Coulomb blockade and single electron tunnelling

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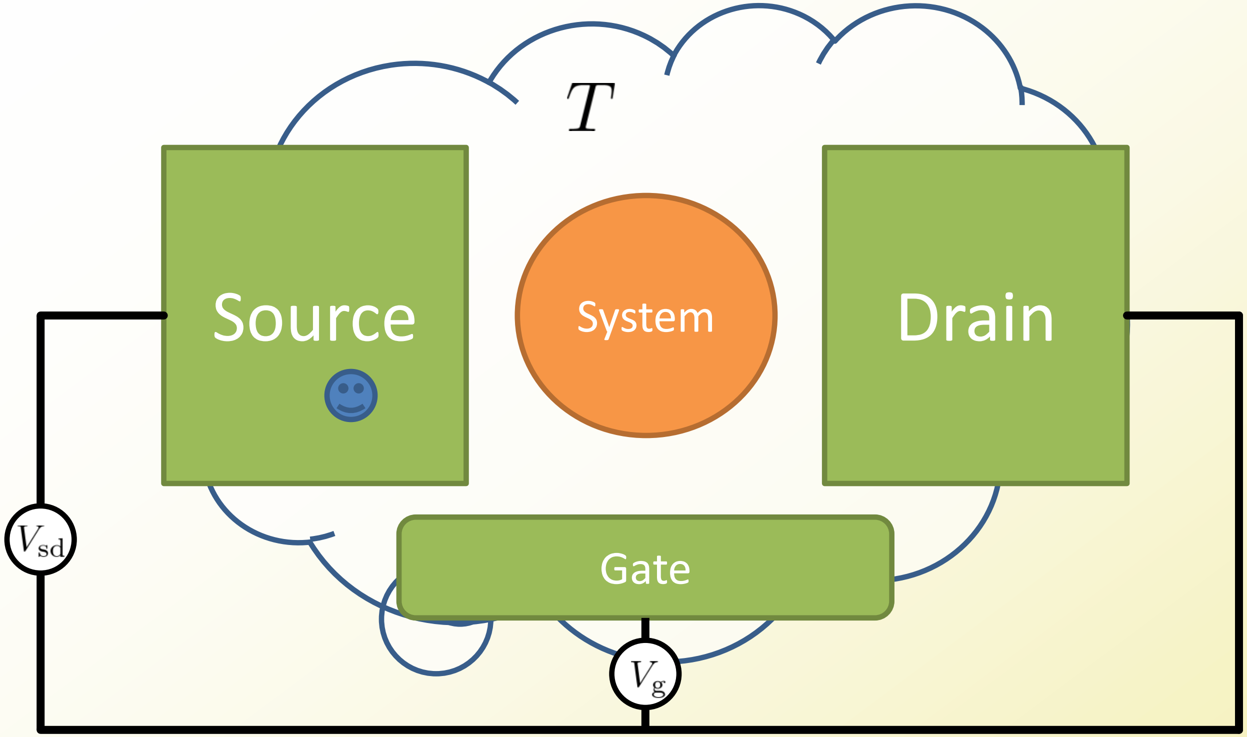
GRK Tutorial

Three terminal device



Variation of the electrostatic energy $\Delta E = \frac{e^2}{C}$

Typical time scale $\delta t = R_t C$

Single electron detection

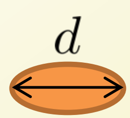
How **small** and how **cold** should the system be such that a **single electron** tunneling has a **measurable** effect ?

