Many-body interference in interacting nanojunctions

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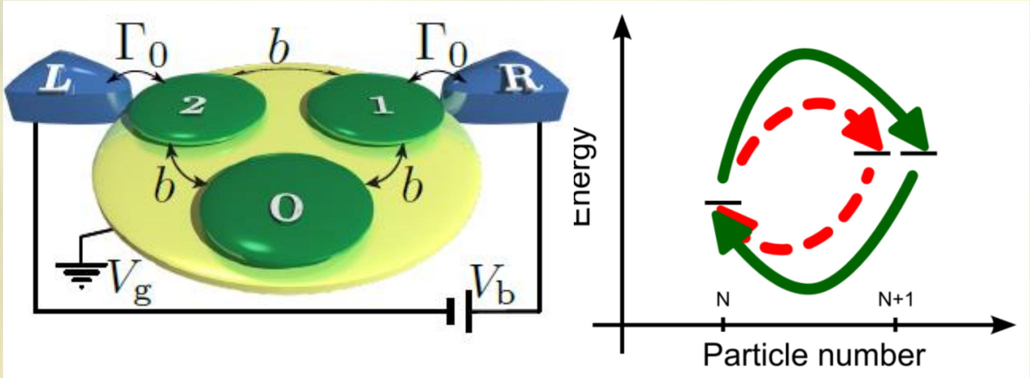


Dresden – qctmbs17 - 13.02.2017

Double slit experiment: (London, 1804)

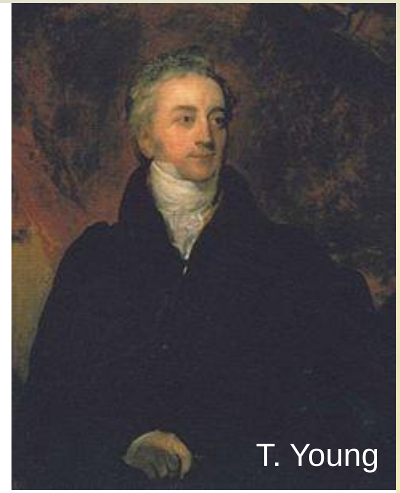
PHILOSOPHICAL

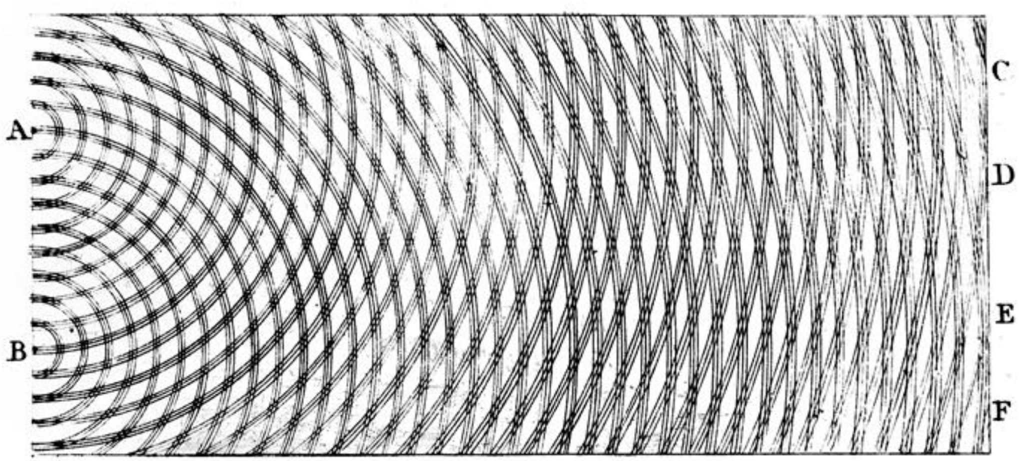
TRANSACTIONS.

I. *The* Bakerian *Lecture*. *Experiments and Calculations relative to physical Optics.* *By* Thomas Young, *M. D. F. R. S.*

Read November 24, 1803.

I. EXPERIMENTAL DEMONSTRATION OF THE GENERAL LAW OF THE INTERFERENCE OF LIGHT.





Phil. Trans. R. Soc. Lon., **94,** 12 (1804)

Single electron interference (Bologna, 1974)

On the statistical aspect of electron interference phenomena

P. G. Merli
CNR–LAMEL, Bologna, Italy

G. F. Missiroli and G. Pozzi
CNR–GNSM, Istituto di Fisica, Laboratorio Microscopia Elettronica, Bologna, Italy
(Received 29 May 1974; revised 17 October 1974)

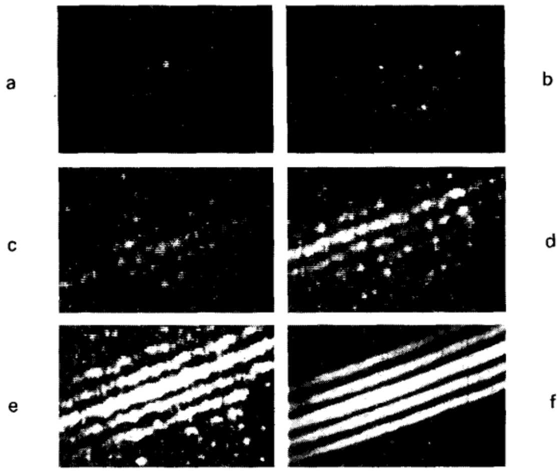


Fig. 1. (a–f) Electron interference fringe patterns filmed from a TV monitor at increasing current densities.

Am. J. Phys., **44**, 306 (1976)

In mesoscopic rings (Rehovot, 1995)

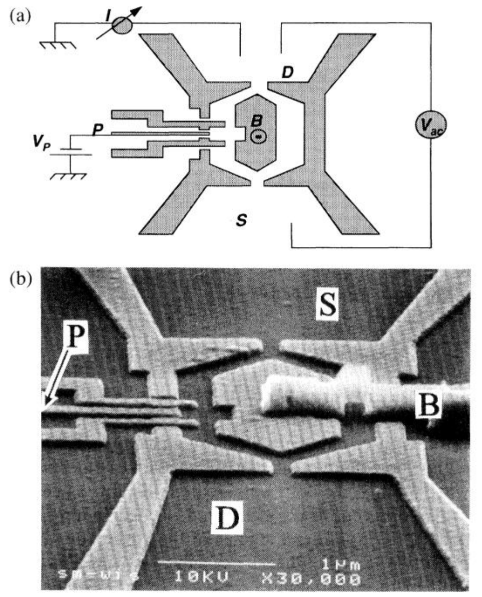
VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

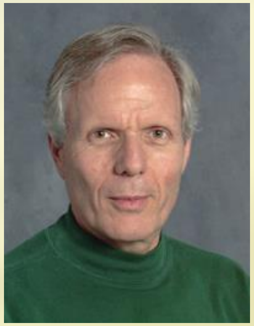
Coherence and Phase Sensitive Measurements in a Quantum Dot

A. Yacoby, M. Heiblum, D. Mahalu, and Hadas Shtrikman
Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel
(Received 10 November 1994)

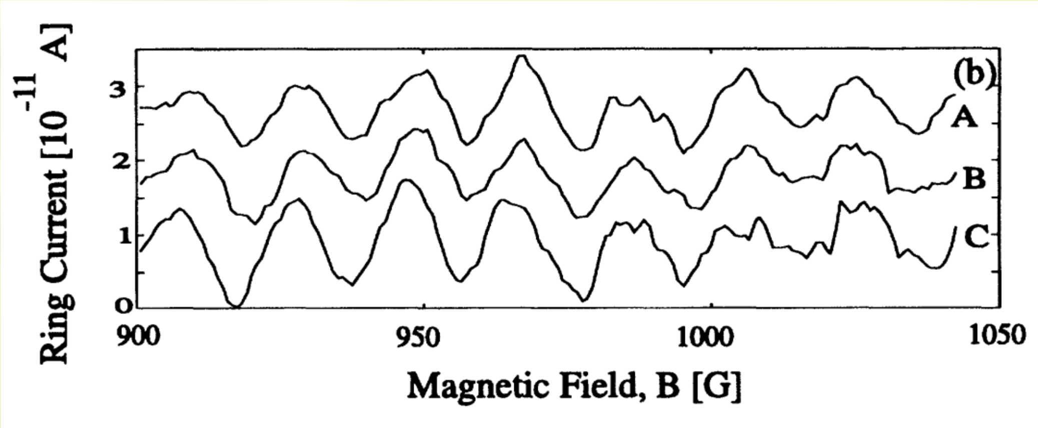
Via a novel interference experiment, which measures magnitude and *phase* of the transmission coefficient through a quantum dot in the Coulomb regime, we prove directly, for the first time, that transport through the dot has a coherent component. We find the same phase of the transmission coefficient at successive Coulomb peaks, each representing a different number of electrons in the dot; however, as we scan through a single Coulomb peak we find an *abrupt* phase change of π. The observed behavior of the phase cannot be understood in the single particle framework.

PACS numbers: 73.20.Dx, 71.45.–d, 72.80.Ey, 73.40.Gk



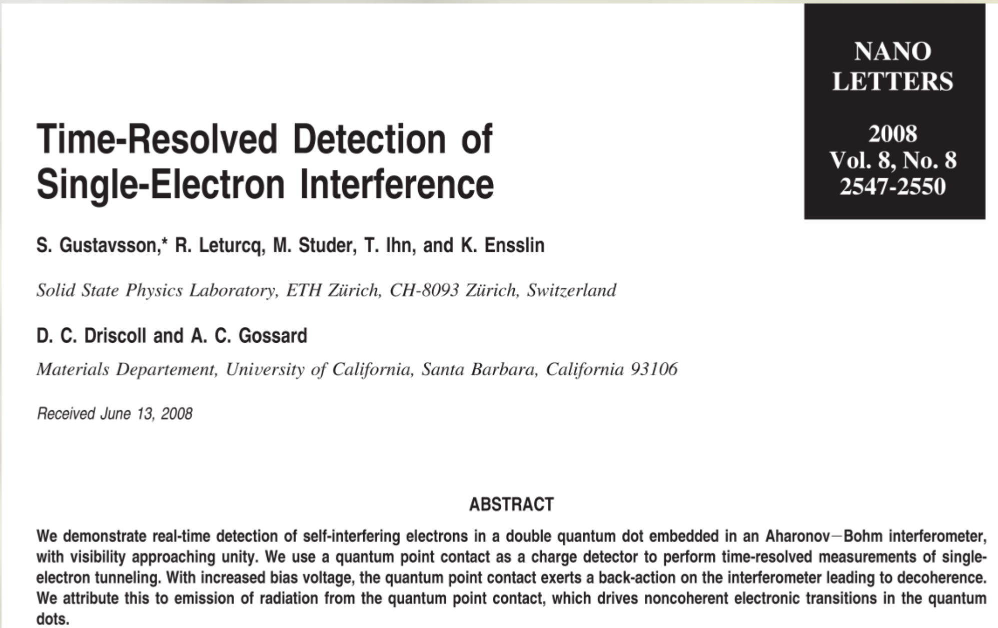


M. Heiblum



Phys. Rev. Lett., **74**, 4047 (1995)

…counting single electrons (Zürich, 2008)


