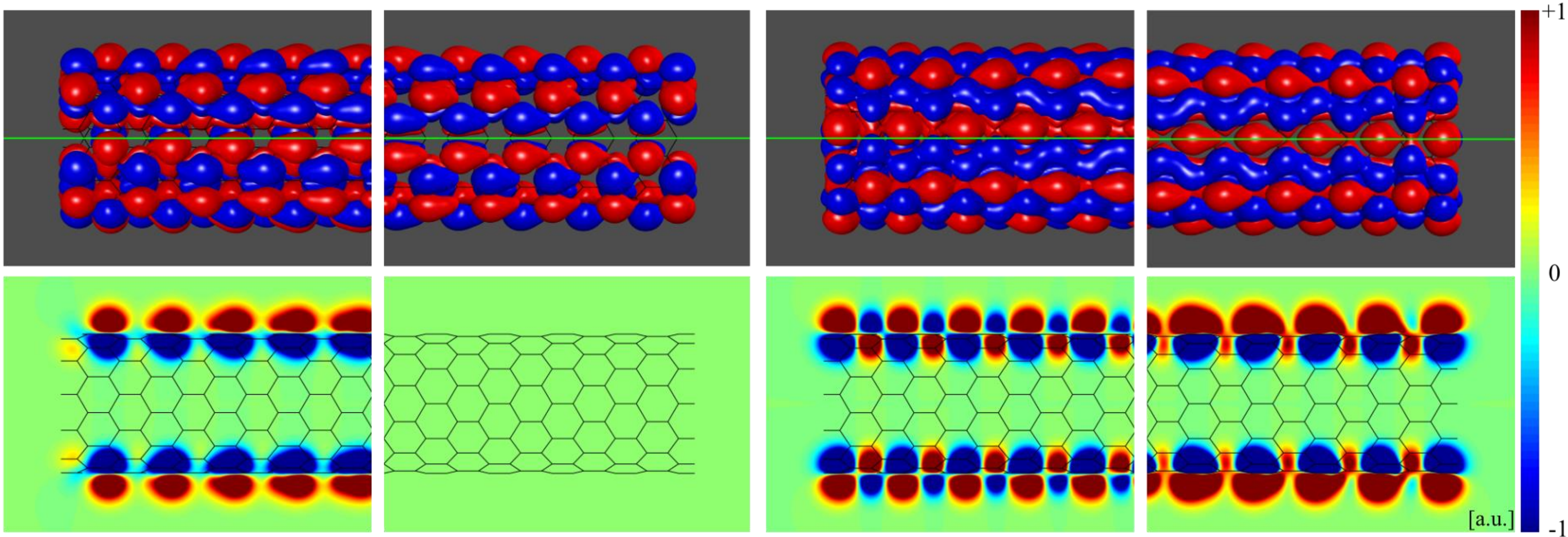
Dark states in CNTs



(12,0) CNT

Dark state

Coupled state



Quantum transport

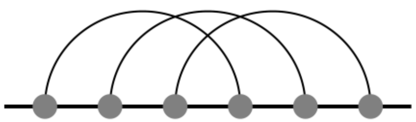
Density matrix approach

Weak coupling

Increase coupling

Strong coupling

$\mathcal{O}(\Gamma) \quad \cdots \quad \mathcal{O}(\Gamma^2) \quad \cdots$



diagrammatics

Current

$\langle \dot{N} \rangle$

Statistics

Counting field: χ

Full counting statistics

Current cumulants

$$\dot{\rho} = \mathcal{L}\rho$$

Single electron transistor

Quantum transport

Master equation:

$$0 = \mathcal{L}\rho^{\infty} = -\frac{i}{\hbar}\left[\hat{H}_{\mathrm{CNT}} + \hat{H}_{\mathrm{LS}}, \rho^{\infty}\right] + \mathcal{L}_{\mathrm{tun}}\rho^{\infty} + \mathcal{L}_{\mathrm{rel}}\rho^{\infty}$$

Reduced density matrix — Lamb shift — tunneling — relaxation

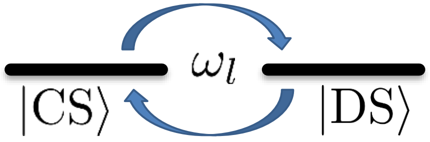
Minimal model for $N = 0 \leftrightarrow 1$ (no extrinsic relaxation):

$$0 = \dot{\rho}_1 = -\frac{i}{\hbar}\left[\hat{H}_{\mathrm{LS}}, \rho_1\right] + 2\Gamma_{\mathrm{L}}\mathcal{R}_{\mathrm{L}}\rho_0 - \frac{\Gamma_{\mathrm{R}}}{2}\left\{\mathcal{R}_{\mathrm{R}}, \rho_1\right\}$$

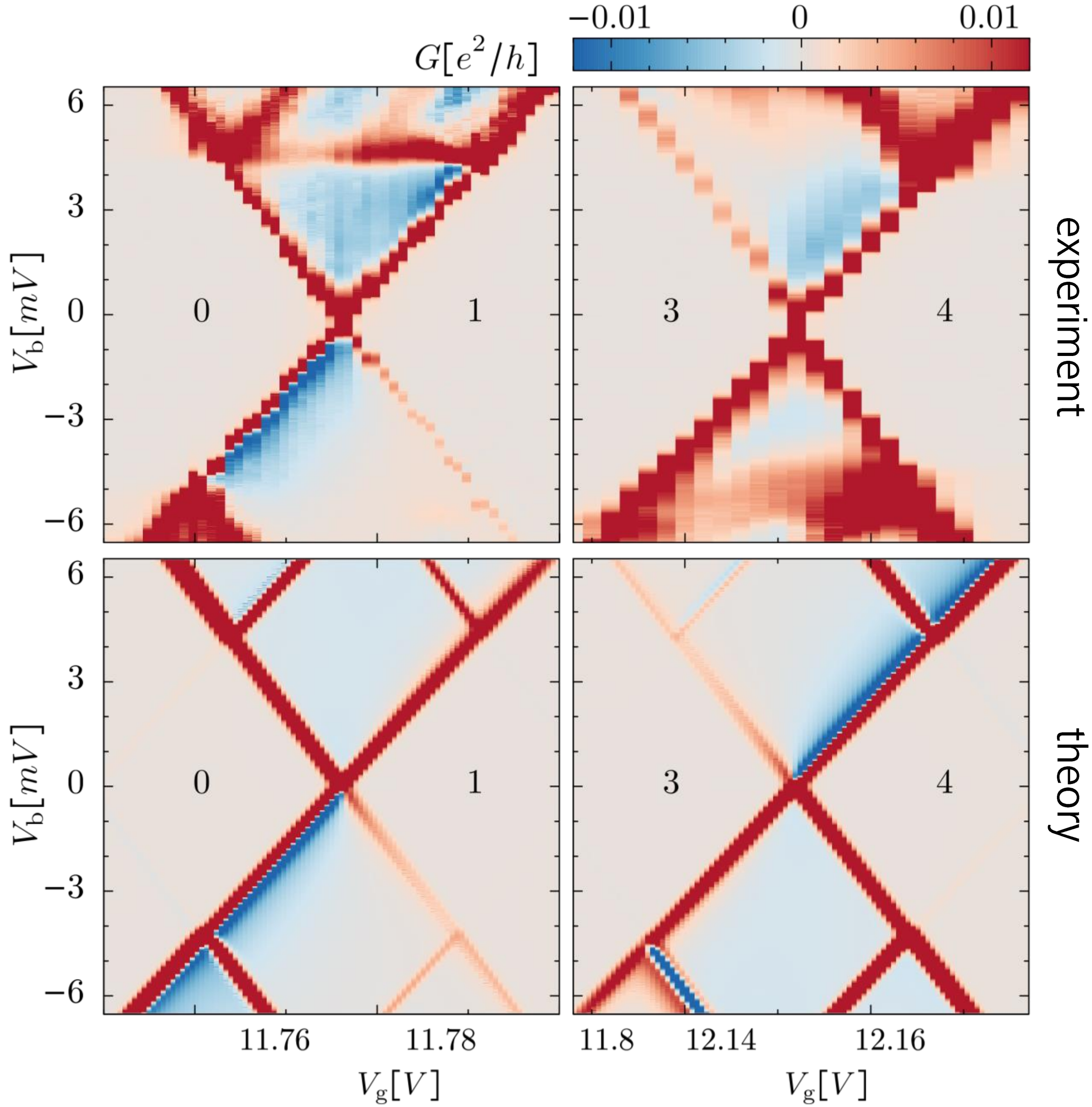
$$0 = \dot{\rho}_0 = \Gamma_{\mathrm{R}}\mathrm{tr}\left\{\mathcal{R}_{\mathrm{R}}\rho_1\right\} - 4\Gamma_{\mathrm{L}}\rho_0$$

Lamb shift:

