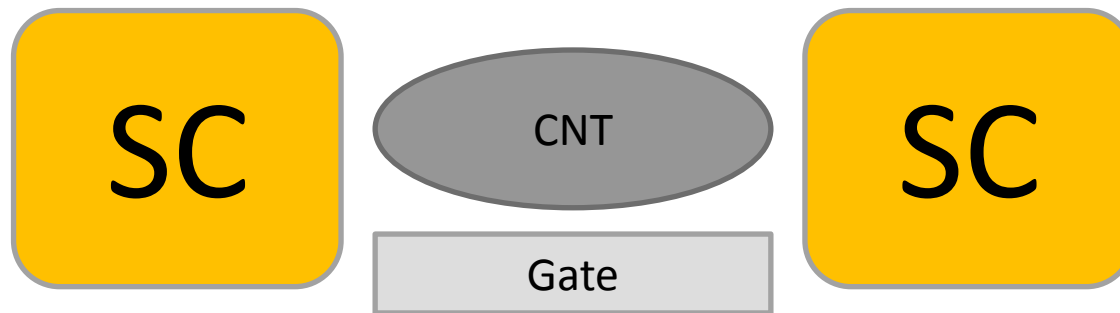


# Thermally induced subgap features in the cotunneling spectroscopy of a carbon nanotube

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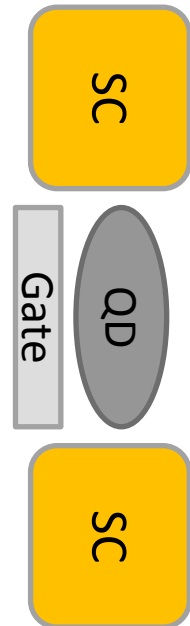
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# Several phenomena

Hybrid SC quantum dot devices in the Coulomb Blockade regime show several transport phenomena:

- Supercurrent transport carried by **Cooper pairs**
  - L. I. Glazman, and K. A. Matveev, *JETP Lett.* **49**, 659 (1989)
  - J. Baselmans, A. F. Morpurgo, et al., *Nature* **43**, 397 (1999)
  - A. V. Rozhkov, D. P. Arovas, and F. Guinea, *Phys. Rev. B* **64**, 233301 (2001)
  - P. Jarillo-Herrero, J. A. van Dam, and L. Kouwenhoven, *Nature* **439**, 953 (2006)
- Coherent electron transport in terms of **multiple Andreev reflections**
  - E. Scheer, W. Belzig, et al., *Phys. Rev. Lett.* **86**, 284 (2001)
  - M. R. Buitelaar, W. Belzig, et al., *Phys. Rev. Lett.* **91**, 057005 (2003)
  - B. M. Andersen, K. Flensberg, et al., *Phys. Rev. Lett.* **107**, 256802 (2011)
  - F. Deon, V. Pellegrini, et al., *Phys. Rev. B* **84**, 100506 (2011)
- **Quasi-particle** transport
  - A. Levy Yeyati, J. C. Cuevas, et al., *Phys. Rev. B* **55**, 6137 (1997)
  - V. N. Golovach and D. Loss, *Phys. Rev. B* **69**, 245327 (2004)
  - J. A. van Dam, Y. V. Nazarov, et al., *Nature* **442**, 667 (2006)
  - A. Eichler, M. Weiss, et al., *Phys. Rev. Lett.* **99**, 126602 (2007)
  - K. Grove-Rasmussen, H. I. Jørgensen et al., *Phys. Rev. B* **79**, 134518 (2009)
  - S. De Franceschi, L. Kouwenhoven, et al., *Nat. Nanotechnol.* **5**, 703 (2010)
  - S. Pfaller, A. Donarini, and M. Grifoni, *Phys. Rev. B* **87**, 155439 (2013)
  - M. Gaass, S. Pfaller, et al., *Phys. Rev. B* **89**, 241405 (2014)



# A specific transport regime

A few energy scales define the hybrid junction:

Temperature	$T$	24 – 1700 mK
Lead-dot (bare) tunnelling rate	$\hbar\Gamma$	0.1 meV
Superconducting gap	$\Delta$	0.26 (1.2) meV
Charging energy	$U$	15 meV