NANO LETTERS
2008
Vol. 8, No. 8
2547-2550

Time-Resolved Detection of Single-Electron Interference

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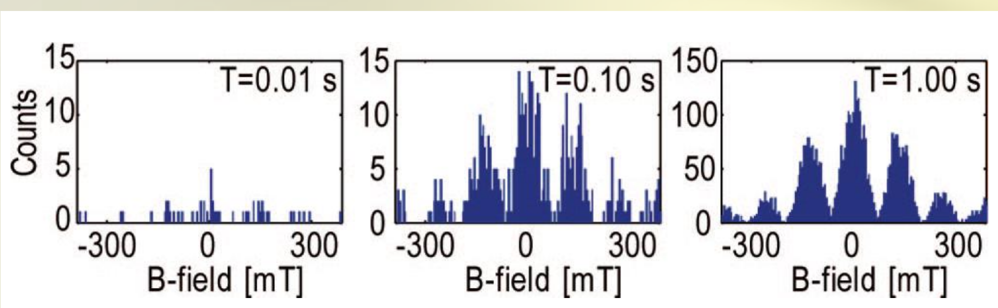
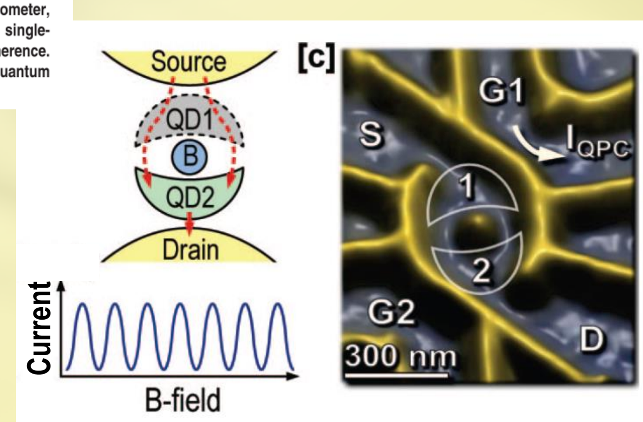
Received June 13, 2008

ABSTRACT

We demonstrate real-time detection of self-interfering electrons in a double quantum dot embedded in an Aharonov−Bohm interferometer, with visibility approaching unity. We use a quantum point contact as a charge detector to perform time-resolved measurements of single-electron tunneling. With increased bias voltage, the quantum point contact exerts a back-action on the interferometer leading to decoherence. We attribute this to emission of radiation from the quantum point contact, which drives noncoherent electronic transitions in the quantum dots.

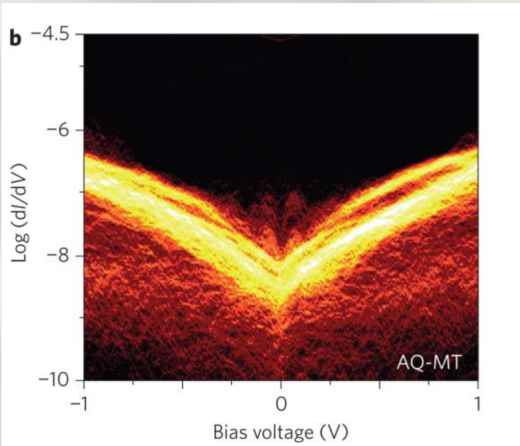

K. Ensslin



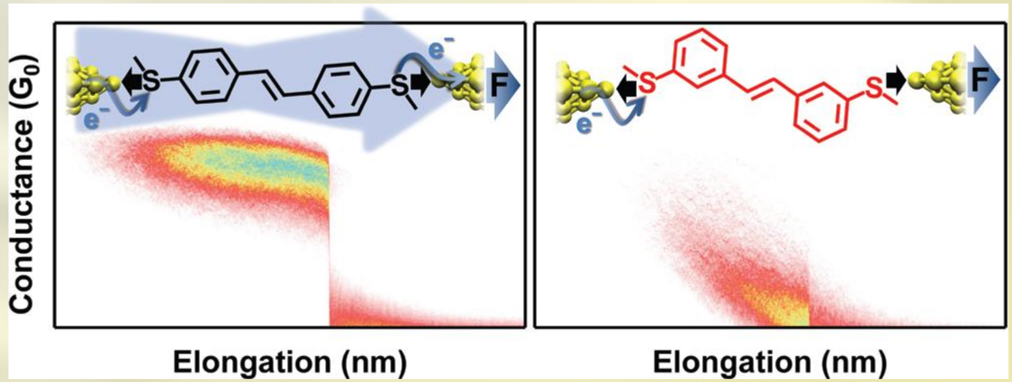


Nano Lett., **8**, 2547 (2008)

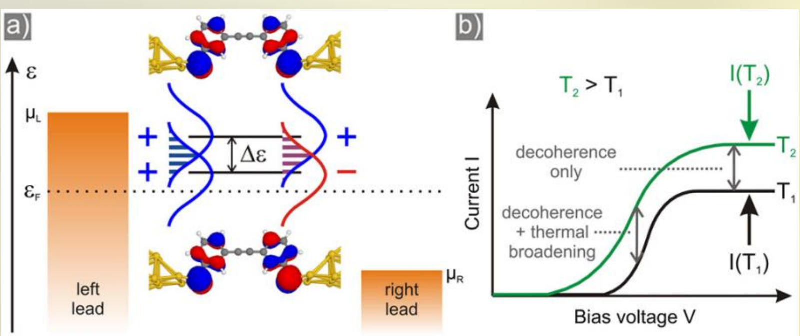
Intramolecular interference



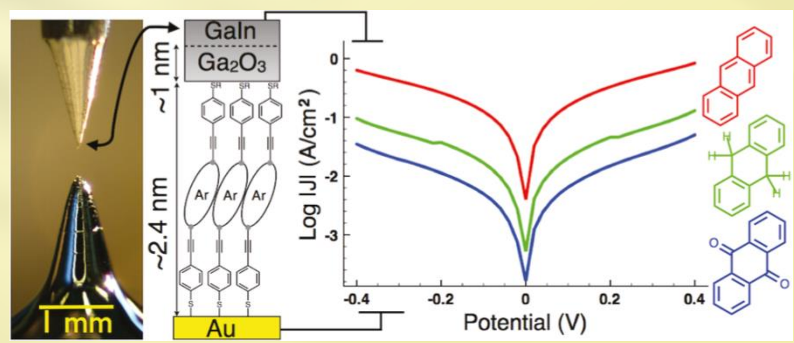
Guédon et al. *Nature Nanotech*. **7**, 305 (2012)



Aradhya et al. *Nano Lett.*, **12**, 1643 (2012)

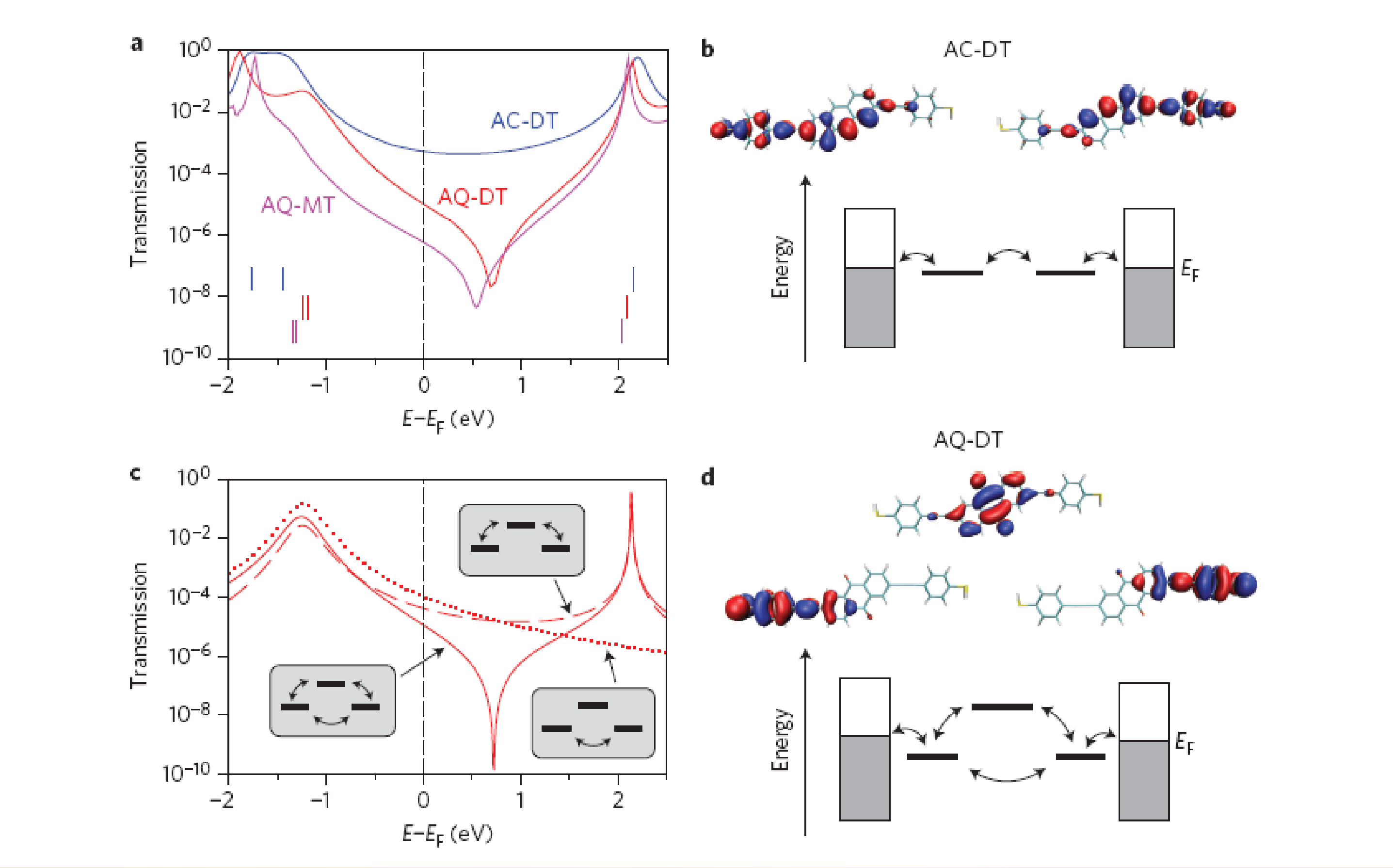


Ballman et al. *PRL* 109, 056801 (2012)