$$\hat{H}_{\mathrm{LS}} = \frac{\hbar}{2}\sum_{l}\omega_l\mathcal{R}_l$$

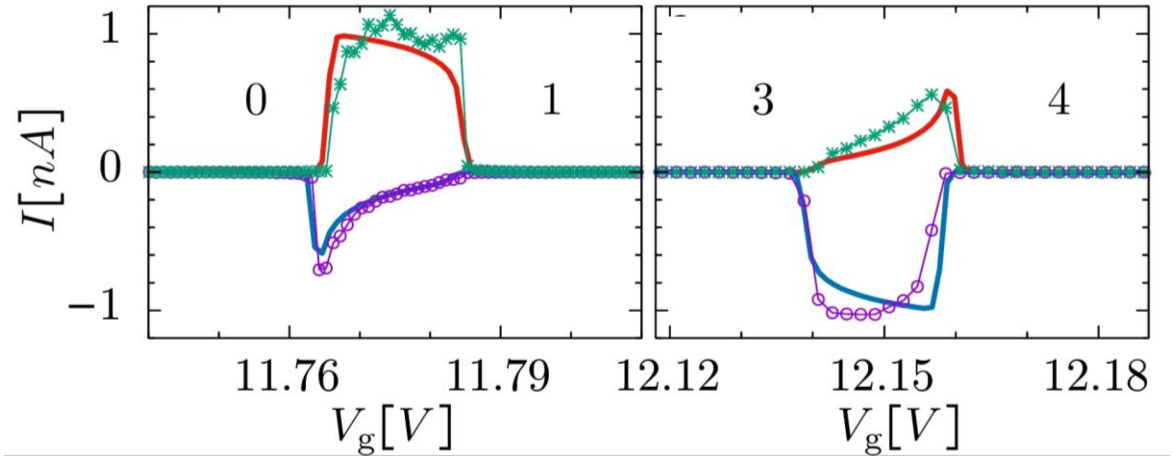


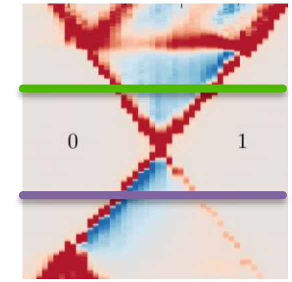
Compare to experiment



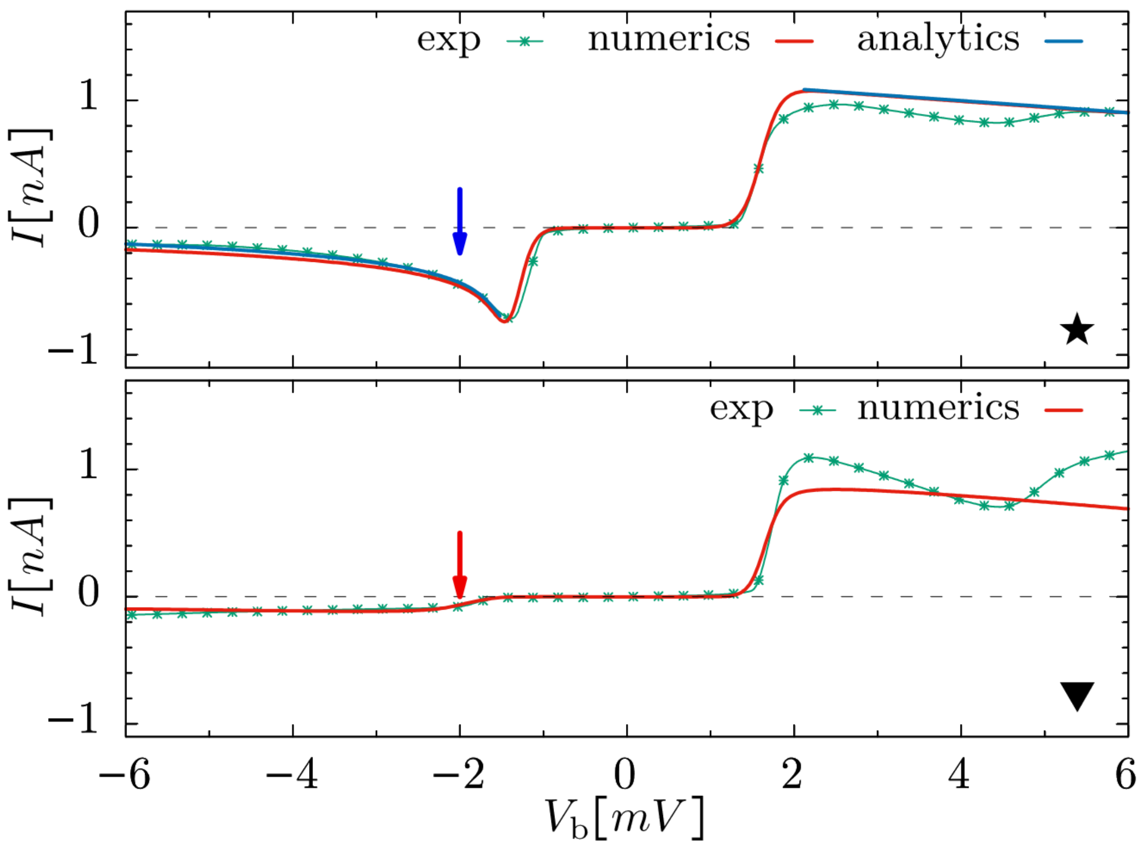
Compare to experiment

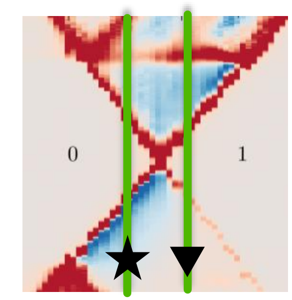
Gate traces:



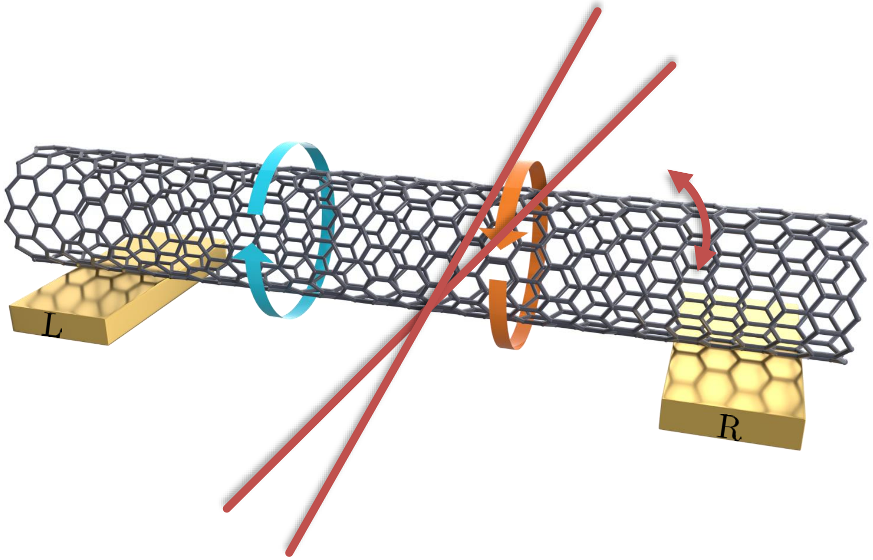


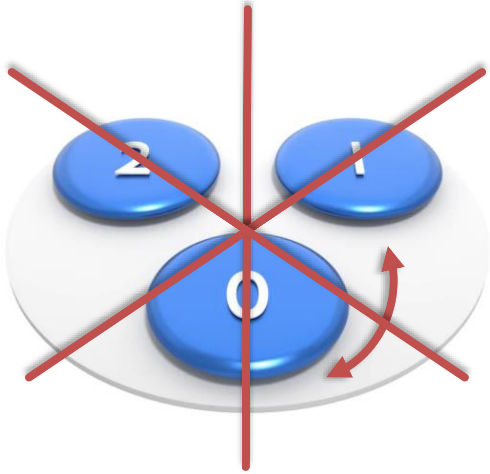
Bias traces:





Dark states in a TQD





D_n - symmetry

(rotations by $\frac{2\pi}{n}$ and dihedral rotations $\mathrm{C}_2', \mathrm{C}_2''$)

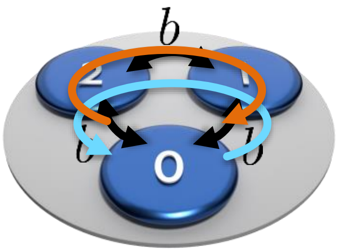
C_{3v} - symmetry

(rotations by $\frac{2\pi}{3}$ and mirror planes σ_v)

1. Angular momentum is a good quantum number
2. Angular momentum degeneracies

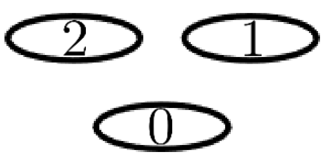
- Study of the interactions
- Influence on statistics

TQD Hamiltonian

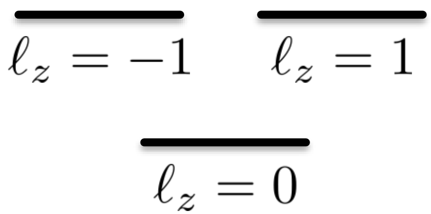


$$\hat{H}_{\mathrm{TQD}} = \alpha_{\mathrm{g}} V_{\mathrm{g}} \sum_{i,\sigma} \hat{n}_{i\sigma} + b \sum_{i \neq j,\sigma} d^{\dagger}_{i\sigma} d_{j\sigma}$$

Single particle part is diagonal in angular momentum basis:

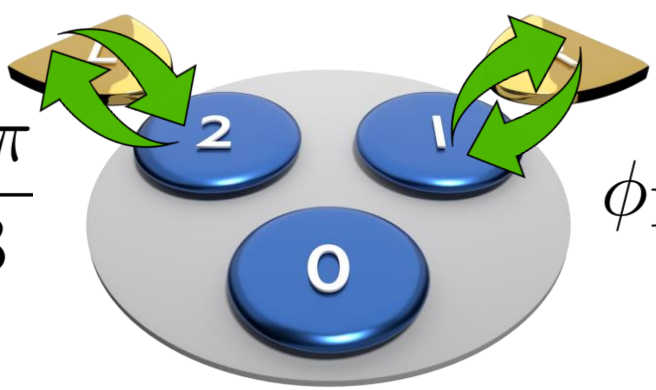


$$|\ell_z\rangle = \frac{1}{\sqrt{3}} \sum_j e^{-ij\ell_z 2\pi/3} |j\rangle$$



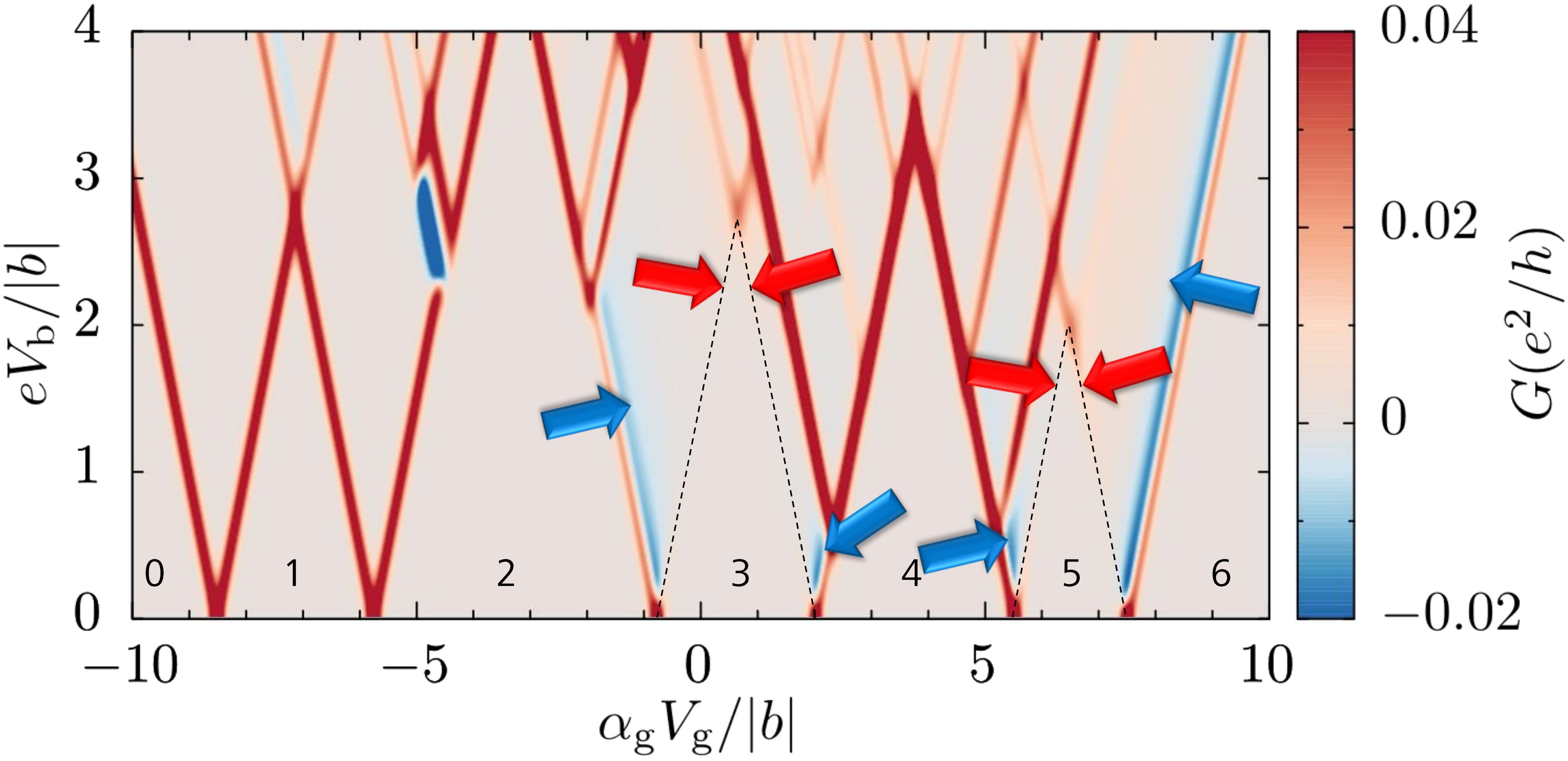
Tunneling phase:

$$\phi_{\mathrm{L}} = -\frac{2\pi}{3}$$



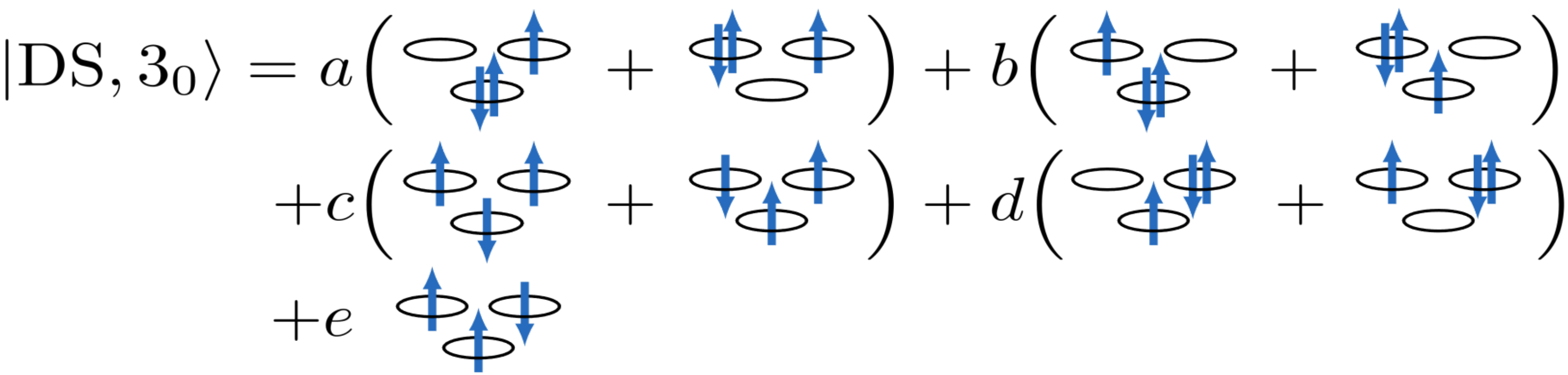
$$\phi_{\mathrm{R}} = \frac{2\pi}{3}$$

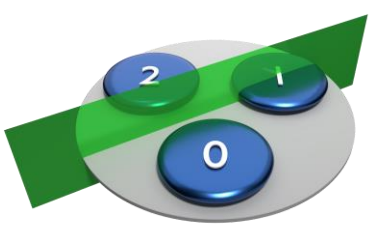
Stability diagram



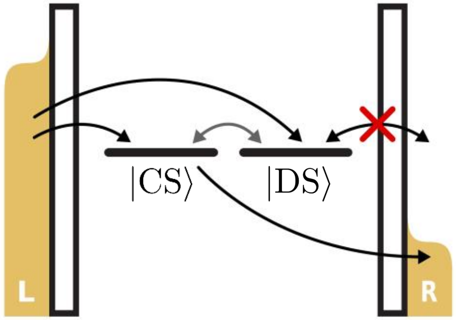
Dark states in a TQD

$$|\mathrm{DS}\rangle = \frac{1}{\sqrt{2}}\left(e^{i2\pi/3}|L_z = 1\rangle - e^{-i2\pi/3}|L_z = -1\rangle\right)$$

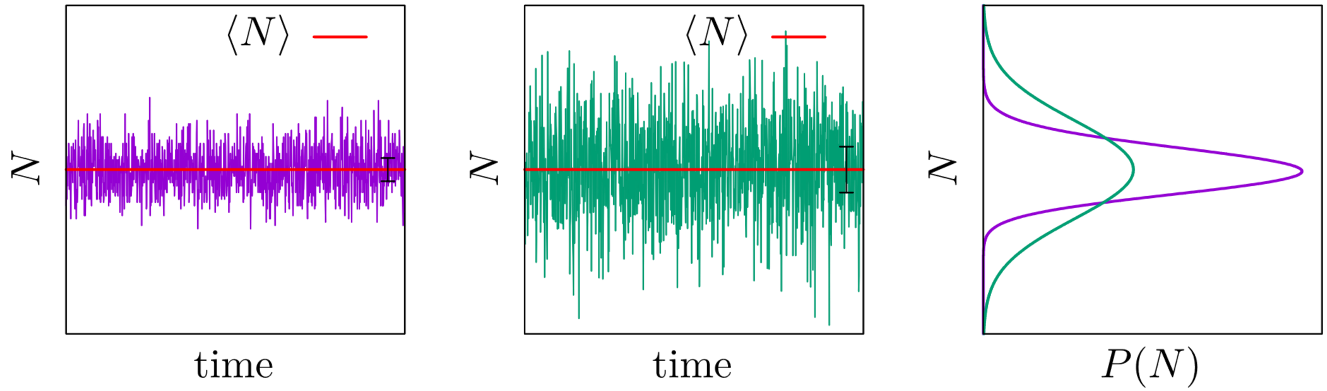




$$\langle 2_0|\, d_{\mathrm{R}\sigma}\, |\mathrm{DS}, 3_0\rangle = 0$$



Shot noise



$$I = e\,\langle \dot{N} \rangle$$

$$S = e^2 \frac{\mathrm{d}}{\mathrm{d}t}\left(\langle N^2 \rangle - \langle N \rangle^2\right)$$