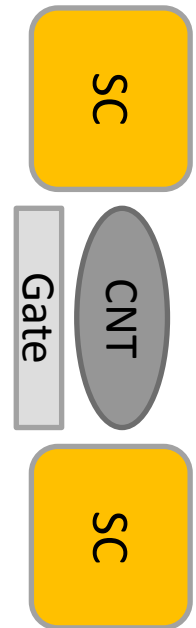
$\hbar\Gamma < \Delta$  → Suppression of Andreev reflection

$\hbar\Gamma \ll U$  → Suppression of multiple quasiparticle tunnelling

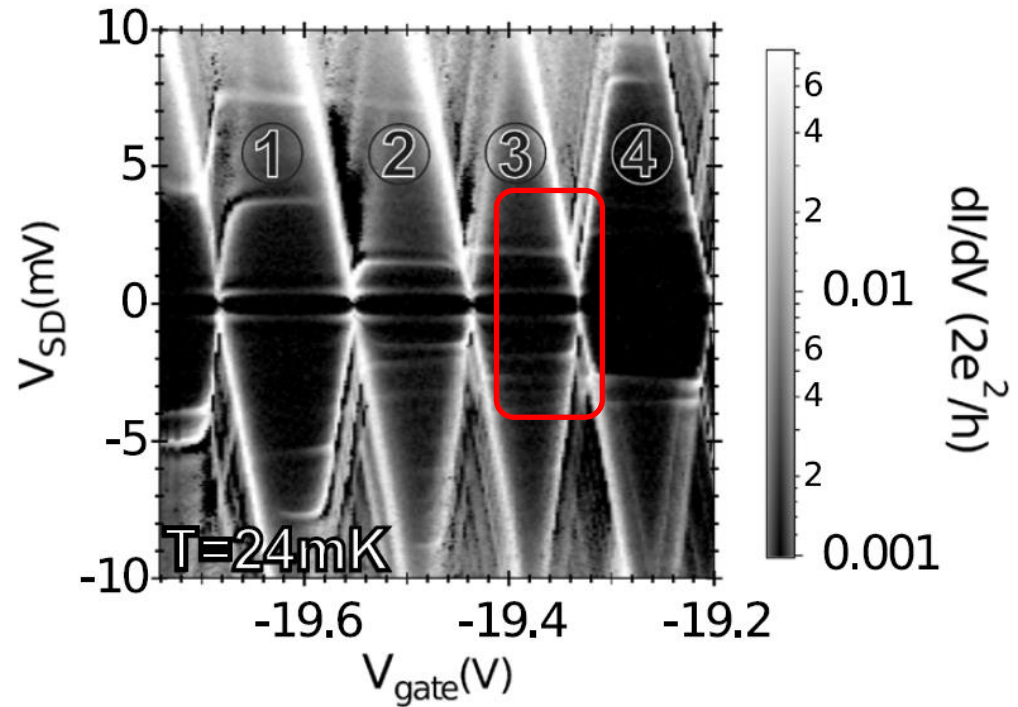
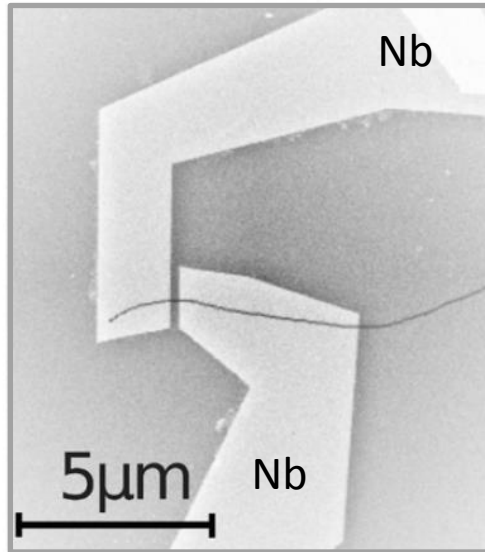
$k_B T \approx \Delta$  → Thermal excitation of quasiparticles