Thermal fluctuations

$$\Delta E \gg k_{\mathrm{B}} T$$

$$e^2/C \gg k_{\mathrm{B}} T$$

$$d = 100\mathrm{nm}$$



$$C = 1\mathrm{pF}$$

$$T \ll 1\mathrm{K}$$

Quantum uncertainty

$$\Delta E \delta t \gg \delta E \delta t \geq h$$

$$R_t \gg h/e^2$$

$$R_t \gg 13\mathrm{k}\Omega$$

First device (1987)

Aluminum- Aluminum oxide, Au-Cr gate electrode

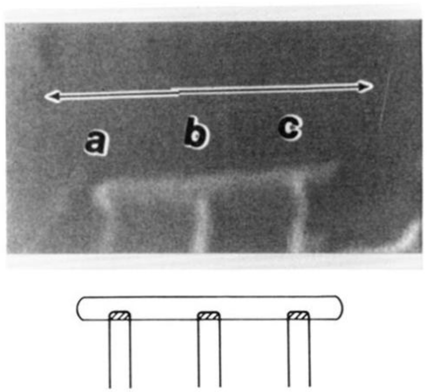


FIG. 2. A scanning-electron micrograph of a typical sample. Junctions labeled a, b, and c are formed where the vertical electrodes overlap and contact the longer horizontal central electrode. The bar is 1 μm long. The configuration is also shown in the accompanying drawing.

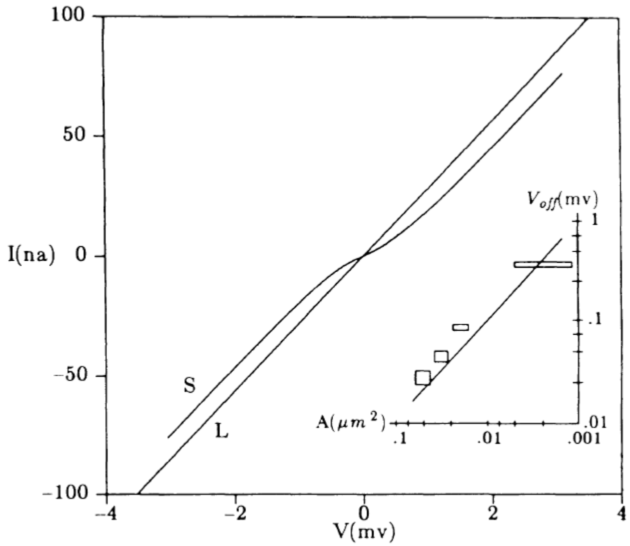


FIG. 3. I-V curves S and L for junctions corresponding to small and large C' at 1.7 K. Inset: Offset voltage vs junction areas (as determined from scanning-electron-microscopy photographs) for four different samples. The boxes represent the estimated uncertainties.

Fulton and Dolan PRL, **59** 109 (1987)

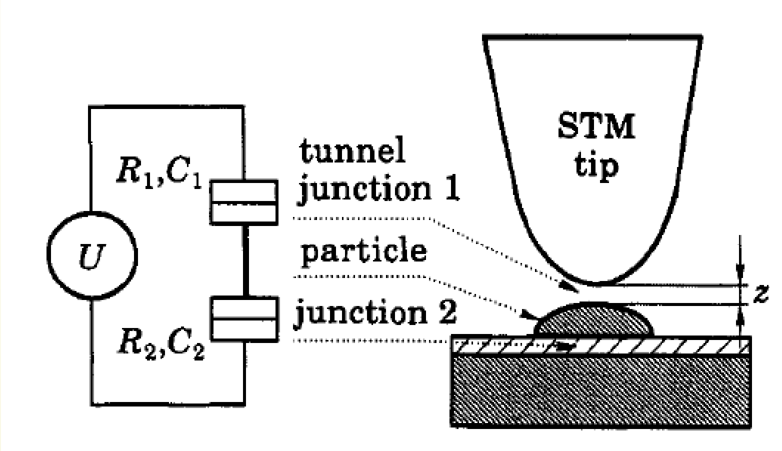
STM set-up (1992)