Fano factor $$F = \frac{S}{|eI|}$$

Anti-bunching

$F < 1$

Sub-Poissonian

Uncorrelated events

$F = 1$

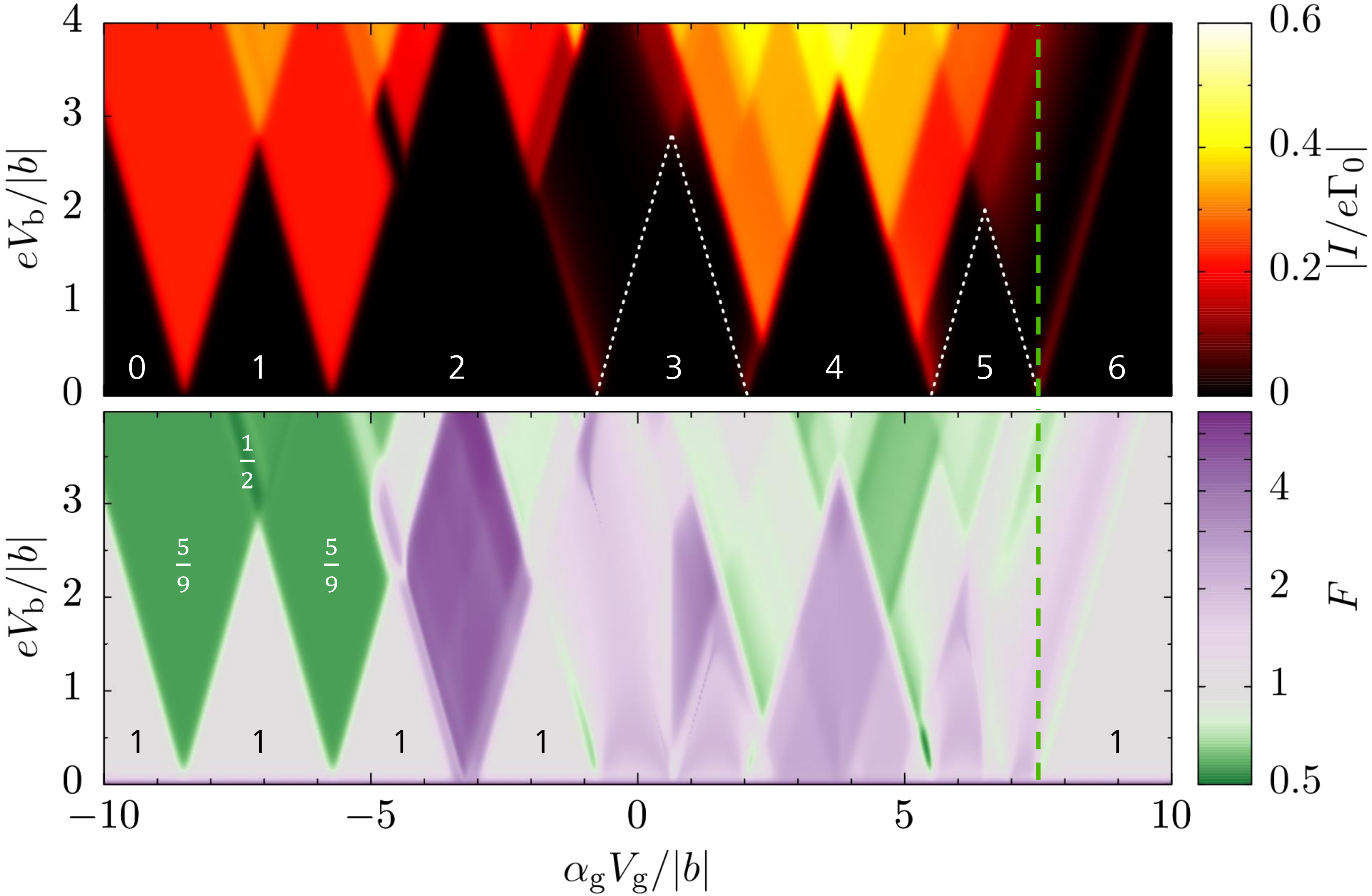
Poissonian

Bunching

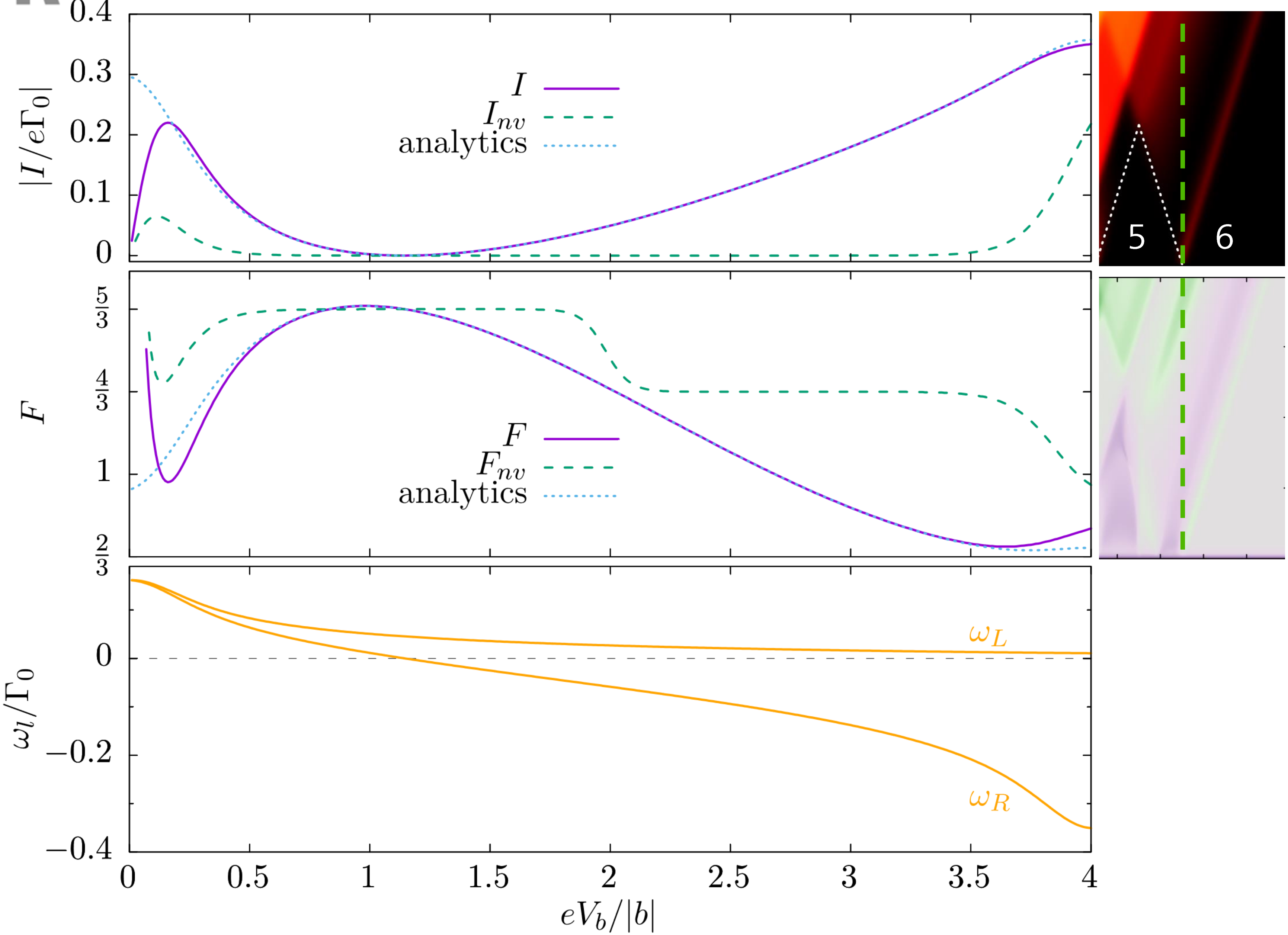
$F > 1$

Super-Poissonian

Fano map



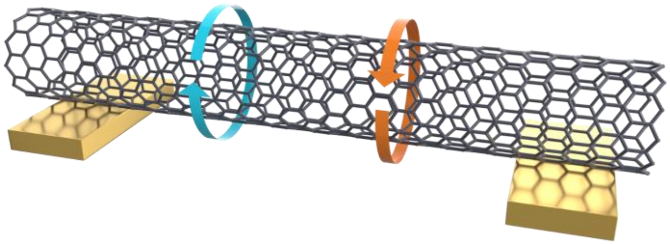
Bias traces



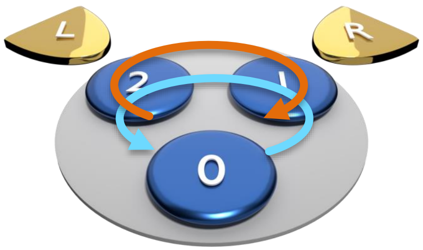
0.4
0.3
0.2
0.1
0
$|I/e\Gamma_0|$
I
I_{nv}
analytics
5
6
$\frac{5}{3}$
$\frac{4}{3}$
1
$\frac{2}{3}$
F
F_{nv}
analytics
0
−0.2
−0.4
ω_l/Γ_0
ω_L
ω_R
0
0.5
1
1.5
2
2.5
3
3.5
4
$eV_b/|b|$

Conclusions

- Orbital degeneracy and lead dependent tunneling phase ϕ_l:



$\ell_z = -\ell$ $\quad$ $\ell_z = \ell$



Dark states

E.g. the simple dark states

$$|\mathrm{DS}\rangle = \frac{1}{\sqrt{2}}\left(e^{i\phi_l}|L_z = \ell\rangle - e^{-i\phi_l}|L_z = -\ell\rangle\right)$$

- Dark state creates bottleneck process: super-Poissonian noise

A. Trottmann

M. Niklas

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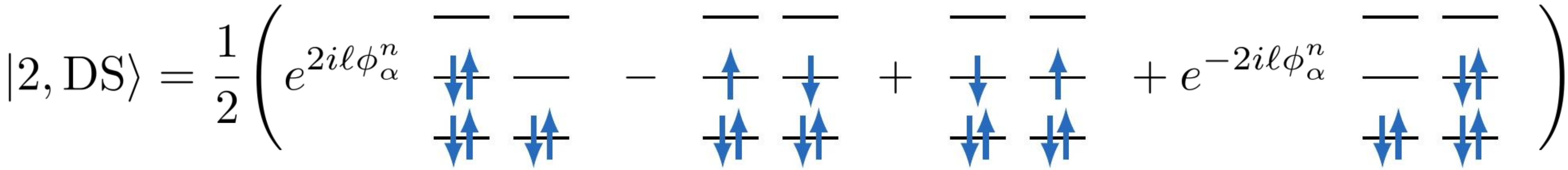
Exchange affects CNT DSs

$$\hat{H}_{\mathrm{J}} = J \sum_m \left(\hat{\mathbf{S}}_{m\ell} \cdot \hat{\mathbf{S}}_{m-\ell} + \frac{1}{4} \hat{n}_{m\ell} \hat{n}_{m-\ell} \right)$$

Without J: 2-particle groundstate is 6-fold degenerate

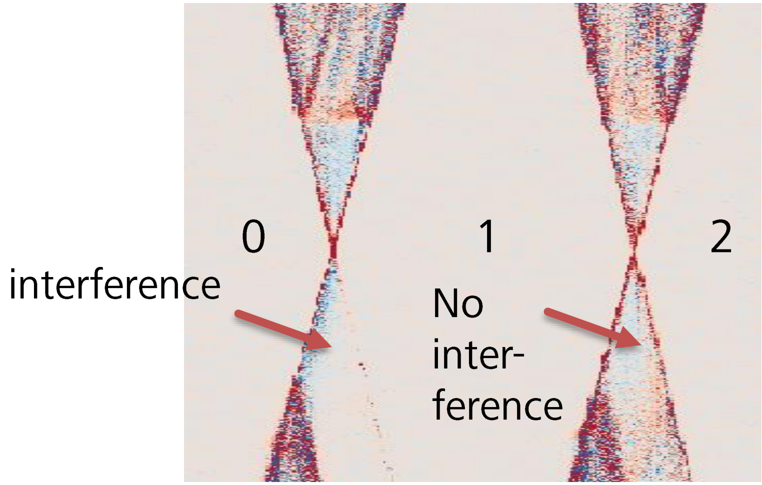
→ new dark state possible



→ Interference also at 1↔2 transitions!?!

→ Include $J \approx \Gamma$

- No interference
- No excitation lines



Parameters

CNT experiment, Regensburg

parameter	shell 0	shell 1	shell 2
ε_0		4.35meV	
U		20meV	
J		10μeV	
$k_{\mathrm{B}}T$		50μeV	
$\hbar\Gamma_{\mathrm{R}}$	$2\mu eV$	10μeV	10μeV
$\hbar\Gamma_{\mathrm{L}}$		4μeV	
$\hbar\Gamma_{\mathrm{rel}}$		0.1μeV	
$\Delta\phi$	0.01π	0.11π	0.07π
η		0.55	

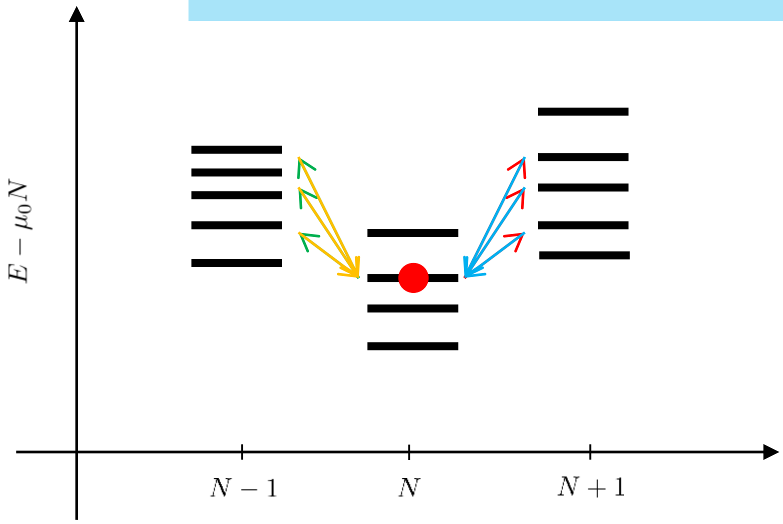
TQD

parameter	value
b	$-1meV$
U	$5meV$
V	$2meV$
$k_{\mathrm{B}}T$	$0.05meV$
$\hbar\Gamma$	$0.01meV$
η	0.5