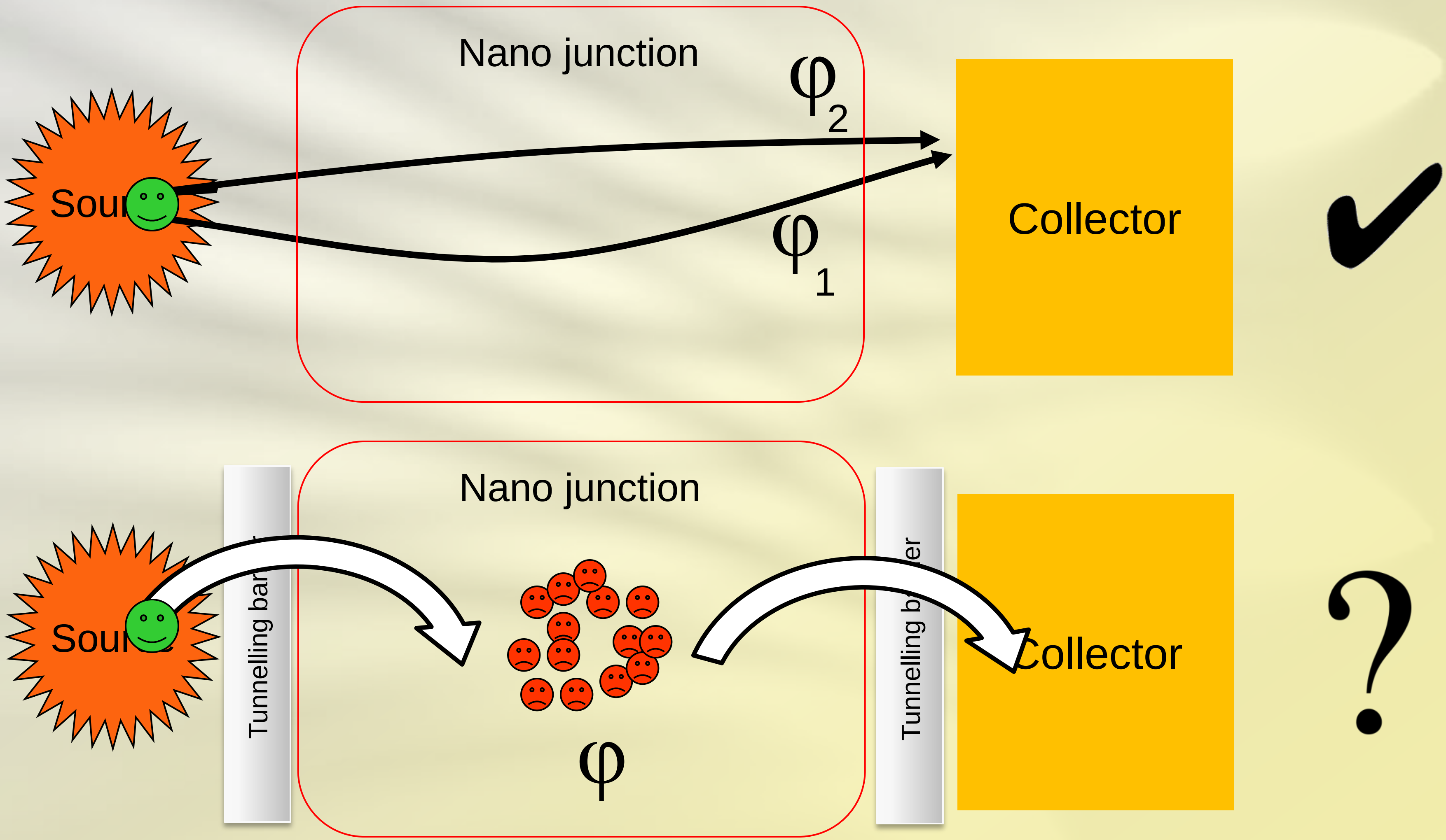
Fracasso et al. *JACS*, **133**, 9556 (2011)

Multiple paths



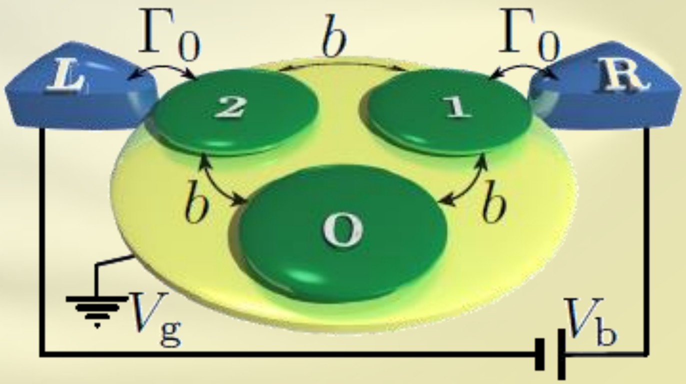
C. M. Guédon, H. Valkenier et al. *Nature Nanotech*. **7**, 305 (2012)

Interference and dephasing



Symmetric triple quantum dot

$$H = H_{\mathrm{TQD}} + H_{\mathrm{leads}} + H_{\mathrm{tun}}$$



$$H_{\mathrm{TQD}} = \xi \sum_{i\sigma} n_{i\sigma} + b \sum_{i\neq j,\sigma} d^{\dagger}_{i\sigma} d_{j\sigma} + U \sum_{i} \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) + V \sum_{i<j} (n_i - 1)(n_j - 1)$$

$$H_{\mathrm{leads}} = \sum_{\alpha \mathbf{k} \sigma} n_{\alpha \mathbf{k} \sigma} \epsilon_{\mathbf{k}}$$

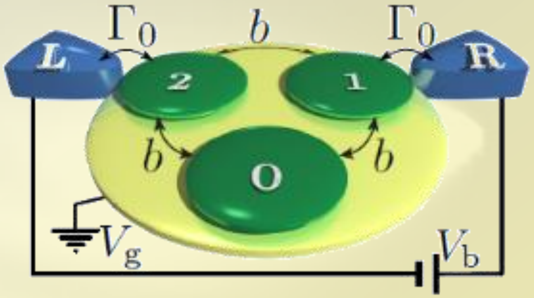
Chemical potential μ_α

Temperature T

$$H_{\mathrm{tun}} = \sum_{\mathbf{k}\sigma} t(d^{\dagger}_{2\sigma} c_{L\mathbf{k}\sigma} + d^{\dagger}_{1\sigma} c_{R\mathbf{k}\sigma}) + h.c.$$

$$\Gamma_{0\alpha} = 2\pi |t|^2 D_\alpha / \hbar$$

Spectrum of the TQD



The single particle component of $H_{\rm TQD}$ is diagonalized by the angular momentum states

$$|l\rangle = \frac{1}{\sqrt{3}} \sum_{j=0}^{2} e^{-i\frac{2\pi}{3}lj} |j\rangle, \quad l = 0, \pm 1$$

The many-body states can be written in the basis

$$|n_{0\uparrow}, n_{1\uparrow}, n_{-1\uparrow}; n_{0\downarrow}, n_{1\downarrow}, n_{-1\downarrow}\rangle$$

$H_{\rm TQD}$ commutes with the operators

$$N = \sum_{l\sigma} n_{l\sigma}$$

$$S^2 = \sum_{i\, l\sigma\sigma'} (d^\dagger_{l\sigma} s^i_{\sigma\sigma'} d_{l\sigma'})^2$$

$$S_z = \frac{\hbar}{2} \sum_{l\sigma} \sigma n_{l\sigma}$$

$$L_z = \hbar \sum_{l\sigma} l n_{l\sigma} |_{\text{mod } 3}$$

By exploiting these symmetries we diagonalize analytically $H_{\rm TQD}$ and obtain the eigenstates

$$|N, E; S, S_z, L_z\rangle$$

Symmetry protected degeneracies are associated to the group of the Hamiltonian

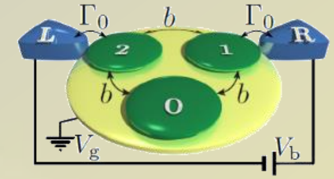
$$\mathrm{C}_{3v} \otimes \mathrm{SU}(2)$$