**Sequential** tunnelling and **cotunnelling** processes suffice for the description of the system dynamics



# Motivation



Cotunnelling lines are visible at:

$$V_b = \pm 2\Delta/e$$

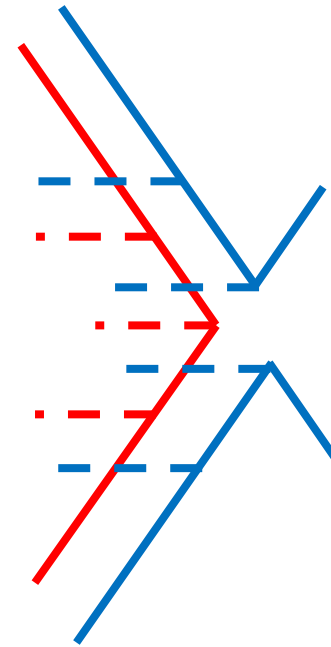
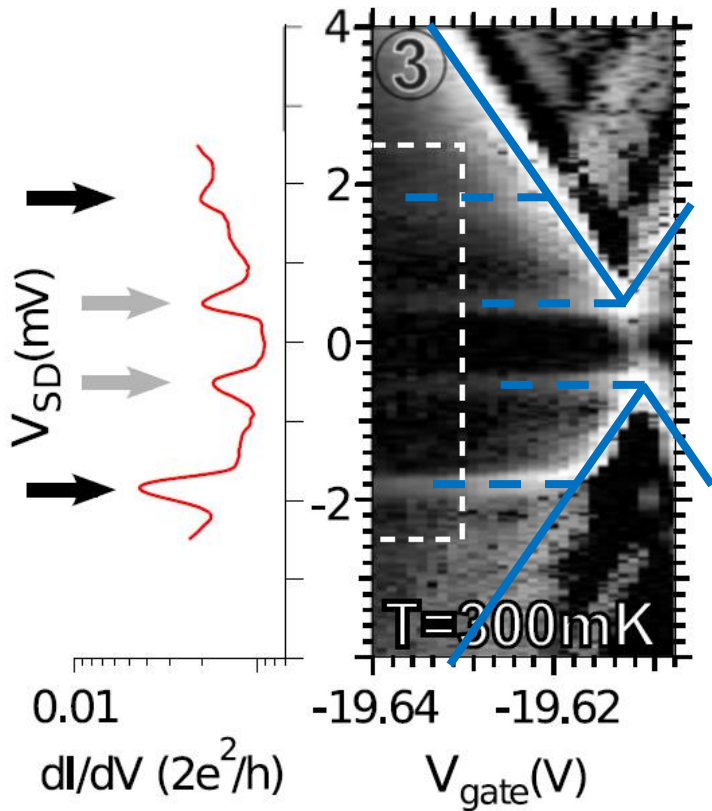
**Elastic** cotunnelling

$$V_b = \pm (2\Delta + \delta_m)/e$$

**Inelastic** cotunnelling

$\{\delta_m\}$  is the set of excitation energies for the quantum dot with  $N$  particles

# Thermally induced effects



- Standard sequential tun.
- - - Standard cotunnelling

- Therm. induced sequential tun.
- - - Therm. induced cotunnelling.

M. Gaass, et al. *Phys. Rev. B* **89**, 241405(R) (2014)

S. Ratz et al., *New J. Phys.* **16**, 123040 (2014)

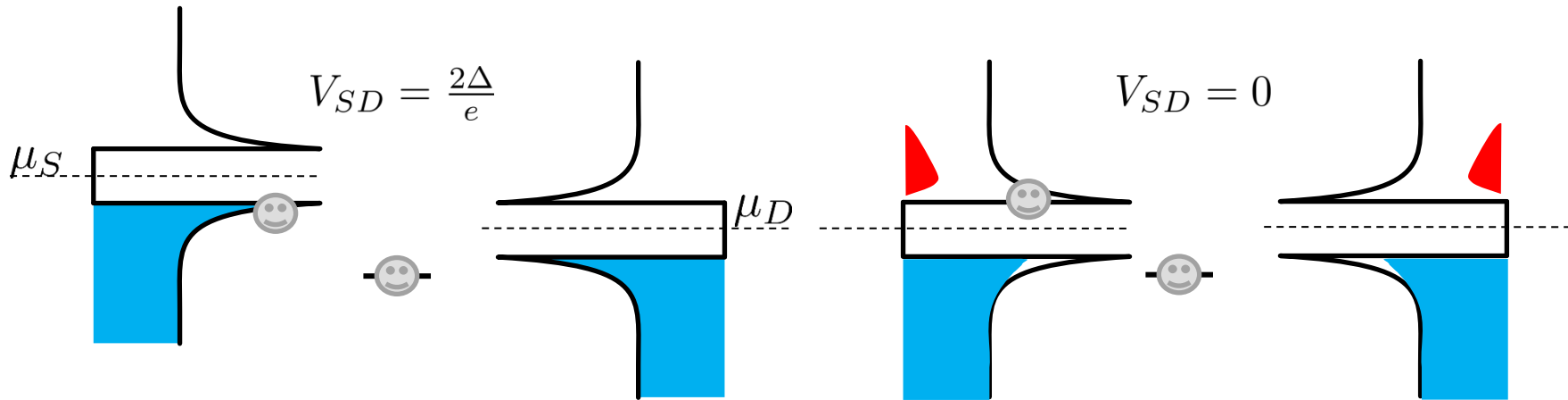
S. Pfaller et al., *Phys. Rev. B* **87**, 155439 (2013)