Tunnelling Liouvillean

$$\mathcal{L}_{\mathrm{tun}}\rho^{NE} = -\frac{1}{2}\sum_{\alpha\sigma}\sum_{ij}\Big\{\mathcal{P}_{NE}\Big[d^\dagger_{i\sigma}\Gamma^\alpha_{ij}(E-H_{\mathrm{CNT}})f^-_\alpha(E-H_{\mathrm{CNT}})d_{j\sigma}+ +d_{j\sigma}\Gamma^\alpha_{ij}(H_{\mathrm{CNT}}-E)f^+_\alpha(H_{\mathrm{CNT}}-E)d^\dagger_{i\sigma}\Big]\rho^{NE}+H.c.\Big\}$$

$$+\sum_{\alpha\sigma}\sum_{ijE'}\mathcal{P}_{NE}\Big[d^\dagger_{i\sigma}\Gamma^\alpha_{ij}(E-E')\rho^{N-1E'}f^+_\alpha(E-E')d_{j\sigma}+ +d_{j\sigma}\Gamma^\alpha_{ij}(E'-E)\rho^{N+1E'}f^-_\alpha(E'-E)d^\dagger_{i\sigma}\Big]\mathcal{P}_{NE}$$



$$\mathcal{P}_{NE} := \sum_i |NEi\rangle\langle NEi|$$

Tunnelling rate matrix

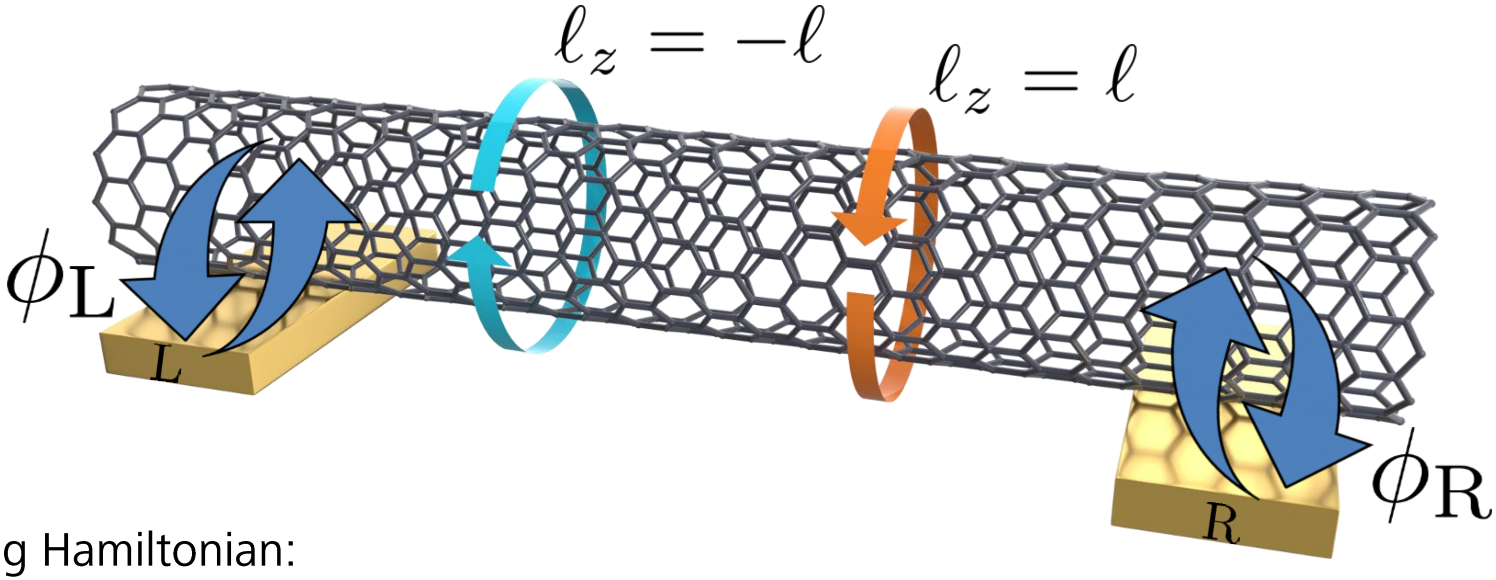
$$I_\alpha = \sum_{NE\sigma ij} \mathcal{P}_{NE} \Big[d_{j\sigma} \Gamma^\alpha_{ij} (H_{\text{CNT}} - E) f^+_\alpha (H_{\text{CNT}} - E) d^\dagger_{i\sigma}$$
$$- d^\dagger_{i\sigma} \Gamma^\alpha_{ij} (E - H_{\text{CNT}}) f^-_\alpha (E - H_{\text{CNT}}) d_{j\sigma} \Big] \mathcal{P}_{NE}$$

Current operator

$$\Gamma^\alpha_{ij}(\Delta E) = \frac{2\pi}{\hbar} \sum_{\mathbf{k}} (t_{\alpha \mathbf{k} i})^* t_{\alpha \mathbf{k} j} \delta(\epsilon_{\mathbf{k}} - \Delta E)$$

Single particle tunnelling rate matrix

Surface-Γ point approx.



Tunneling Hamiltonian:

$$H_{\text{tun}} = \sum_{l\boldsymbol{k}\ell_z\sigma} t_l e^{i\ell_z\phi_l} d^\dagger_{\ell_z\sigma} c_{l\boldsymbol{k}\sigma} + \text{h.c.}$$

Rate matrix:

$$(\boldsymbol{\Gamma}_l)_{\ell_z\ell'_z} \propto |t_l|^2 \langle \ell_z | d^\dagger_l | 0 \rangle\langle 0 | d_l | \ell'_z \rangle \rightarrow \Gamma_l \underbrace{\begin{pmatrix} 1 & e^{i2\ell\phi_l} \\ e^{-i2\ell\phi_l} & 1 \end{pmatrix}}_{\mathcal{R}_l\text{: coherence matrix}}$$

Shot noise

Iterative scheme for current cumulants:

$$0 = \dot{\rho} = \mathcal{L}\rho \qquad \text{Solve for } \rho$$

⇓

$$I = ec_1 = e\ \mathrm{tr}_S\left\{(\mathcal{J}^+ - \mathcal{J}^-)\rho\right\}$$

⇓

$$0 = \dot{X}_1 = \mathcal{L}X_1 + (\mathcal{J}^+ - \mathcal{J}^- - c_1)\rho \qquad \text{Solve for } X_1$$

⇓

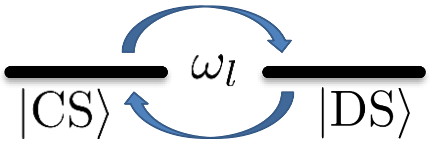
$$S = e^2 c_2 = e^2\ 2\mathrm{tr}_S\left\{(\mathcal{J}^+ - \mathcal{J}^-)X_1\right\} + e^2\ \mathrm{tr}_S\left\{(\mathcal{J}^+ + \mathcal{J}^-)\rho\right\}$$

⇓

$$F = \frac{S}{|eI|}$$

Lamb shift

$$\hat{H}_{\mathrm{LS}} = \frac{\hbar}{2}\sum_l \omega_l \mathcal{R}_l$$



$$\omega_l(V_{\mathrm{g}}, V_{\mathrm{b}}) = \frac{\Gamma_l}{\pi}\left[p_l(\alpha_{\mathrm{g}}V_{\mathrm{g}}) + p_l(U - \frac{J}{2} + \alpha_{\mathrm{g}}V_{\mathrm{g}})\right]$$

$$p_l(\Delta E) = -\mathrm{Re}\psi\left(\frac{1}{2} + i\frac{\Delta E - \mu_l}{2\pi k_{\mathrm{B}}T}\right)$$

$$\mu_{\mathrm{L}} = \eta e V_{\mathrm{b}}$$
$$\mu_{\mathrm{R}} = (\eta - 1)eV_{\mathrm{b}}$$

Digamma function

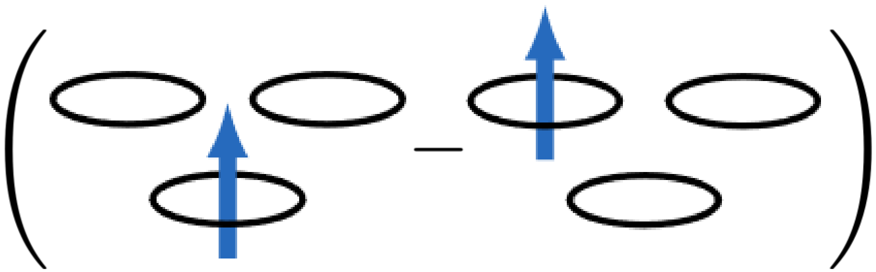
Terms come from principal value integrals over the Fermi functions:

$$p.v. \int_{-\infty}^{\infty} \mathrm{d}\epsilon \frac{f^{\pm}(\epsilon)}{\epsilon - \Delta E}$$

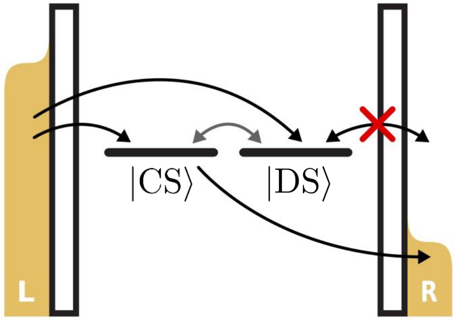
Dark states in a TQD

$$|\mathrm{DS}\rangle = \frac{1}{\sqrt{2}}\left(e^{i2\pi/3}|L_z = 1\rangle - e^{-i2\pi/3}|L_z = -1\rangle\right)$$

$$|\mathrm{DS}, 1_1\rangle = \frac{1}{\sqrt{2}}$$



$$\langle 0|\, d_{\mathrm{R}\sigma}\, |\mathrm{DS}, 1_1\rangle = 0$$



Minimal model

Minimal model for $N = 5_g \leftrightarrow 6$:

$$0 = \dot{\rho}_5 = -\frac{i}{\hbar}\left[\hat{H}_{\mathrm{LS}}, \rho_5\right] + 2\Gamma\mathcal{R}_{\mathrm{R}}\rho_6 - \frac{\Gamma}{2}\left\{\mathcal{R}_{\mathrm{L}}, \rho_5\right\}$$

$$0 = \dot{\rho}_6 = \Gamma \mathrm{tr}\left\{\mathcal{R}_{\mathrm{L}}\rho_5\right\} - 4\Gamma\rho_6$$

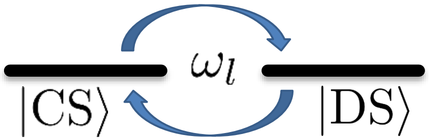
Coherence matrices:

$$\mathcal{R}_{\mathrm{R}} = \begin{pmatrix} 1 & e^{-i2\pi/3} \\ e^{i2\pi/3} & 1 \end{pmatrix} \quad \mathcal{R}_{\mathrm{L}} = \begin{pmatrix} 1 & e^{i2\pi/3} \\ e^{-i2\pi/3} & 1 \end{pmatrix}$$ Angular momentum

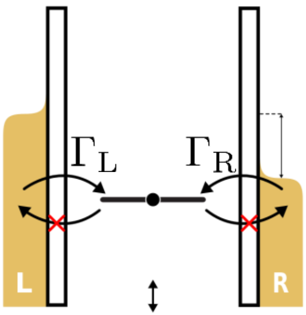
$$\mathcal{R}_{\mathrm{R}} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \quad \mathcal{R}_{\mathrm{L}} = \frac{1}{2}\begin{pmatrix} 3 & -i\sqrt{3} \\ i\sqrt{3} & 1 \end{pmatrix}$$ Dark/Coupled state