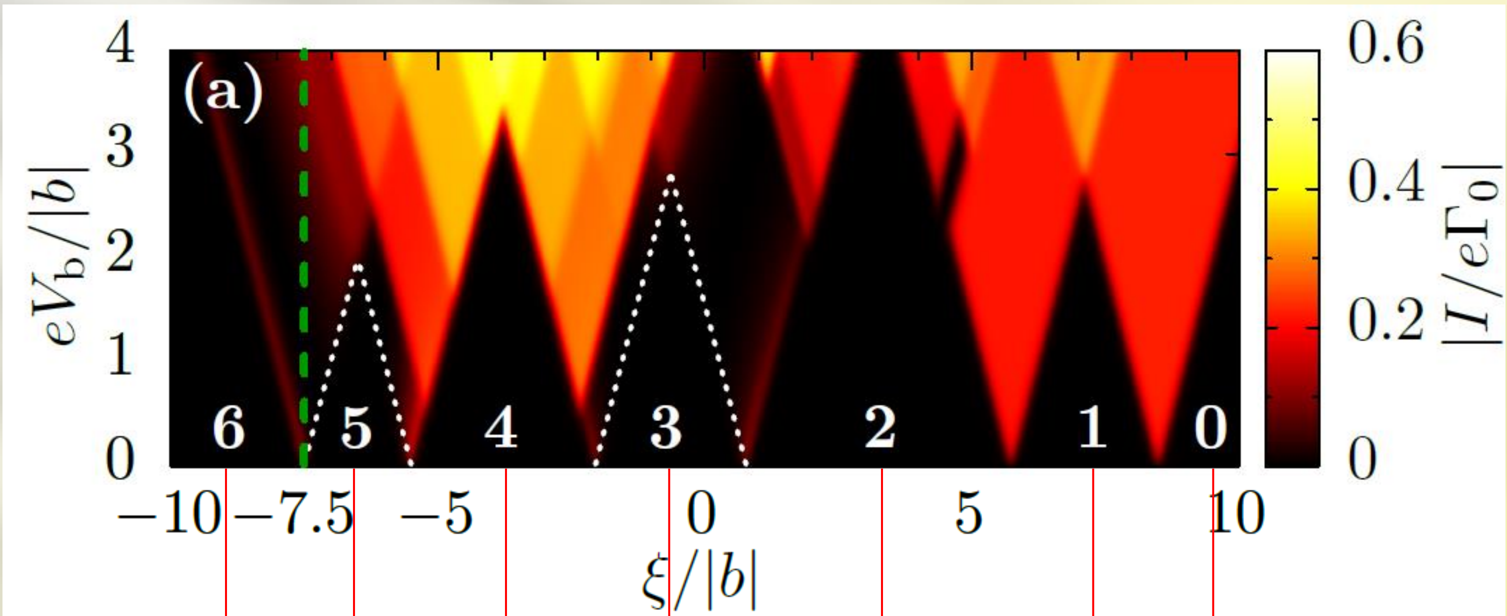
Spectrum of the TQD

N	Eigenenergy	S	S_z	L_z	Eigenstate in the basis $\{\lvert n_{0\uparrow}, n_{1\uparrow}, n_{-1\uparrow}; n_{0\downarrow}, n_{1\downarrow}, n_{-1\downarrow}\rangle\}$
0	$E_0 = 0$	0	0	0	$\lvert 000, 000\rangle$
1	$E_{1_0} = \xi - \frac{U}{2} - 2V + 2b$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\lvert 000, 100\rangle$
			$\frac{1}{2}$		$\lvert 100, 000\rangle$
	$E_{1_1} = \xi - \frac{U}{2} - 2V - b$	$\frac{1}{2}$	$-\frac{1}{2}$	-1	$\lvert 000, 001\rangle$
				1	$\lvert 000, 010\rangle$
			$\frac{1}{2}$	-1	$\lvert 001, 000\rangle$
				1	$\lvert 010, 000\rangle$
2	$E_{2_0} = 2\xi - U - 3V + b + \frac{U-V}{2} - s_{-2}$	0	0	0	$\cos(\phi_{-2})\lvert 100, 100\rangle - \sin(\phi_{-2})\frac{1}{\sqrt{2}}(\lvert 010, 001\rangle + \lvert 001, 010\rangle)$
	$E_{2_1} = 2\xi - U - 3V + b$	1	-1	-1	$\lvert 000, 101\rangle$
				1	$\lvert 000, 110\rangle$
			0	-1	$\frac{1}{\sqrt{2}}(\lvert 100, 001\rangle - \lvert 001, 100\rangle)$
				1	$\frac{1}{\sqrt{2}}(\lvert 100, 010\rangle - \lvert 010, 100\rangle)$
			1	-1	$\lvert 101, 000\rangle$
				1	$\lvert 110, 000\rangle$
	$E_{2_2} = 2\xi - U - 3V - \frac{b}{2} + \frac{U-V}{2} - s_1$	0	0	-1	$\cos(\phi_1)\lvert 010, 010\rangle - \sin(\phi_1)\frac{1}{\sqrt{2}}(\lvert 100, 001\rangle + \lvert 001, 100\rangle)$
				1	$\cos(\phi_1)\lvert 001, 001\rangle - \sin(\phi_1)\frac{1}{\sqrt{2}}(\lvert 100, 010\rangle + \lvert 010, 100\rangle)$
	$E_{2_3} = 2\xi - U - 3V - 2b$	1	-1	0	$\lvert 000, 011\rangle$
			0		$\frac{1}{\sqrt{2}}(\lvert 010, 001\rangle - \lvert 001, 010\rangle)$
			1		$\lvert 011, 000\rangle$
	$E_{2_4} = 2\xi - U - 3V - \frac{b}{2} + \frac{U-V}{2} + s_1$	0	0	-1	$\sin(\phi_1)\lvert 010, 010\rangle + \cos(\phi_1)\frac{1}{\sqrt{2}}(\lvert 100, 001\rangle + \lvert 001, 100\rangle)$
				1	$\sin(\phi_1)\lvert 001, 001\rangle + \cos(\phi_1)\frac{1}{\sqrt{2}}(\lvert 100, 010\rangle + \lvert 010, 100\rangle)$
	$E_{2_5} = 2\xi + b - U - 3V + \frac{U-V}{2} + s_{-2}$	0	0	0	$\sin(\phi_{-2})\lvert 100, 100\rangle + \cos(\phi_{-2})\frac{1}{\sqrt{2}}(\lvert 010, 001\rangle + \lvert 001, 010\rangle)$

M. Niklas, A. Trottmann, AD, M. Grifoni, arXiv:1610.08447

Current stability diagram



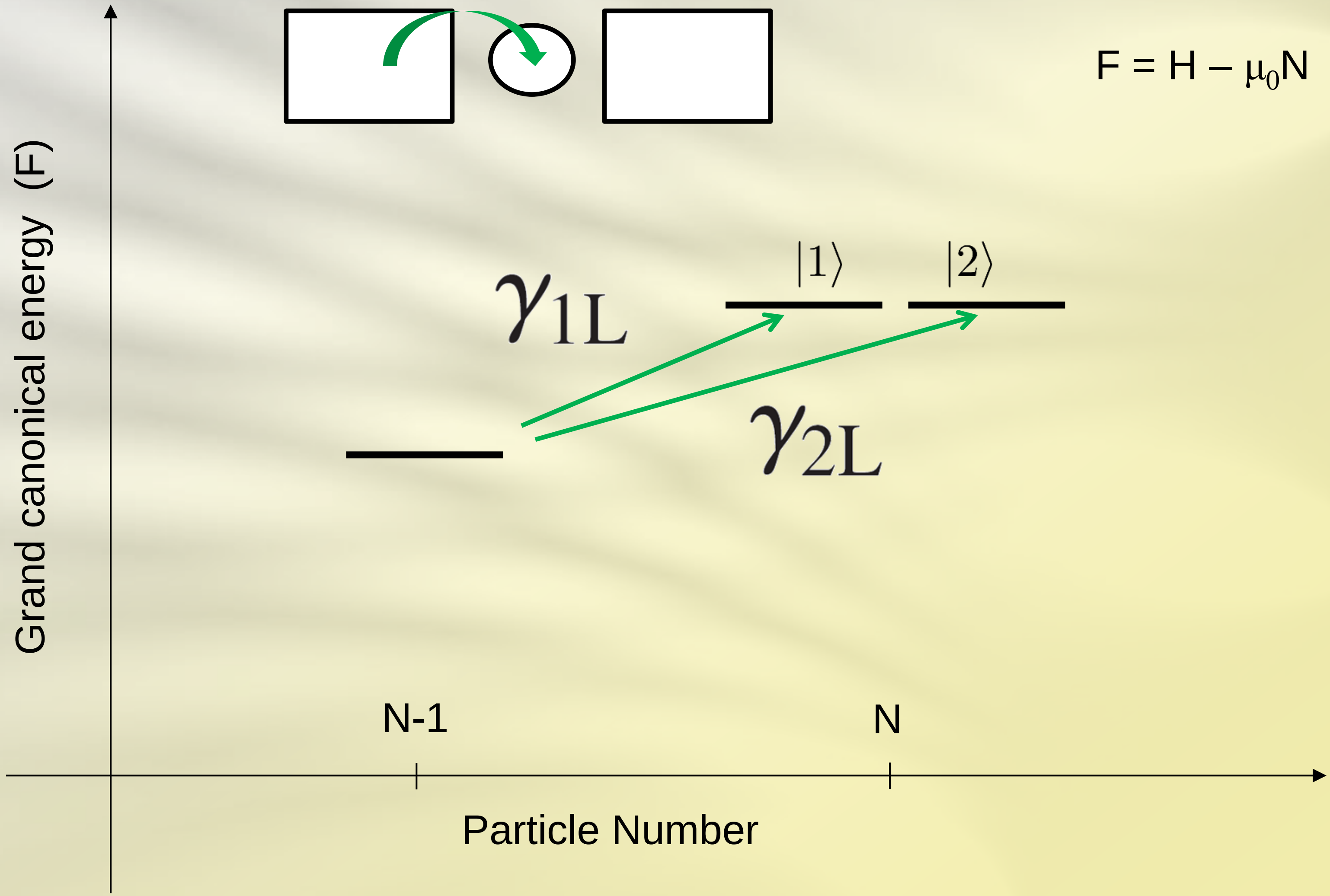


1x1 2x2 1x3 2x2 1x1 1x2 1x1

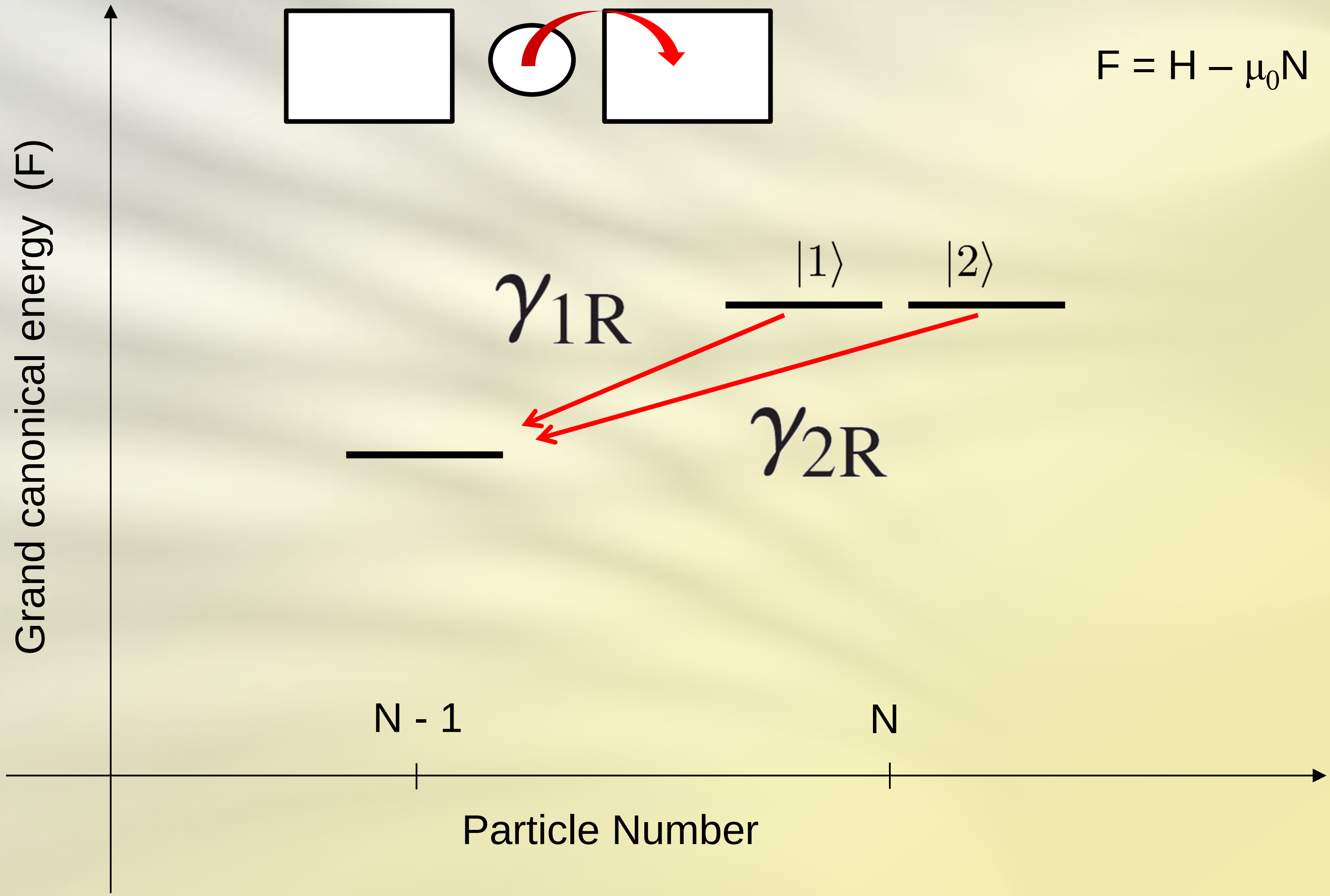
Orbital degeneracy
Spin degeneracy

AD, G. Begemann, and M. Grifoni, *Nano Lett.* **9**, 2897 (2009)

Many-body tunnelling amplitudes



Many-body tunnelling amplitudes



Dark state

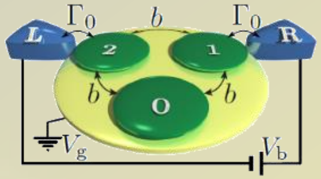
$$|1'\rangle = a|1\rangle + b|2\rangle \quad \Rightarrow \quad \gamma_{1'L} = a\gamma_{1L} + b\gamma_{2L}$$

$$\frac{\gamma_{1L}}{\gamma_{2L}} \neq \frac{\gamma_{1R}}{\gamma_{2R}} \quad \Rightarrow \quad \exists \quad \begin{matrix} |1'\rangle \\ |2'\rangle \end{matrix} \quad \begin{matrix} \gamma_{1'L} \neq 0 \\ \gamma_{1'R} = 0 \\ \gamma_{2'L} \neq 0 \\ \gamma_{2'R} \neq 0 \end{matrix}$$

$|1'\rangle$ is a **dark state** for the system: it suppresses transport.

$|2'\rangle$ is the degenarate **coupled state**. It does not lift the blockade due to Coulomb interaction.