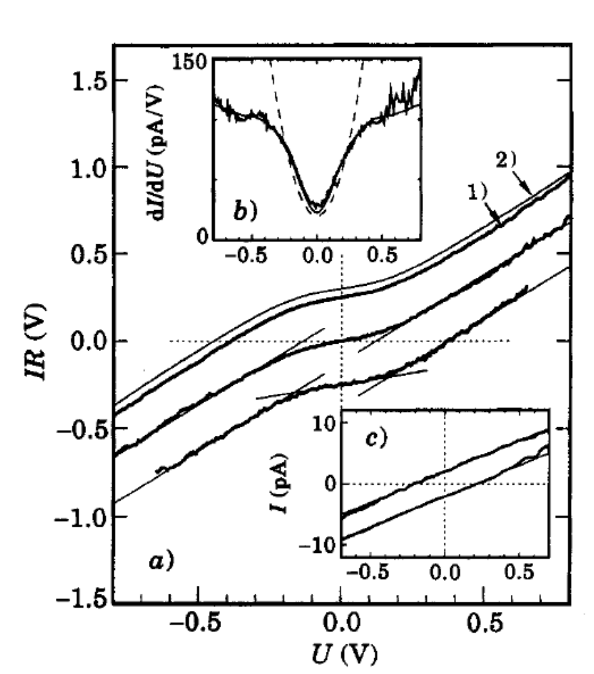
Single-Electron Tunnelling Observed at Room Temperature by Scanning-Tunnelling Microscopy.

C. Schönenberger, H. van Houten and H. C. Donkersloot
Philips Research Laboratories - P.O.Box 80.000, 5600 JA Eindhoven, The Netherlands

(received 22 April 1992; accepted in final form 30 July 1992)

Abstract. – Ultrasmall (≲5 nm in lateral diameter) double-barrier tunnel junctions have been realized using a scanning tunnelling microscope, and an optimized metal particle-oxide-metallic substrate system. Three electrical transport effects, all in good agreement with the semi-classical theory of single-electron tunnelling, have been found at room temperature: the Coulomb gap, the Coulomb staircase and zero-bias conductance oscillations as a function of tip-particle distance.

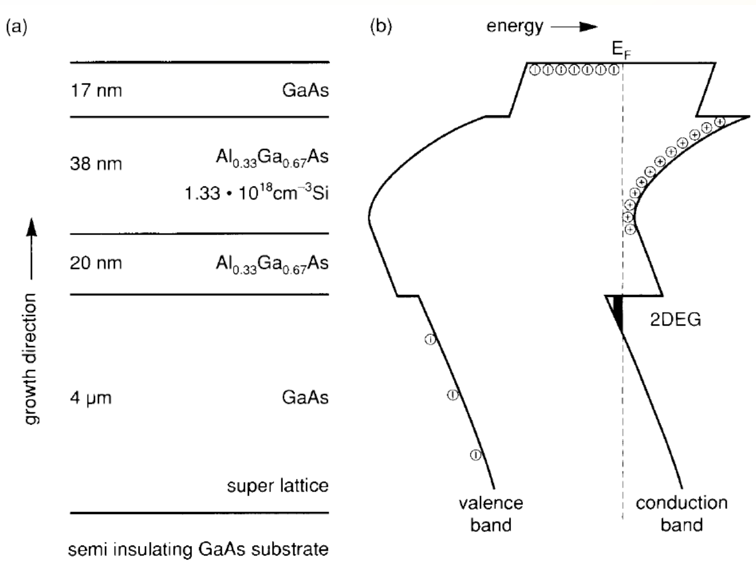


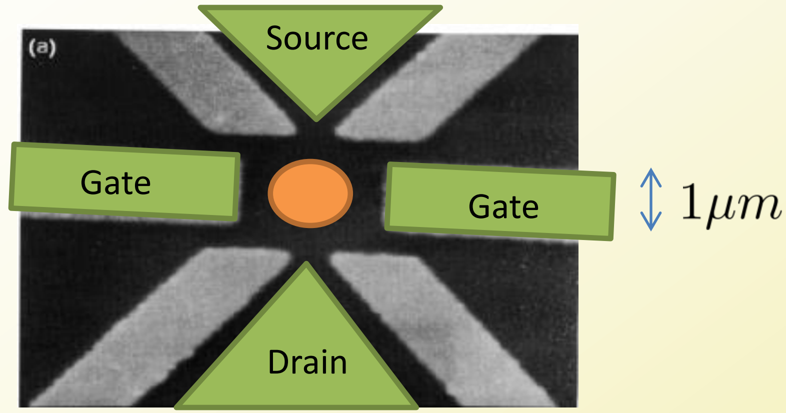


Schönenberger, van Houten and Donkersloot, EPL **20,** 249 (1992)