# Cotunnelling processes

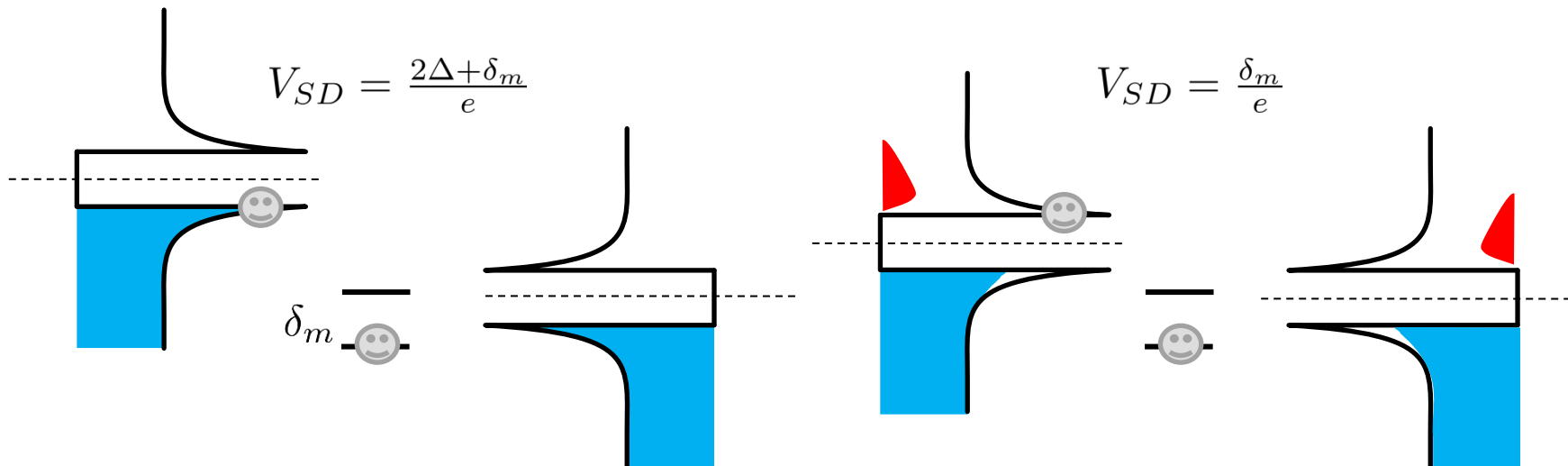
Low temperature

High temperature

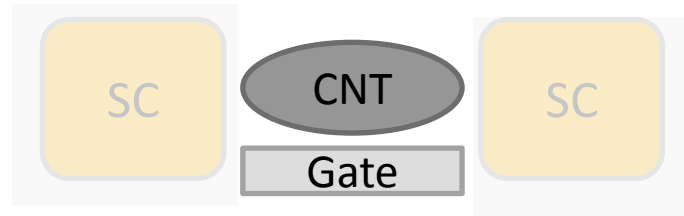
Elastic cotunnelling



Inelastic cotunnelling



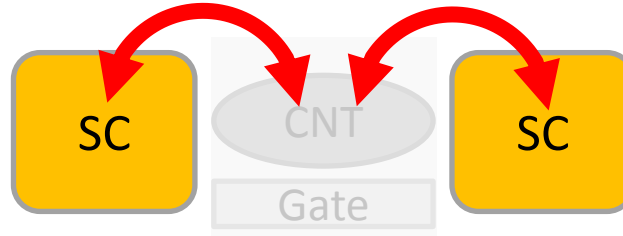
# Model



$$\hat{H}_{\text{CNT}} = \underbrace{\sum_{m\sigma} E_{m\sigma} \hat{d}_{m\sigma}^\dagger \hat{d}_{m\sigma}}_{\text{Single shell + spin orbit and KK' mixing}} + \underbrace{\frac{U}{2} \hat{N} (\hat{N} - 1)}_{\text{Constant interaction}} - \underbrace{\alpha e V_{\text{gate}} \hat{N}}_{\text{Gate shift}}$$