AD, G. Begemann and M. Grifoni *Phys. Rev. B*, **82**, 125451 (2010)



Dark states in the TQD

The generic DS in a triple quantum dot

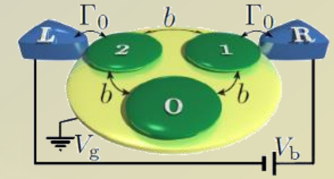
$$|N,\alpha_i;DS\rangle = \frac{1}{\sqrt{2}}\left[e^{i\frac{2\pi}{3}}|N,\alpha_i,1\rangle - e^{-i\frac{2\pi}{3}}|N,\alpha_i,-1\rangle\right]$$

Proof:

$$\begin{aligned}\gamma_R =&\langle N-1;0|d_{1\sigma}|N;DS\rangle = \\ &\langle N-1;0|\sum_l e^{-il2\pi/3}d_{l\sigma}\left[e^{i2\pi/3}|N;1\rangle - e^{-i2\pi/3}|N;-1\rangle\right] \\ &= \langle N-1;0|d_{l=1\sigma}|N;1\rangle - \langle N-1;0|d_{l=-1\sigma}|N;-1\rangle \\ &= 0,\end{aligned}$$

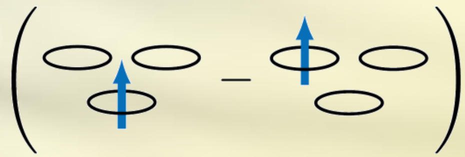
$$\gamma_L = \langle N-1;0|d_{2\sigma}|N;DS\rangle \neq 0$$

Dark states in a TQD

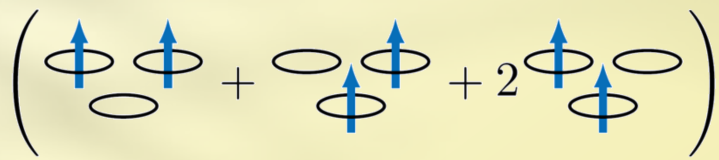


The form of the dark states in the position basis $\{0\uparrow, 1\uparrow, 2\uparrow; 0\downarrow, 1\downarrow, 2\downarrow\}$

$$|1, E_{1_1}; \frac{1}{2}, \frac{1}{2}; DS\rangle = \frac{1}{\sqrt{2}}$$



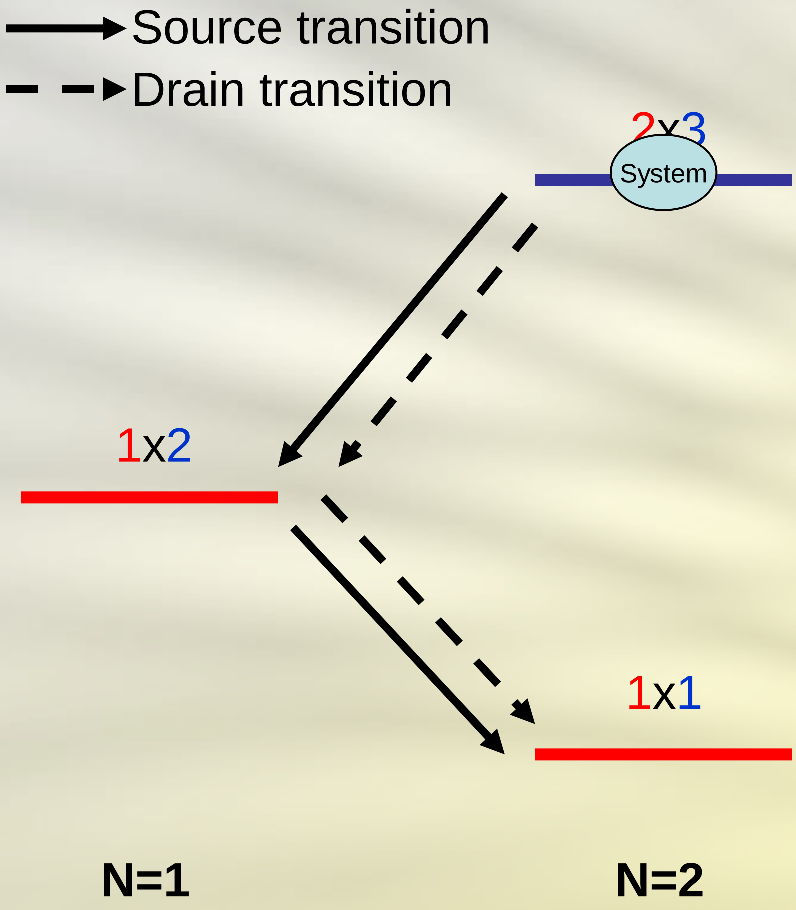
$$|2, E_{2_1}; 1, 1; DS\rangle = \frac{1}{\sqrt{6}}$$



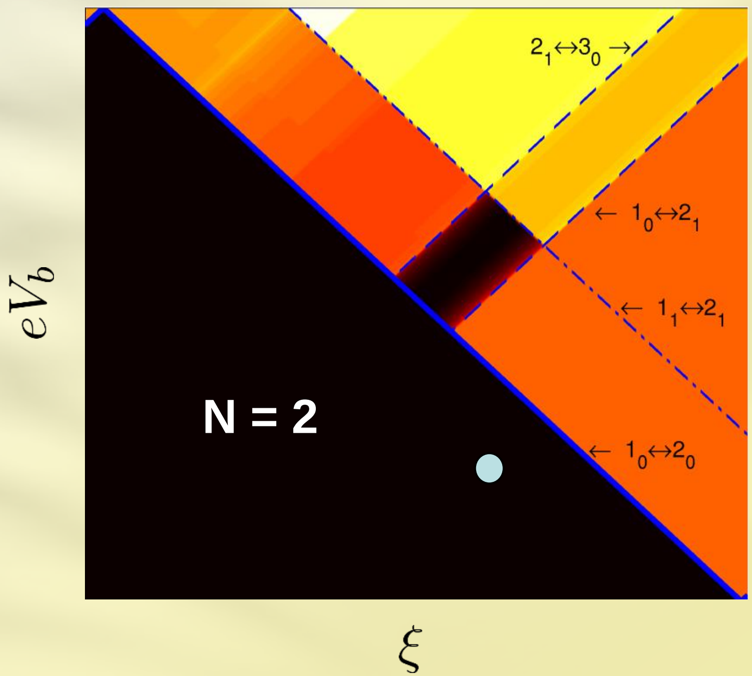
The right coupled dot CAN be occupied. The DS is antisymmetric with respect to

$$\sigma_{v1} : |1\tau\rangle \to |1\tau\rangle, \quad |0\tau\rangle \leftrightarrow |2\tau\rangle$$