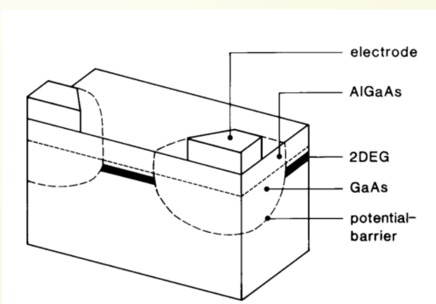
Single electron transistor in 2DEG structures

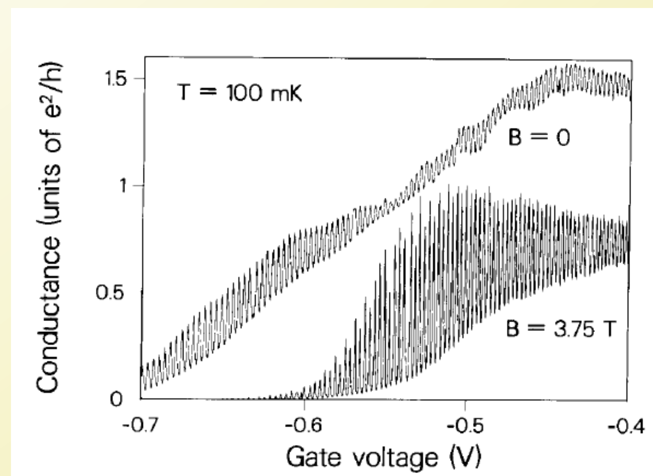
2 dimensional electron gas (2DEG)





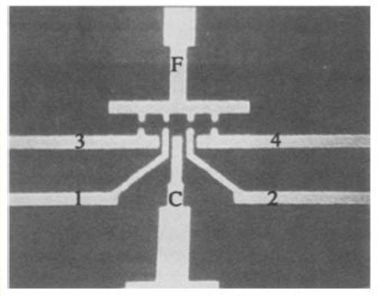
Depleting top metal gates

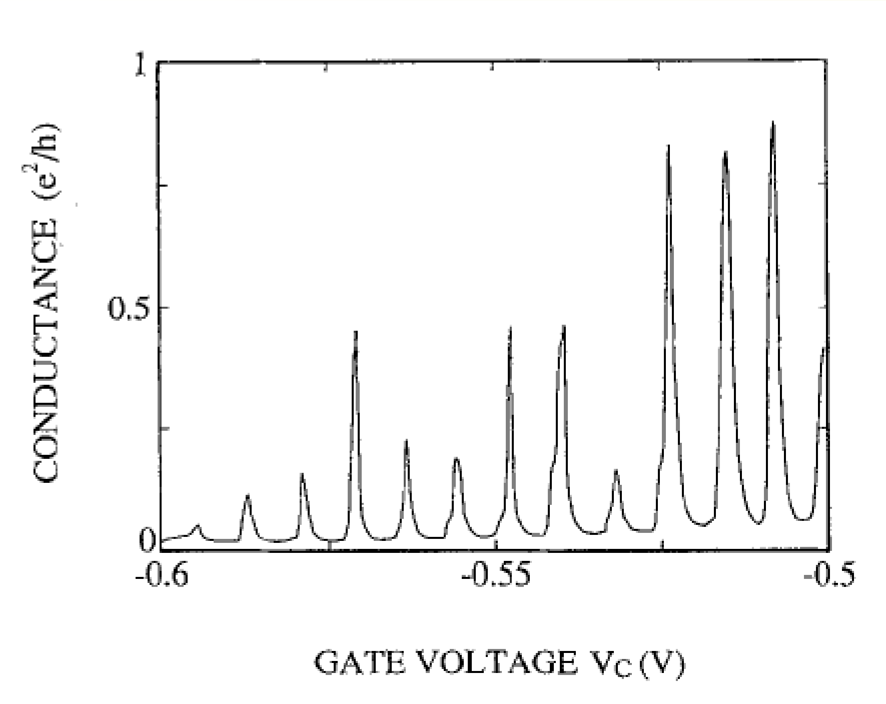




Staring, PhD thesis (1992)