$$E_{m\sigma} = \epsilon_d + \frac{1}{2} m \sigma \delta$$

# Model



$$\hat{H}_l = E_l^0 + \underbrace{\sum_{\vec{k}\sigma} E_{l\vec{k}} \hat{\gamma}_{l\vec{k}\sigma}^\dagger \hat{\gamma}_{l\vec{k}\sigma}}_{\text{Quasiparticle excitations}} + \mu_l \hat{N}_l$$

$$E_{l\vec{k}} = \sqrt{(\epsilon_{\vec{k}} - \mu_l)^2 + \Delta^2}$$

The gap equation

$$\Delta \equiv |V| \sum_{\vec{k}} \left\langle \hat{S}_l^\dagger \hat{c}_{l-\vec{k}\downarrow} \hat{c}_{l\vec{k}\uparrow} \right\rangle$$

$$\hat{H}_{T,l} = T_l \sum_{\vec{k}\sigma m} \left( \hat{d}_{m\sigma}^\dagger \hat{c}_{l\vec{k}\sigma} + \text{h.c.} \right)$$

Bogolioubov – Valatin

particle conserving transformation

$$\hat{c}_{l\vec{k}\sigma}^\dagger = u_{l\vec{k}} \hat{\gamma}_{l\vec{k}\sigma}^\dagger + \sigma v_{l\vec{k}}^* \hat{S}_l^\dagger \hat{\gamma}_{l-\vec{k}\bar{\sigma}}$$

$$\hat{c}_{l\vec{k}\sigma} = u_{l\vec{k}}^* \hat{\gamma}_{l\vec{k}\sigma} + \sigma v_{l\vec{k}} \hat{S}_l \hat{\gamma}_{l-\vec{k}\bar{\sigma}}^\dagger$$