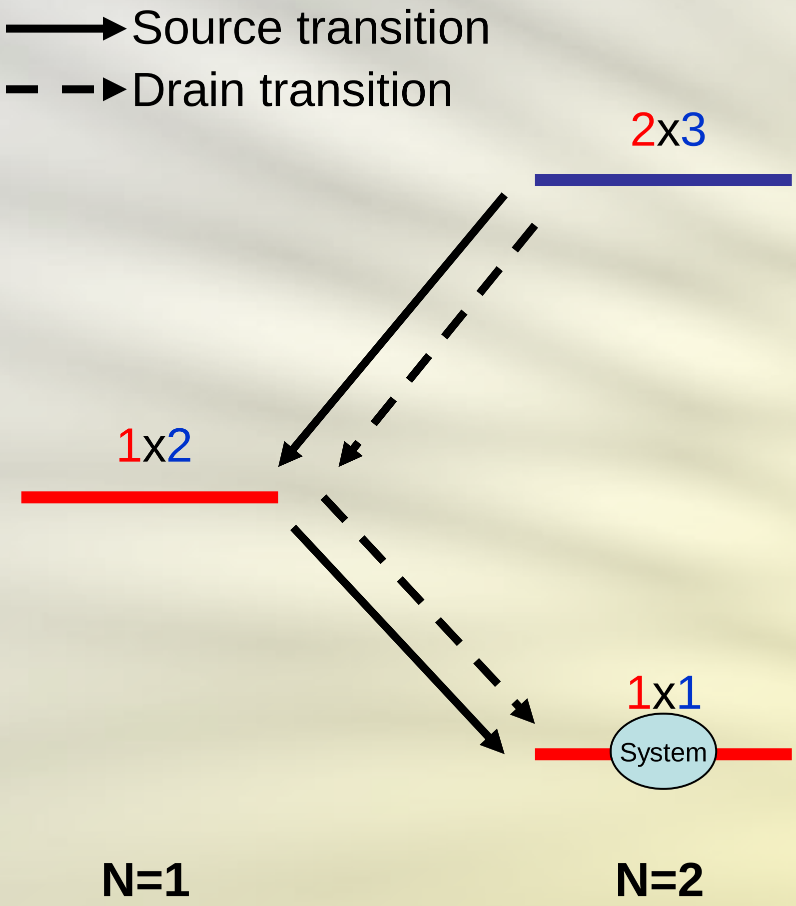
Excited dark states

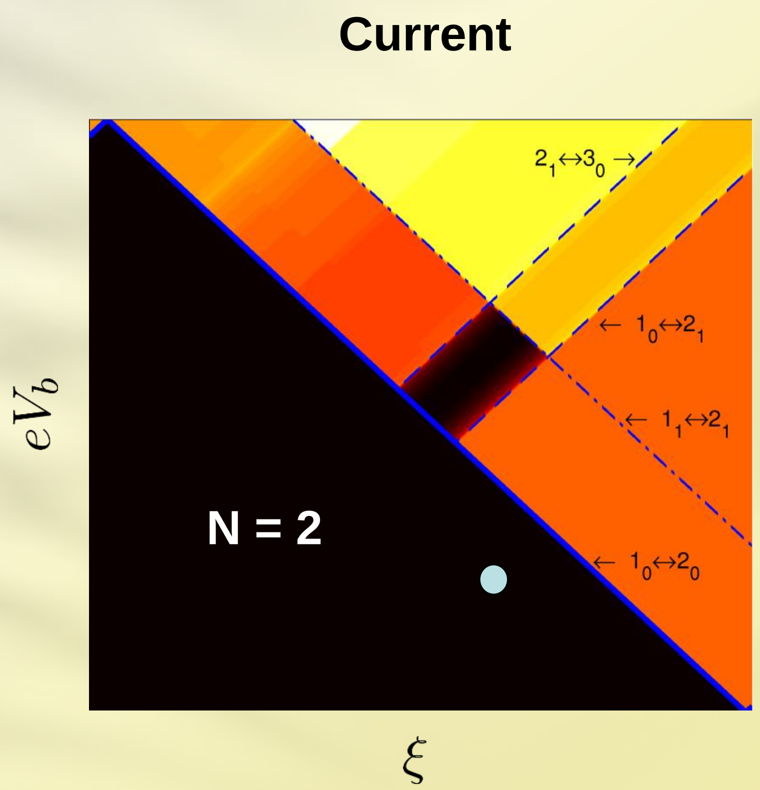


Coulomb Blockade

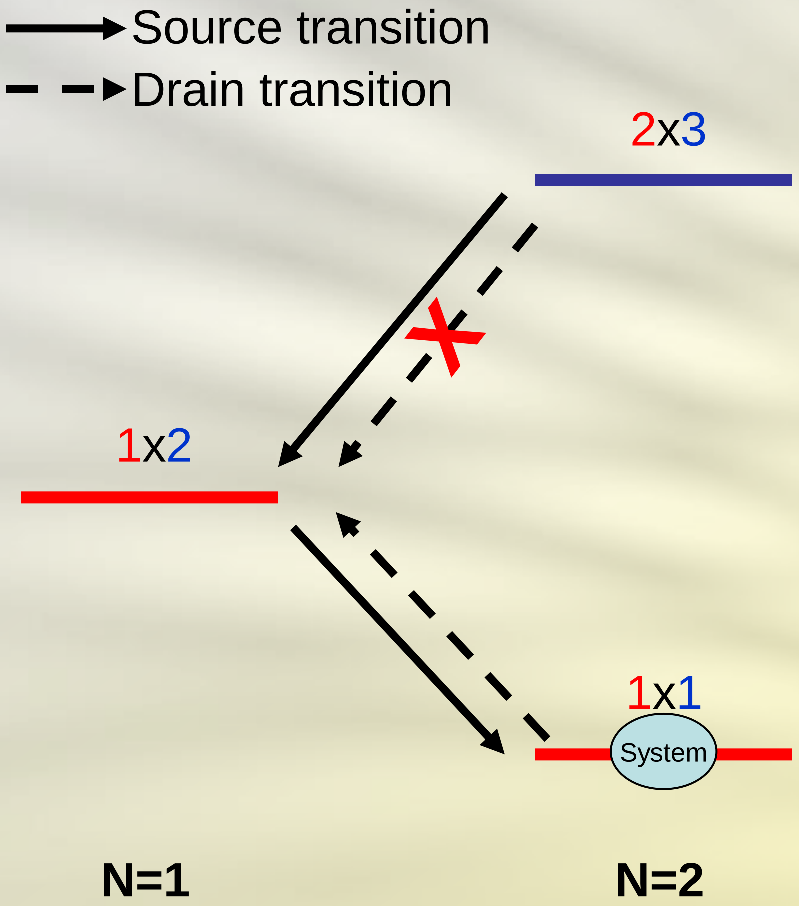


Excited dark states

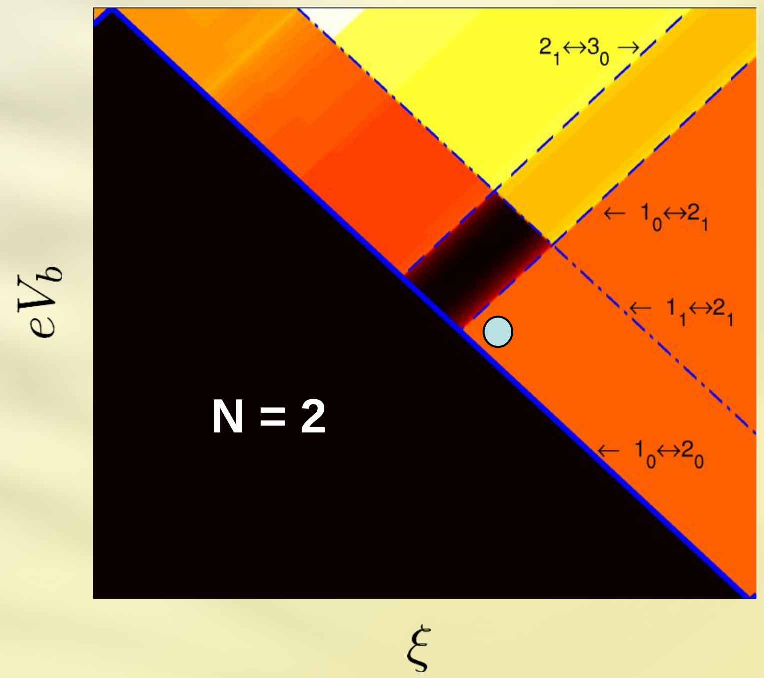




Excited dark states



Interference Blockade



Three linear combinations of 2-particle excited states are coupled **ONLY to the source: excited dark states**.

Reduced density matrix approach

We use a master equation approach for the **generalized reduced density matrix**

$$\rho_\chi = \mathrm{Tr}_{\mathrm{res}}\left\{e^{i\chi N_{\mathrm{R}}}\rho\right\}$$

- χ Counting field
- N_{R} Number operator of the right lead
- ρ Total density matrix

A truncation to the second order in the H_{tun} yields the **generalized master equation**

$$\dot{\rho}_\chi = \left[\mathcal{L} + (e^{i\chi} - 1)\mathcal{J}^{+} + (e^{-i\chi} - 1)\mathcal{J}^{-}\right]\rho_\chi$$

- $\mathcal{L}$ ↓ Liouvillean
- $(e^{i\chi} - 1)\mathcal{J}^{+}$ ↓ R-increasing jump superoperator
- $(e^{-i\chi} - 1)\mathcal{J}^{-}$ ↓ R-decreasing jump superoperator

Reduced density matrix approach

The stationary current and the zero frequency noise can be written as

$$I = -e\,\mathrm{Tr}_{\mathrm{TQD}}\left(\mathcal{J}^{+} - \mathcal{J}^{-}\right)\rho^{\infty},$$
$$S = e^{2}\mathrm{Tr}_{\mathrm{TQD}}\left[2\left(\mathcal{J}^{+} - \mathcal{J}^{-}\right)\mathcal{F}_{1\perp}^{\infty} + \left(\mathcal{J}^{+} + \mathcal{J}^{-}\right)\rho^{\infty}\right]$$

$$F = \frac{S}{e|I|}$$

Fano factor

Where $\rho^{\infty} = \lim_{t\to\infty} \rho_{\chi=0}$ $\qquad \mathcal{F}_{1\perp}^{\infty} = (1 - \rho^{\infty}\mathrm{Tr}_{\mathrm{TQD}})\mathcal{F}_{1}^{\infty}$

$$\mathcal{F}_{1}^{\infty} = \lim_{t\to\infty} d/d(i\chi)\rho_{\chi}|_{\chi=0}$$

Stationary limit
of the first moment

It is sufficient to solve the set of coupled equations

$$\mathcal{L}\rho^{\infty} = -\frac{i}{\hbar}\left[H_{\mathrm{TQD}} + H_{\mathrm{LS}}, \rho^{\infty}\right] + \mathcal{L}_{t}\rho^{\infty} = 0$$
$$\mathcal{L}\mathcal{F}_{1\perp}^{\infty} = \left(-I/e - \mathcal{J}^{+} + \mathcal{J}^{-}\right)\rho^{\infty}$$