Lamb shift:

$$\hat{H}_{\mathrm{LS}} = \frac{\hbar}{2}\sum_l \omega_l \mathcal{R}_l$$



Fano factor

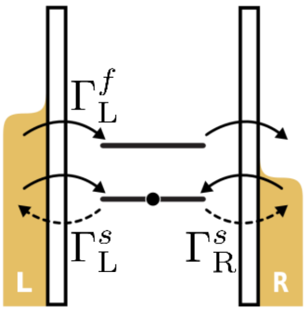


Fano factor of a single resonant level:

$$F = \frac{\Gamma_\mathrm{L}^2 + \Gamma_\mathrm{R}^2}{(\Gamma_\mathrm{L} + \Gamma_\mathrm{R})^2}$$

$\Gamma_\mathrm{L} = \Gamma_\mathrm{R}$ → $F = \frac{1}{2}$

$\Gamma_\mathrm{L} = 2\Gamma_\mathrm{R}$ → $F = \frac{5}{9}$



Fano factor of model with fast and slow channels:

$$F = 1 + \frac{2\Gamma_\mathrm{L}^f}{\Gamma_\mathrm{L}^s + \Gamma_\mathrm{R}^s}$$

$\Gamma_\mathrm{L}^s = 0$, $\Gamma_\mathrm{L} = \Gamma_\mathrm{R}$ → $F = 3$

$\Gamma_\mathrm{R}^s = 0$, $\Gamma_\mathrm{L}^s = 3\Gamma/2$, $\Gamma_\mathrm{L}^f = \Gamma/2$ → $F = \frac{5}{3}$

Tunneling rate matrix

$$\Gamma_{\mathrm{R}} = \Gamma \begin{pmatrix} 1 & e^{-i\frac{2\pi}{3}} \\ e^{i\frac{2\pi}{3}} & 1 \end{pmatrix} \xrightarrow{\text{diagonalize}} \Gamma_{\mathrm{R}} = \Gamma \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\Gamma^{s}_{\mathrm{R}} = 0$$

$$\Gamma_{\mathrm{L}} = \Gamma \begin{pmatrix} 1 & e^{i\frac{2\pi}{3}} \\ e^{-i\frac{2\pi}{3}} & 1 \end{pmatrix} \rightarrow \Gamma_{\mathrm{L}} = \frac{\Gamma}{2} \begin{pmatrix} 3 & -i\sqrt{3} \\ i\sqrt{3} & 1 \end{pmatrix}$$

Cannot be diagonalized at the same time

$$\Gamma^{s}_{\mathrm{L}} = 3\Gamma/2$$

$$\Gamma^{f}_{\mathrm{L}} = \Gamma/2$$

General case (e.g. CNTs):

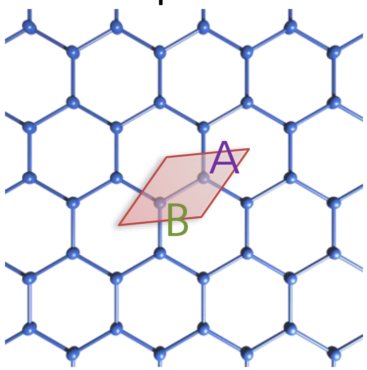
$$\mathcal{R}_{\mathrm{R}} = \begin{pmatrix} 1 & e^{i2\phi_{\mathrm{R}}} \\ e^{-i2\phi_{\mathrm{R}}} & 1 \end{pmatrix} \xrightarrow{\text{diagonalize}} \mathcal{R}_{\mathrm{R}} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$$

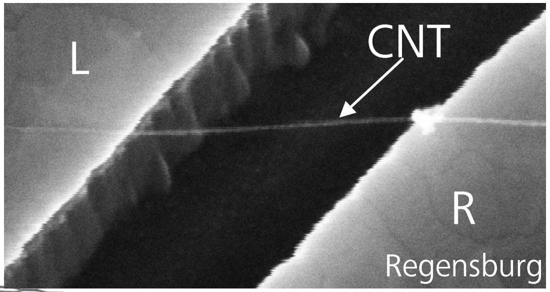
$$\mathcal{R}_{\mathrm{L}} = \begin{pmatrix} 1 & e^{i2\phi_{\mathrm{L}}} \\ e^{-i2\phi_{\mathrm{L}}} & 1 \end{pmatrix} \rightarrow \mathcal{R}_{\mathrm{L}} = 2 \begin{pmatrix} \sin^2\Delta\phi & -i\sin\Delta\phi\cos\Delta\phi \\ \sin\Delta\phi\cos\Delta\phi & \cos^2\Delta\phi \end{pmatrix}$$

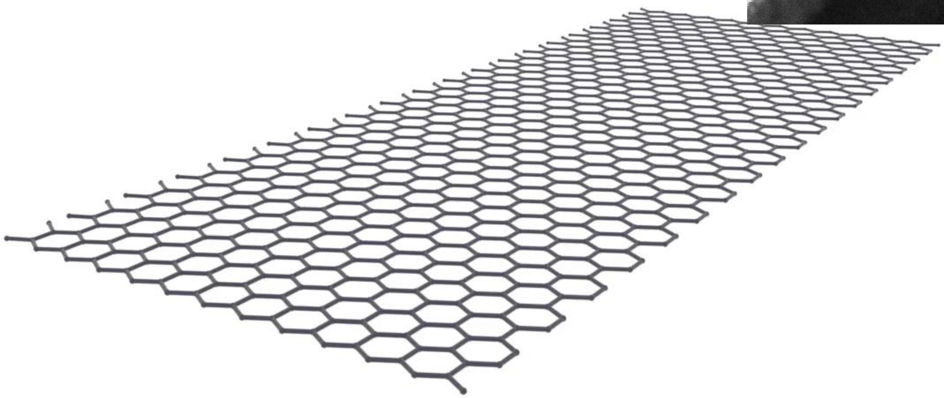
$$\Delta\phi = \phi_{\mathrm{L}} - \phi_{\mathrm{R}}$$