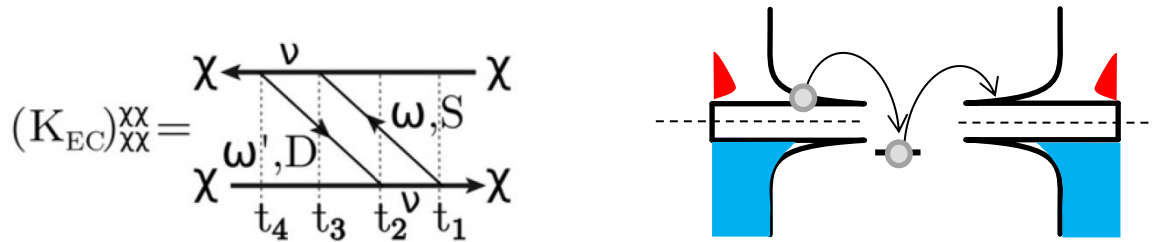


# Transport calculation

The dynamics is described by the generalized master equation

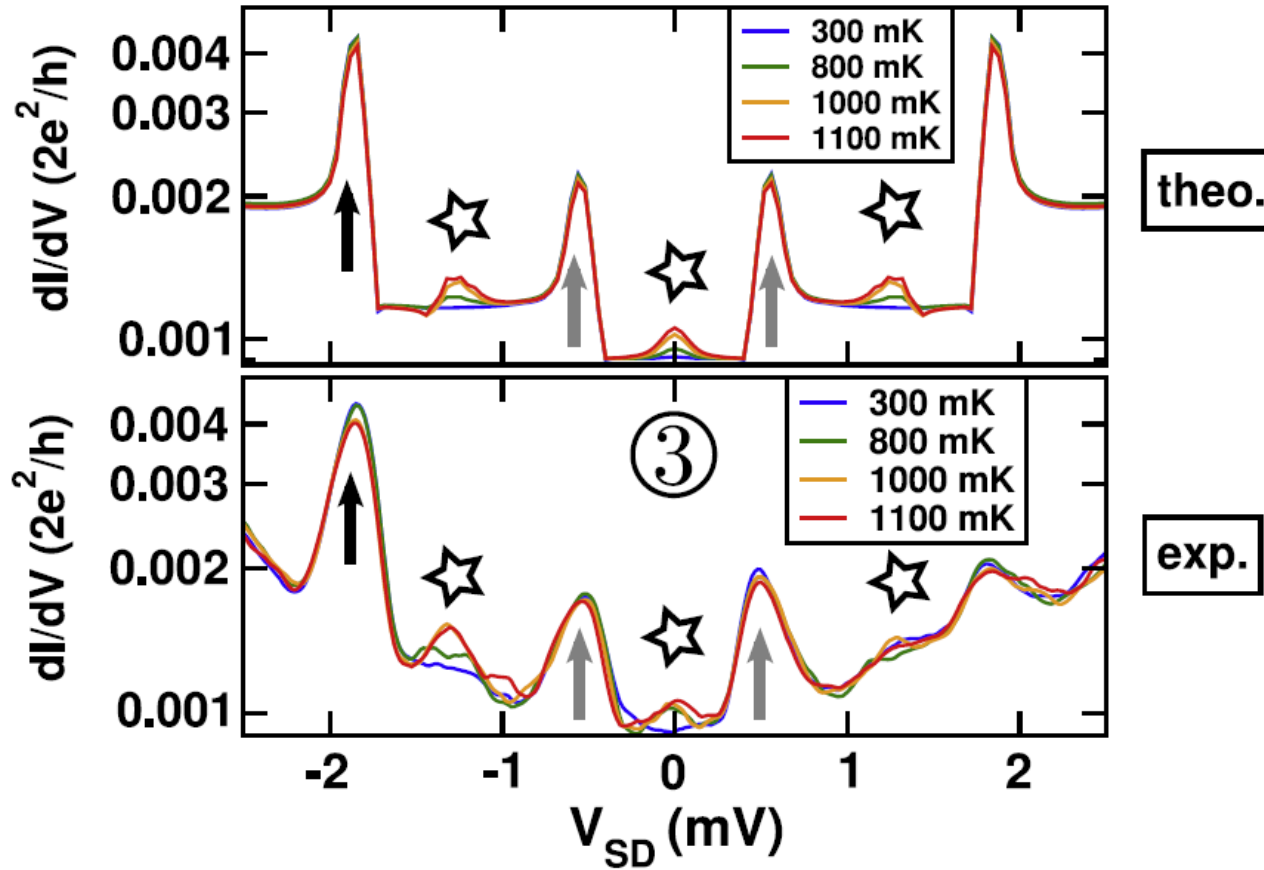
$$\dot{\hat{\rho}}_{\text{red}}(t) = -\frac{i}{\hbar} \left[ \hat{H}_{\text{CNT}}, \hat{\rho}_{\text{red}}(t) \right] + \int_{t_0}^t d\tau \hat{K}(t - \tau) \hat{\rho}_{\text{red}}(\tau)$$

Exemplarily, one elastic cotunnelling contribution to the evolution kernel reads:



$$\begin{aligned} \left( \hat{K}_{EC} \right)_{\chi\chi}^{\chi\chi} &\equiv -i\hbar\Gamma_S\Gamma_D \sum_{\nu} \int \frac{d\omega}{2\pi} \frac{d\omega'}{2\pi} D_S(\omega, \Delta) D_D(\omega', \Delta) \\ &\times \frac{f_S(\omega)(1 - f_D(\omega'))}{(-\omega + \delta E + i0^+)(\omega' - \omega + i0^+)(\omega' - \delta E + i0^+)} \end{aligned}$$

# Theory vs. Experiment



# Conclusions and outlook

- We report on **new transport properties** of a CNT contacted with two superconducting Nb leads.
- **Thermal replicas** of the elastic and inelastic cotunnelling resonances are observed for a temperature above 600 mK ( $k_B T \approx \Delta$ ): i.e. an extra **zero-bias peak** and **inelastic peak** corresponding to the lower excitation energy.
- We develop a generalized master equation for the reduced density matrix in the charge-conserved regime applicable to **any intradot interaction** and **finite** superconducting gap.
- A model of the CNT with a low energy interacting spectrum gives **remarkable agreement** with the experimental results.
- Further developments of the theory to include multiple Andreev reflections and supercurrent transport carried by Cooper pairs is envisaged.