Transport statistics

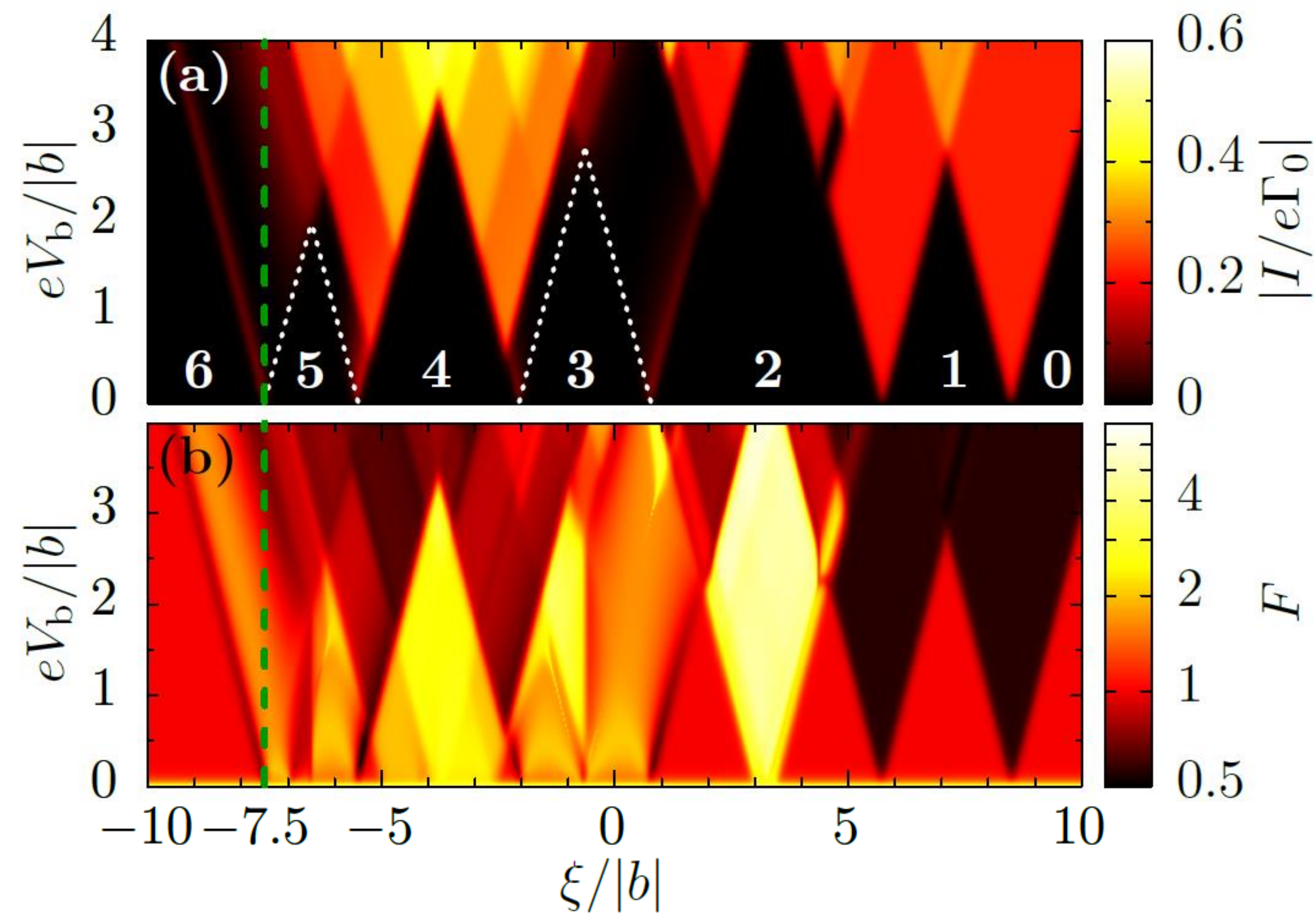
Fano factor

$$F = \frac{S}{e|I|}$$

$F < 1$ | 1 | $F > 1$

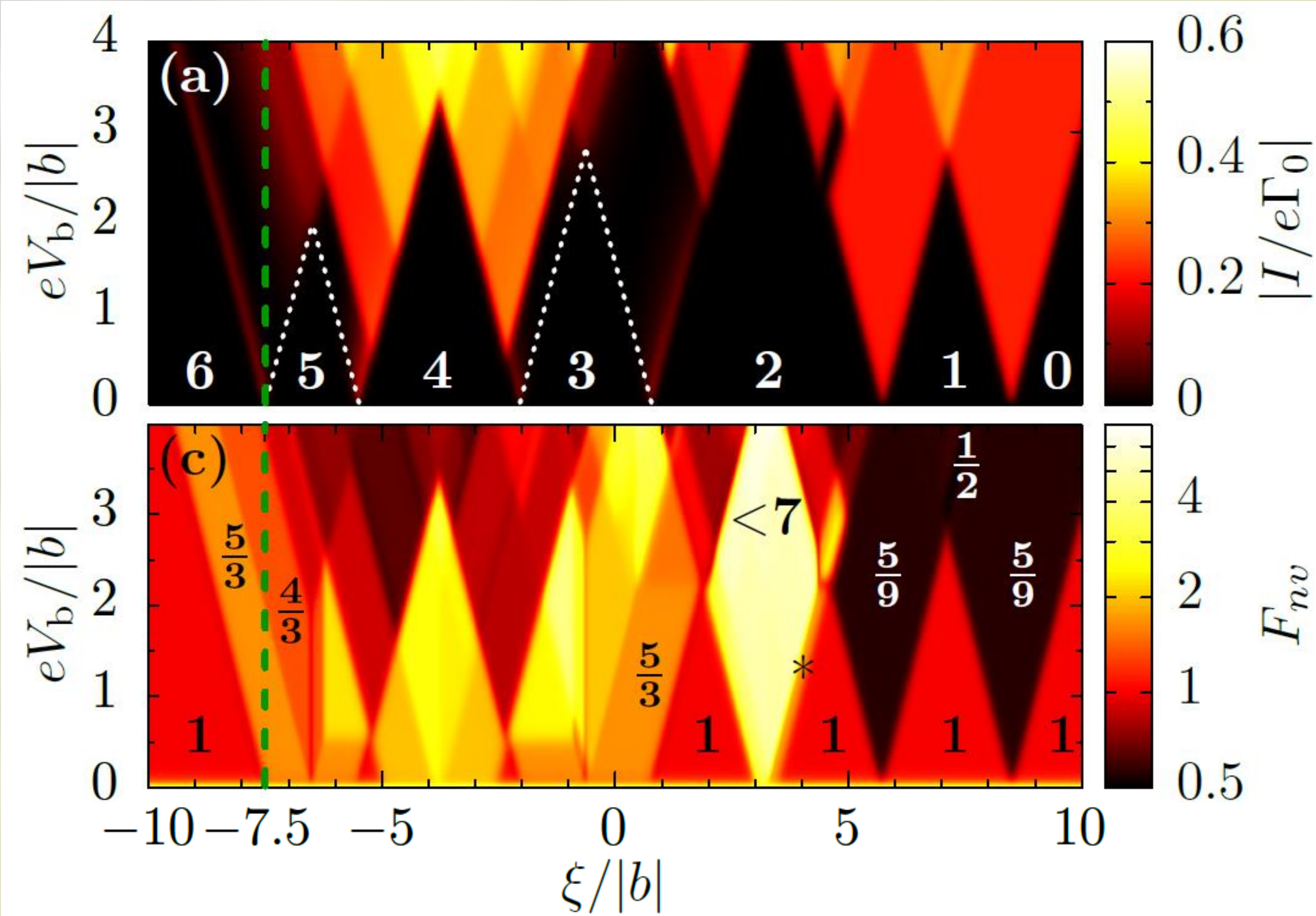
$F < 1$	$F = 1$	$F > 1$
Anti-bunching	Independent Tunnelling events	Bunching
Sub-poissonian noise	Poissonian noise	Super-poissonian noise

Fano stability diagram



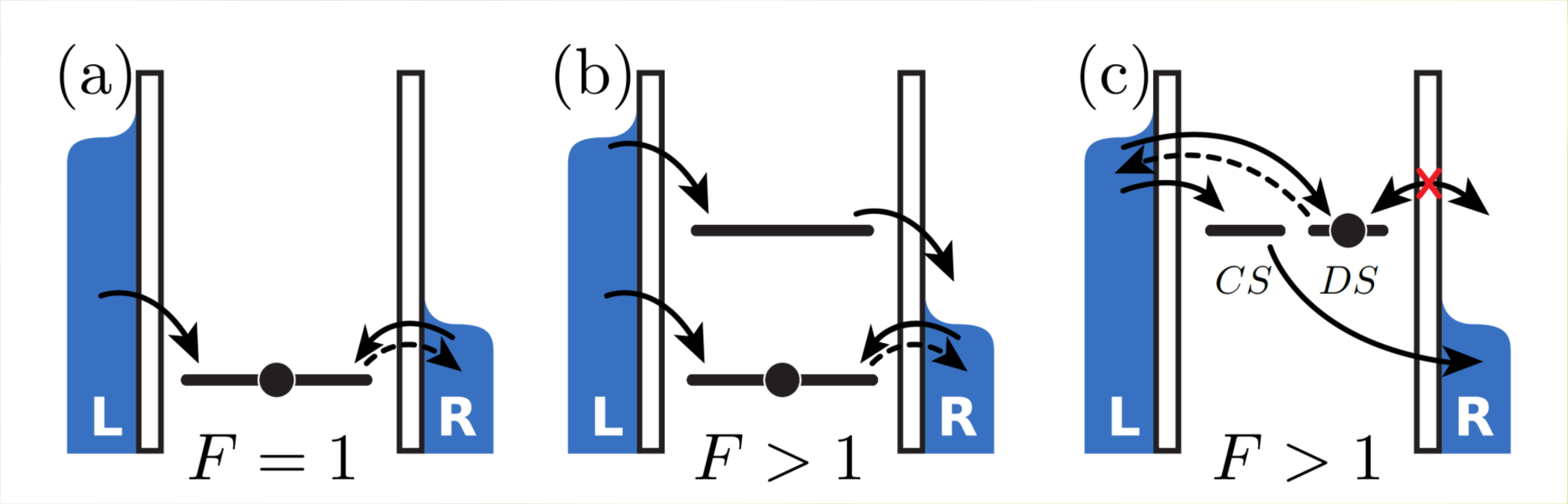
M. Niklas, A. Trottmann, AD, and M. Grifoni, arXiv:1610.08447

Fano stability diagram



M. Niklas, A. Trottmann, AD, and M. Grifoni, arXiv:1610.08447

Blockade mechanisms



Coulomb blockade	Channel blockade	Interference blockade

In both cases a **two effective channels model** provides the Fano factor

$$F_{nv} = 1 + \frac{2\Gamma_L^f}{\Gamma_L^s + \Gamma_R^s}, \quad \mu_L > \mu_R$$

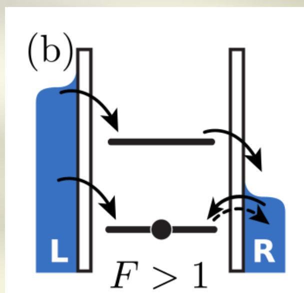
M. Niklas, A. Trottmann, AD, and M. Grifoni, arXiv:1610.08447

Different bunching mechanisms

$$F_{nv} = 1 + \frac{2\Gamma_L^f}{\Gamma_L^s + \Gamma_R^s}, \quad \mu_L > \mu_R$$

$$\Gamma_\alpha^p = R_\alpha^p \Gamma_{0\alpha}$$

$$\Gamma_{0\alpha} = 2\pi |t|^2 D_\alpha / \hbar$$



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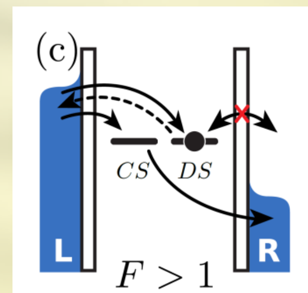


$$\Gamma_{0R} \ll \Gamma_{0L}$$

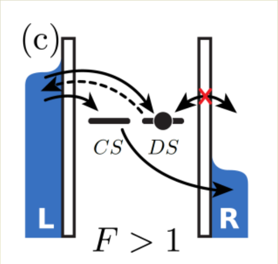
Assuming $\Gamma_L^f = \Gamma_L^s$

$$F_{nv} = 3$$

$$F_{nv} = 5/3$$

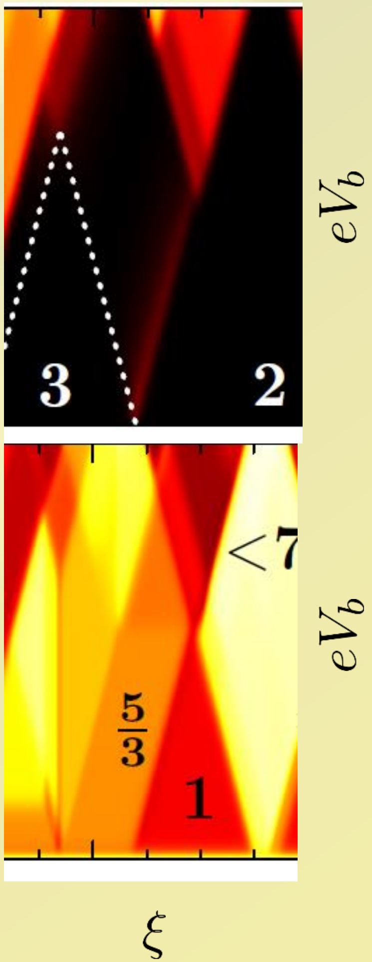
M. Niklas, A. Trottmann, AD, and M. Grifoni, arXiv:1610.08447

Fingerprints of interference



Let us consider the $2_0 \leftrightarrow 3_0$ resonance. For unidirectional transport with $\mu_L > \mu_R$

$$\dot{\rho}^3 = -\frac{i}{\hbar}\left[H_{\mathrm{LS}}, \rho^3\right] + 2\Gamma \mathcal{R}_L \rho^2 - \frac{\Gamma}{2}\left\{\mathcal{R}_R, \rho^3\right\}$$



Angular momentum basis

$$\mathcal{R}_L = \begin{pmatrix} 1 & e^{i\frac{2\pi}{3}} \\ e^{-i\frac{2\pi}{3}} & 1 \end{pmatrix}$$

$$\mathcal{R}_R = \begin{pmatrix} 1 & e^{-i\frac{2\pi}{3}} \\ e^{i\frac{2\pi}{3}} & 1 \end{pmatrix}$$