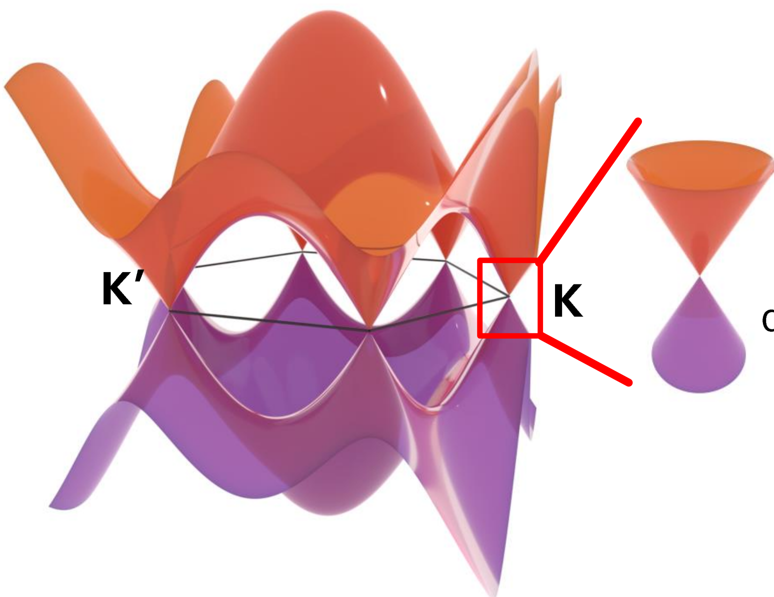
CNT quantum dots

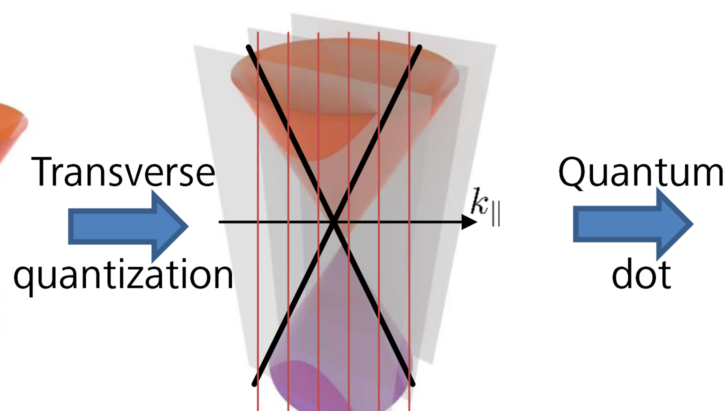
Graphene

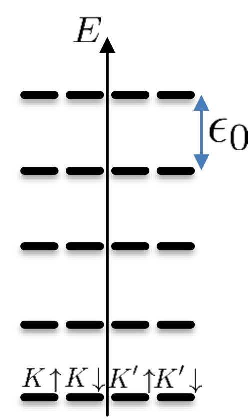










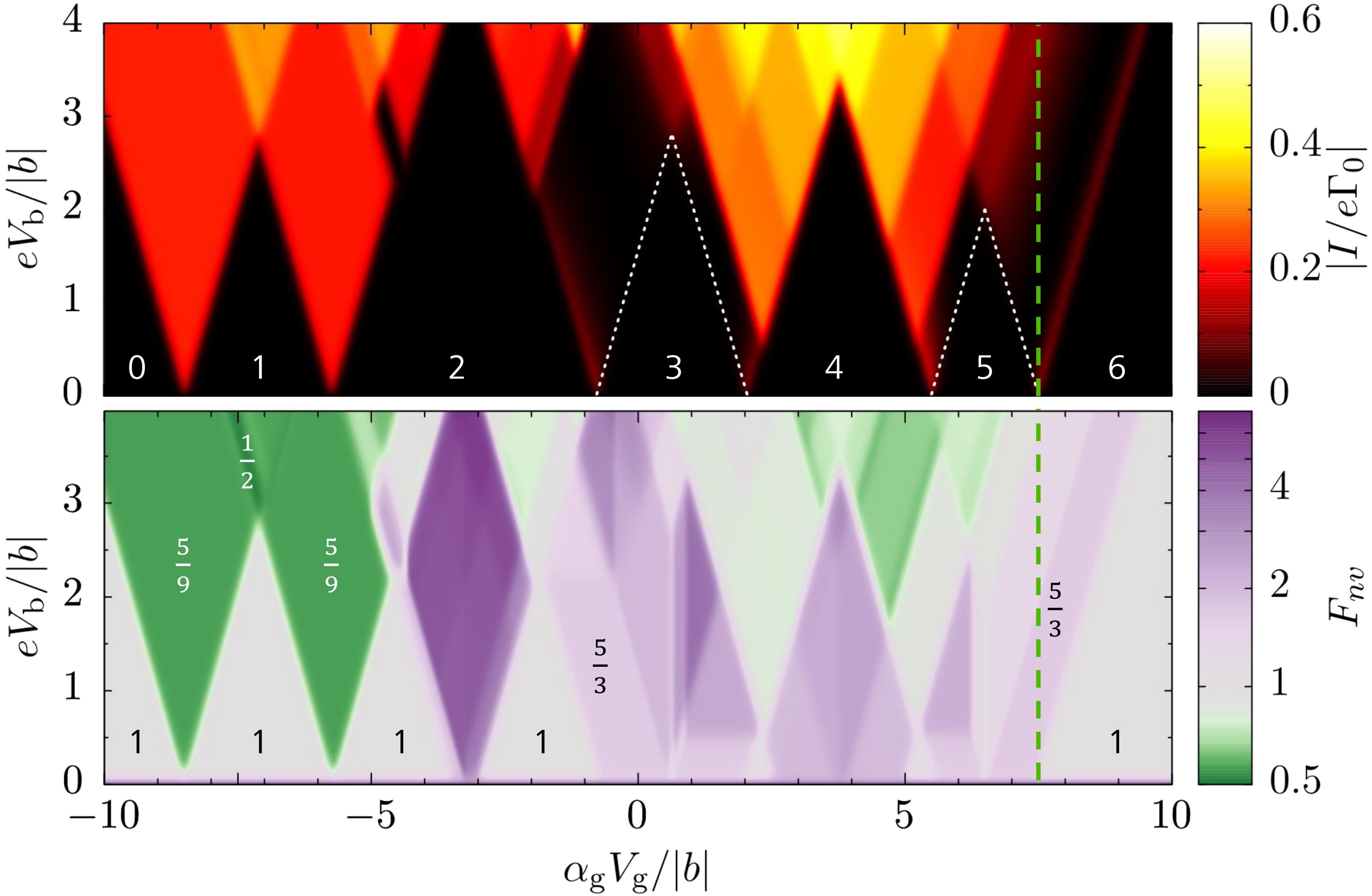


4-fold degenerate:

- 2x spin
- 2x angular momentum (valley)

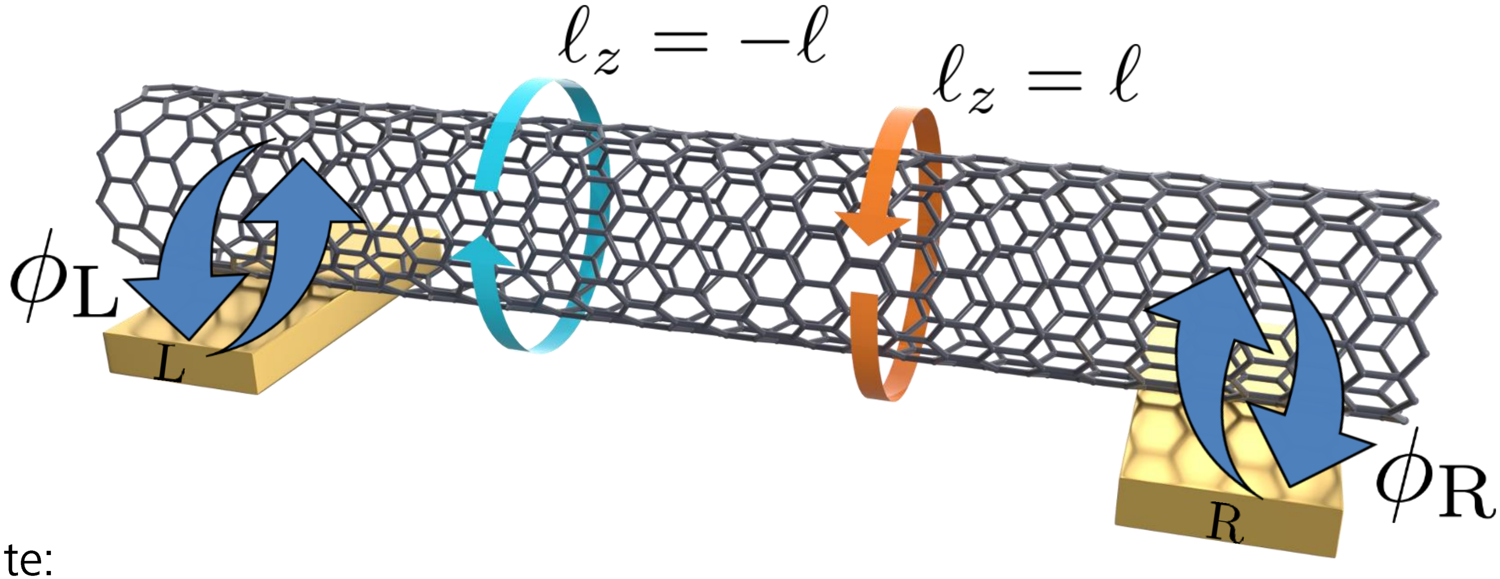
Shell structure

Fano map



4
3
2
1
0
$eV_b/|b|$
0
1
2
3
4
5
6
0.6
0.4
0.2
0
$|I/e\Gamma_0|$
$\frac{1}{2}$
$\frac{5}{9}$
$\frac{5}{9}$
$\frac{5}{3}$
$\frac{5}{3}$
1
1
1
1
1
4
2
1
0.5
F_{nv}
−10
−5
0
5
10
$\alpha_g V_g/|b|$

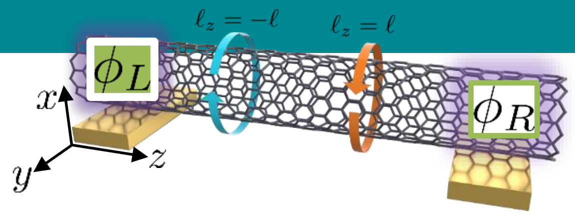
Dark states in CNTs

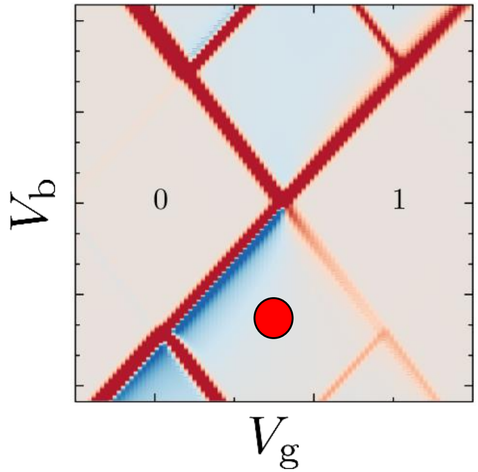


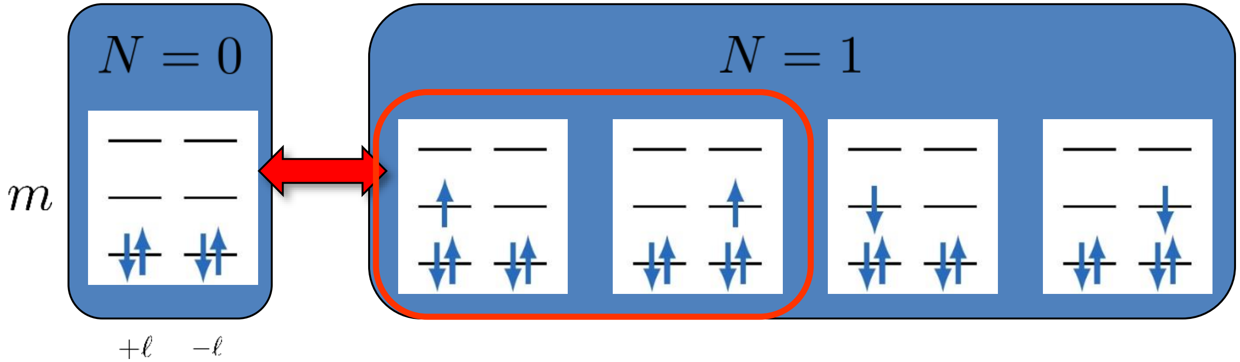
Dark state:

$$|\mathrm{DS}\rangle = \frac{1}{\sqrt{2}}\left(e^{i\ell\phi_l}|\ell_z = \ell\rangle - e^{-i\ell\phi_l}|\ell_z = -\ell\rangle\right)$$

One particle dark state



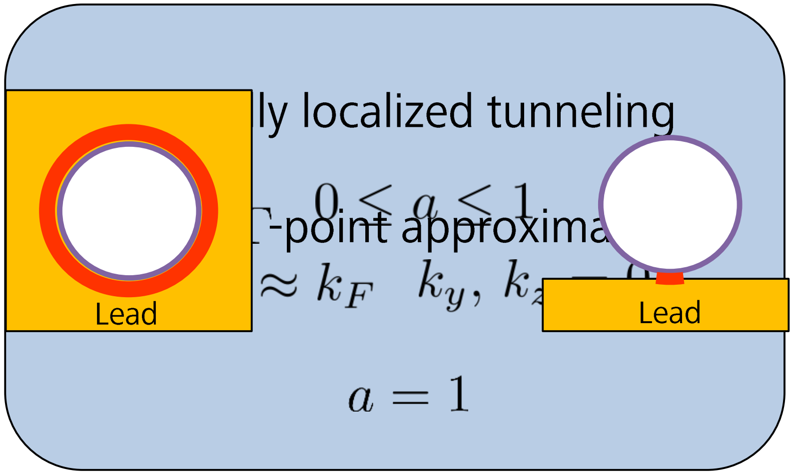




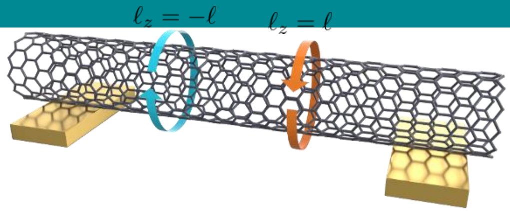
$$\Gamma^{\alpha}_{\ell_z \ell'_z}(E_1 - E_0) = \frac{2\pi}{\hbar} \sum_{\mathbf{k}} (t_{\alpha \mathbf{k} \ell_z})^* t_{\alpha \mathbf{k} \ell'_z} \delta(\epsilon_{\mathbf{k}} - E_1 - E_0) \langle \ell_z | d_l^\dagger | 0 \rangle \langle 0 | d_l | \ell'_z \rangle$$

$$\mathbf{\Gamma}^{\alpha} = \Gamma^{\alpha} \underbrace{\begin{pmatrix} 1 & a e^{2i\ell\phi_\alpha} \\ a e^{-2i\ell\phi_\alpha} & 1 \end{pmatrix}}_{}$$

$\mathcal{R}_l$: coherence matrix



One particle dark state



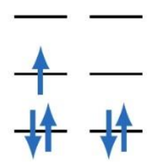
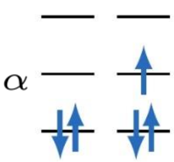
Angular momentum basis

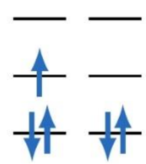
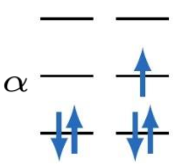
$$\Gamma^\alpha \underbrace{\begin{pmatrix} 1 & e^{2i\ell\phi_\alpha} \\ e^{-2i\ell\phi_\alpha} & 1 \end{pmatrix}}_{\mathcal{R}_\alpha}$$