Dark state basis

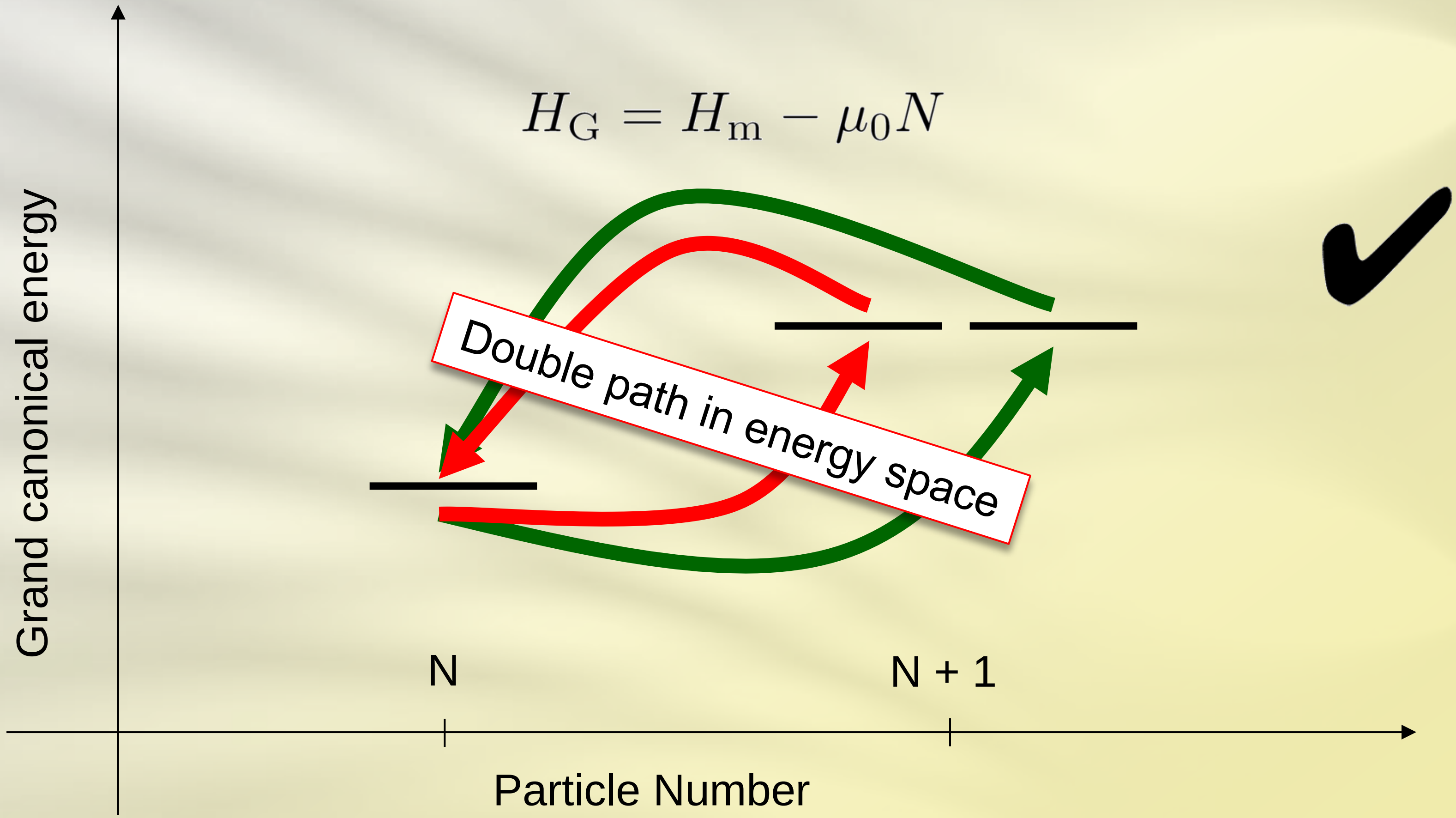
$$\mathcal{R}_L = \frac{1}{2}\begin{pmatrix} 3 & -i\sqrt{3} \\ i\sqrt{3} & 1 \end{pmatrix}$$

$$\mathcal{R}_R = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$$

$$F_{nv} = 1 + \frac{2\Gamma_L^f}{\Gamma_L^s + \Gamma_R^s} = 1 + \frac{2}{3} = \frac{5}{3}$$

M. Niklas, A. Trottmann, AD, and M. Grifoni, arXiv:1610.08447

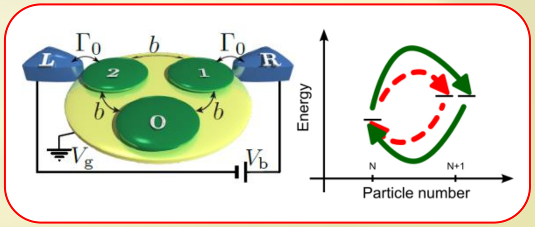
Interfernce + interaction



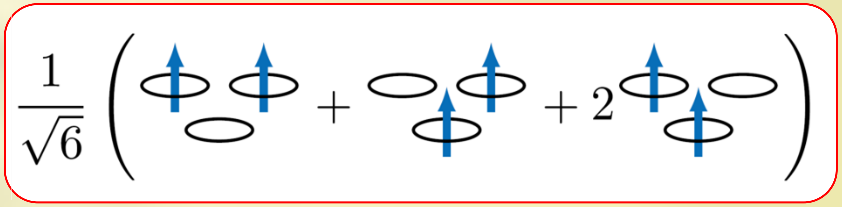
AD, G. Begemann, M. Grifoni, *Phys. Rev. B* **82,** 125451 (2010)

Conclusions

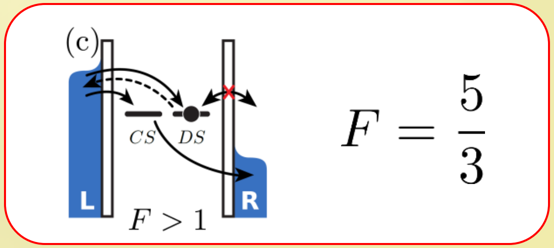
Interference does occur in the single-electron tunnelling regime when energetically **equivalent paths** involving **degenerate states** contribute to the dynamics



Interference blockade of the current appears due to the existance of many-body **dark states**



Fingerprints of the interference effects are **super-Poissonian** Fano factors (e.g. F = 5/3) which indicate a characteristic bunching dynamics.



Aknowledgments

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SFB 689 Spinphänomene in reduzierten Dimensionen

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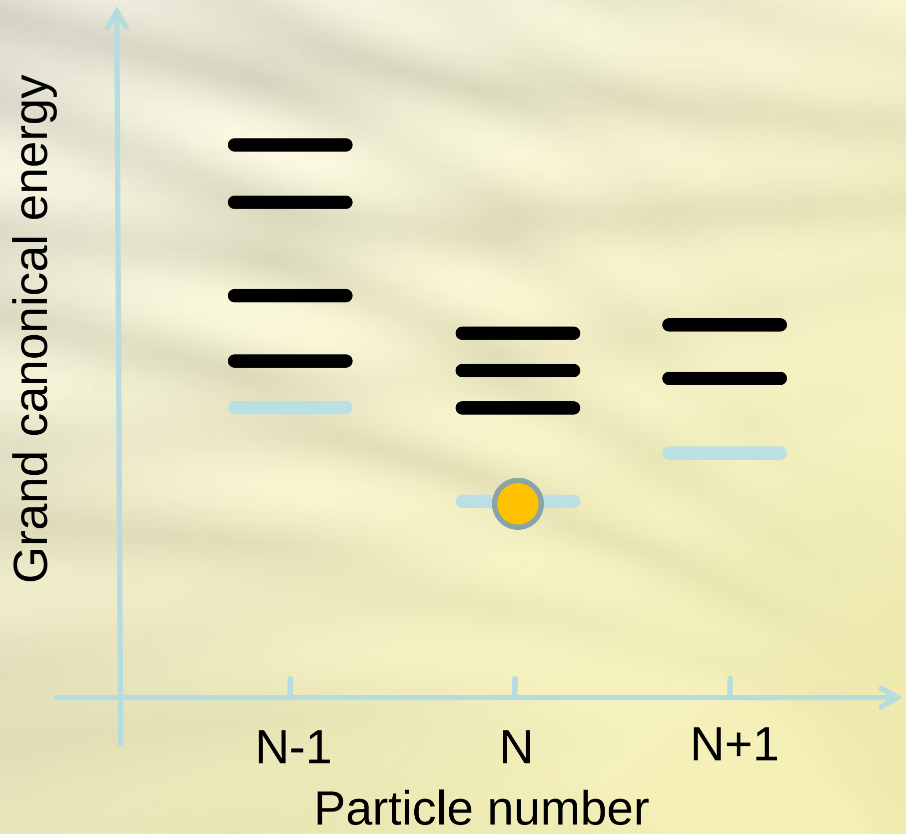
Thank you for your attention !



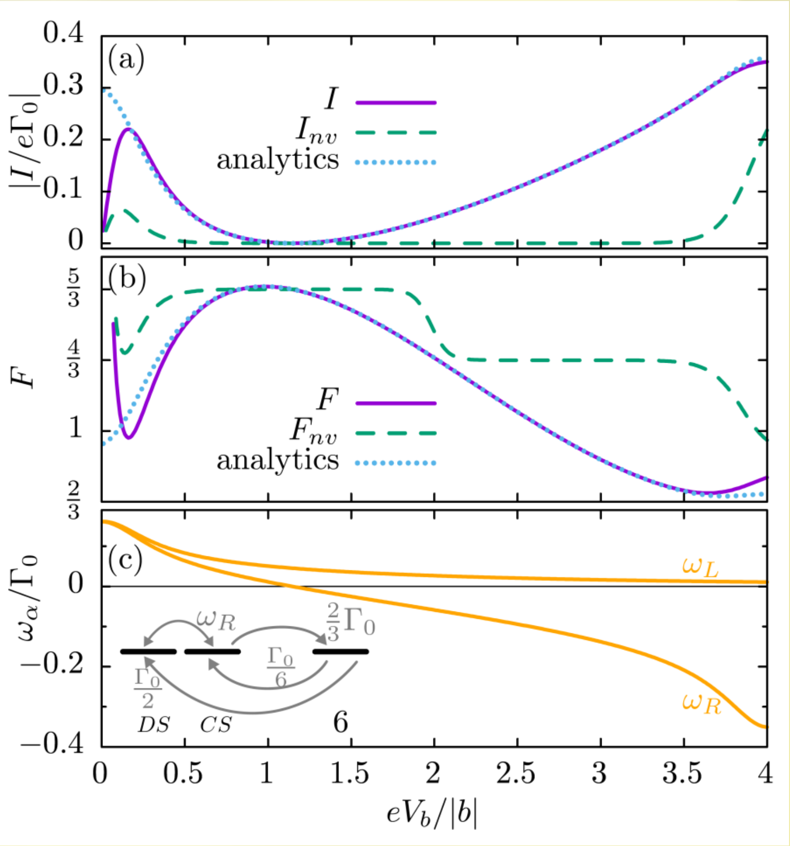
Interference dynamics

$$0 = -\frac{i}{\hbar}\left[H_{\rm LS}, \rho^5\right] + 2\Gamma\mathcal{R}_R\rho^6 - \frac{\Gamma}{2}\left\{\mathcal{R}_L, \rho^5\right\}$$

$$H_{\rm LS} = \hbar \sum_\alpha \omega_\alpha \mathcal{R}_\alpha$$

$$\omega_\alpha = \frac{\Gamma_0}{2\pi}\sum_{\tau,E} p_\alpha\left(E - E_{5_0}\right)\times$$
$$\langle 5, E_{5_0}; \tfrac{1}{2}, S_z, L_z | d_{0\tau}\mathcal{P}_{6,E}d_{0\tau}^\dagger | 5, E_{5_0}; \tfrac{1}{2}, S_z, -L_z\rangle$$
$$+\, p_\alpha\left(E_{5_0} - E\right)\times$$
$$\langle 5, E_{5_0}; \tfrac{1}{2}, S_z, L_z | d_{0\tau}^\dagger\mathcal{P}_{4,E}d_{0\tau} | 5, E_{5_0}; \tfrac{1}{2}, S_z, -L_z\rangle$$

$$p_\alpha\left(\Delta E\right) = -\,\mathrm{Re}\,\psi\left[1/2 + i(\Delta E - \mu_\alpha)/(2\pi k_{\rm B}T)\right]$$



Robustness

We tested the robustness of the interference effects on the $5_0 \leftrightarrow 6$ transition against the perturbation

$$H_\Delta = \frac{1}{2}\begin{pmatrix} 0 & \Delta E \\ \Delta E & 0 \end{pmatrix}$$

and kept $\Delta E \ll k_B T$ to neglect modification of the tunnelling Liouvillean