Dark and coupled state basis

$$\Gamma^\alpha \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$$

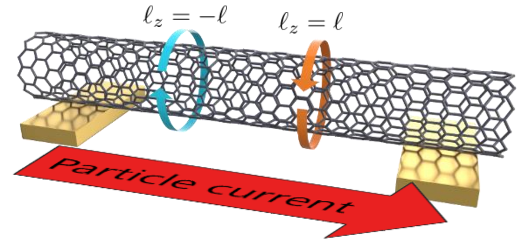
The basis $\{|DS\rangle, |CS\rangle\}$ **diagonalizes** one of the tunnelling rate matrices

$$|DS, \uparrow\alpha\rangle = \frac{1}{\sqrt{2}}\Big(e^{i\ell\phi_\alpha}$$  $$- e^{-i\ell\phi_\alpha}$$  $$\Big)$$

$$|CS, \uparrow\alpha\rangle = \frac{1}{\sqrt{2}}\Big(e^{i\ell\phi_\alpha}$$  $$+ e^{-i\ell\phi_\alpha}$$  $$\Big)$$

The dark state is defined with respect to a **specific** lead

Lamb-shift like precession

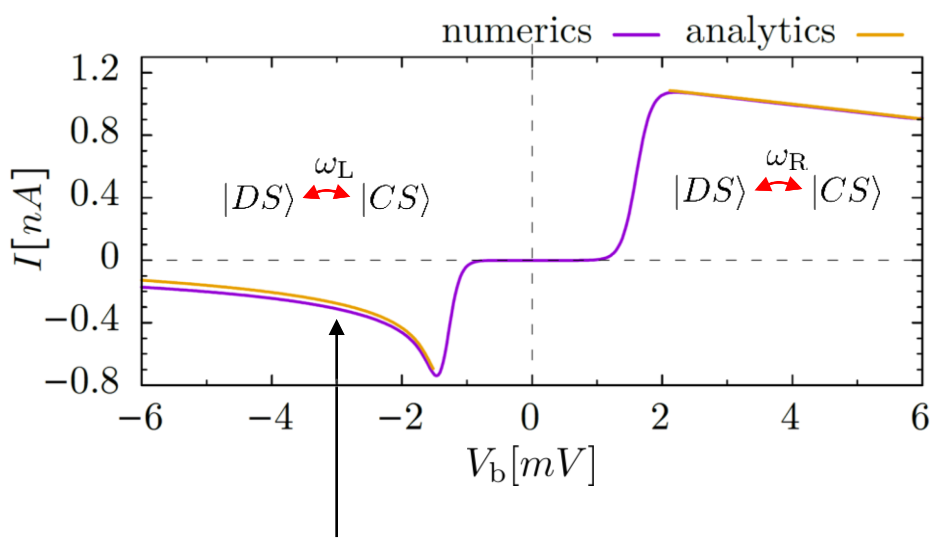


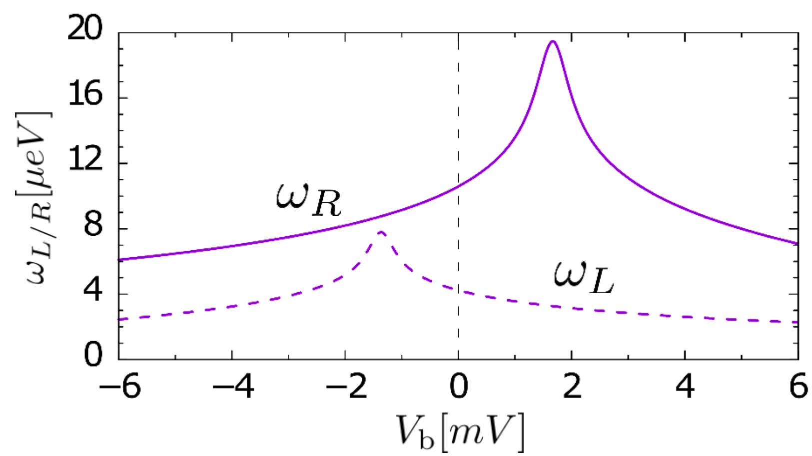
$$\dot{\rho} = -\frac{i}{\hbar}[H_{\mathrm{CNT}}, \rho] - \frac{i}{\hbar}[H_{\mathrm{LS}}, \rho] + \mathcal{L}_{\mathrm{tun}}[\rho] + \mathcal{L}_{\mathrm{rel}}[\rho]$$

$$H_{\mathrm{LS}} = \frac{\hbar}{2}(\omega_L \mathcal{R}_L + \omega_R \mathcal{R}_R)$$

$$\Big[\mathcal{R}_\alpha,\ |DS,\alpha\rangle\langle DS,\alpha|\Big] = 0$$

➡ Only the **source** contribution perturbs the dark state





$$I = \frac{4e\Gamma_{\mathrm{R}}\omega_{\mathrm{L}}^2(1+\cos\Delta\phi)}{\Gamma_{\mathrm{R}}^2 + 4(\omega_{\mathrm{L}} - \omega_{\mathrm{R}})^2 + \omega_{\mathrm{L}}(1+\cos\Delta\phi)\left[\omega_{\mathrm{L}}\Gamma_{\mathrm{R}}/\Gamma_{\mathrm{L}} + 4\omega_{\mathrm{R}}\right]}$$

$$\Delta\phi = \phi_L - \phi_R$$

Closing remarks

Interference occurs between the **degenerate** angular momentum states of a **zig-zag class** carbon nanotube

Spatially confined tunneling mixes angular momentum states providing a non-diagonal **tunnelling rate matrix**

The **dark state** is the eigenvector of the tunneling rate matrix with zero eigenvalue

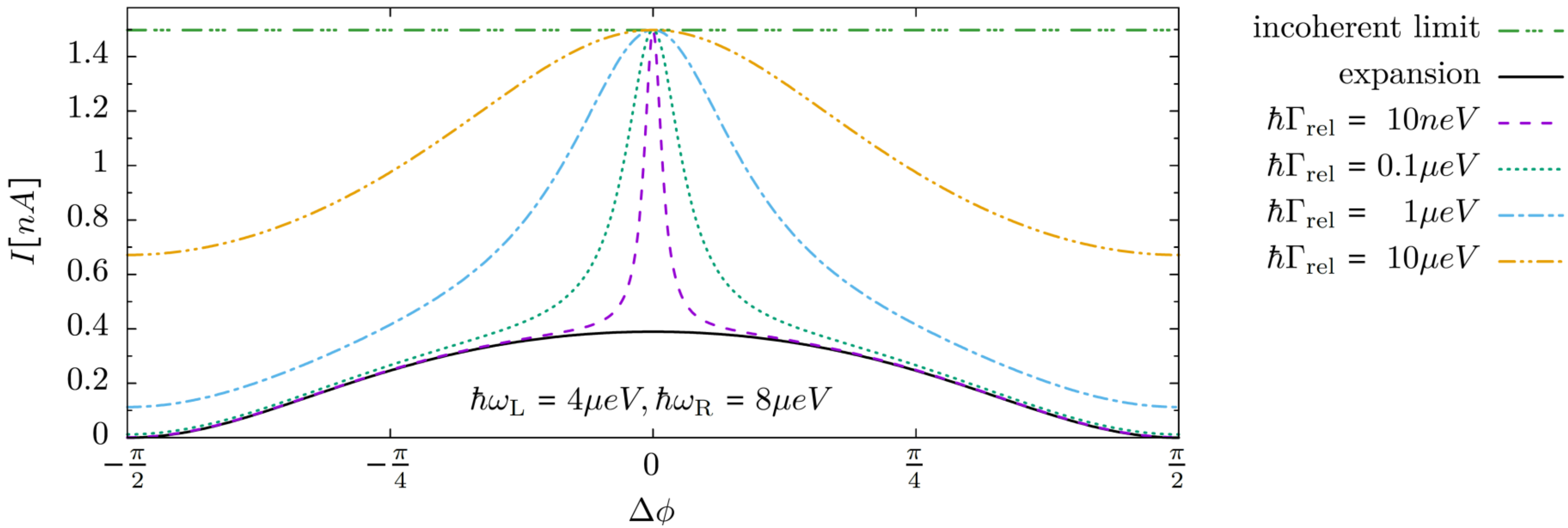
The **Lamb-shift like** precession perturbs the dark state and explains the **bias voltage asymmetry**

Thank you for your attention

Current vs. tunneling phase

$$I = \frac{4e\Gamma_R\omega_L^2(1+\cos\Delta\phi)}{\Gamma_R^2 + 4(\omega_L-\omega_R)^2 + \omega_L(1+\cos\Delta\phi)\left[\omega_L\Gamma_R/\Gamma_L + 4\omega_R\right]}$$

$$\Delta\phi = \phi_L - \phi_R$$



$$I(\Delta\phi = 0) = \frac{4e\Gamma_L\Gamma_R}{4\Gamma_L + \Gamma_R}$$

incoherent current through 4 degenerate levels

Effects of exchange

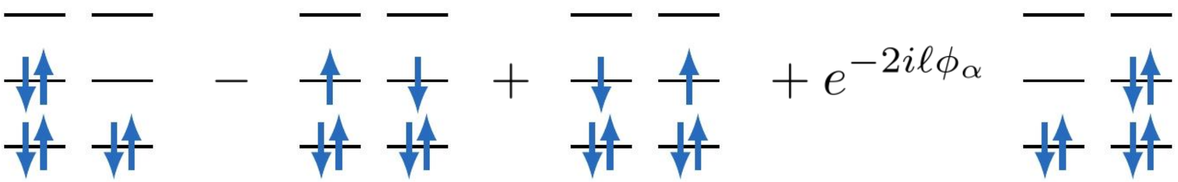
$$H_{\mathrm{CNT}} = \sum_{m\ell_z} (m\epsilon_0 - \xi)\, \hat{n}_{m\ell_z} + \frac{U}{2}\hat{N}^2 + J \sum_m \left(\hat{\mathbf{S}}_{m\ell} \cdot \hat{\mathbf{S}}_{m-\ell} + \frac{1}{4}\hat{n}_{m\ell}\hat{n}_{m-\ell} \right)$$

$J = 0$

2-particle groundstate is 6-fold degenerate

→ new dark state possible

$$|2, \mathrm{DS}\rangle = \frac{1}{2}\Big(e^{2i\ell\phi_\alpha} \quad - \quad + \quad + e^{-2i\ell\phi_\alpha} \Big)$$

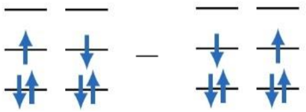


→ interference also at 1↔2 particle resonance

$J > \Gamma$

2-particle groundstate is the inter-valley singlet

$$|2_0\rangle = \frac{1}{\sqrt{2}}\Big(\quad - \quad \Big)$$



→ no 2 particle dark state is possible