Coulomb oscillations





$d = 600\,\mathrm{nm}$

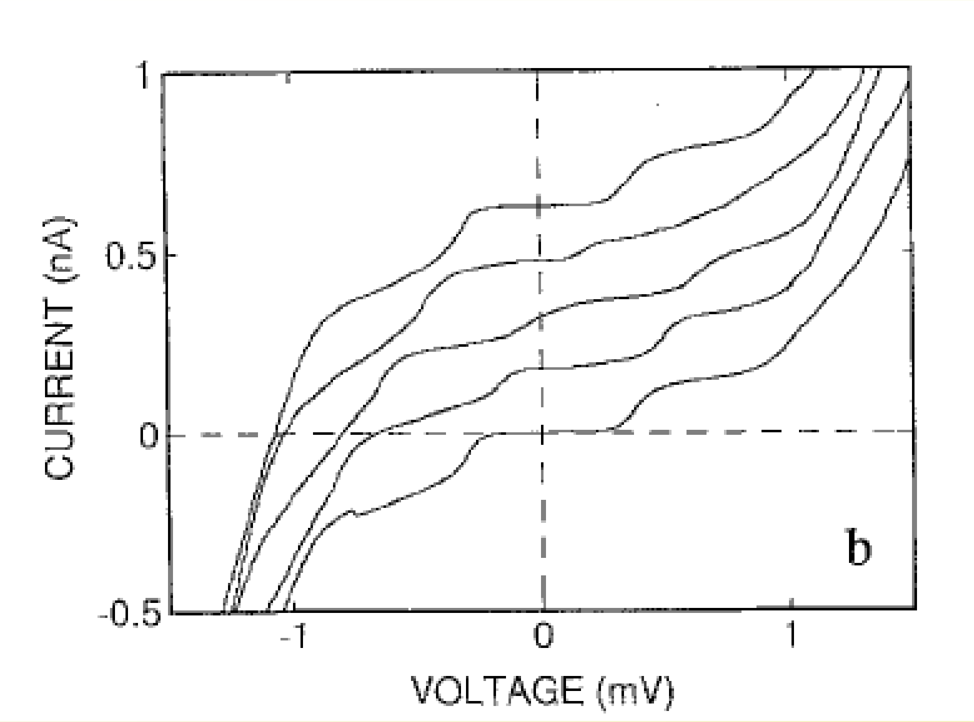
$C = 0.28\,\mathrm{fF}$

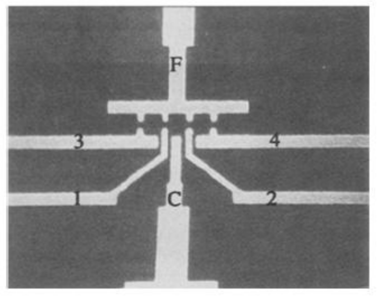
$\Delta E = 0.6\,\mathrm{meV}$

$T = 10\,\mathrm{mK}$

Kouwenhoven *et al.*, Z. Phys. B Condensed Matter **85**, 367 (1991)

Coulomb staircase





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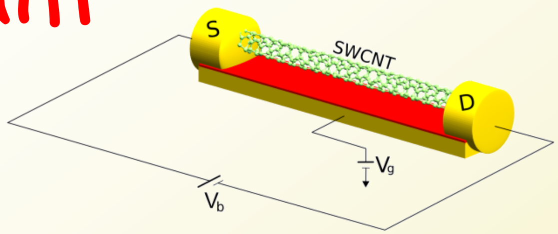
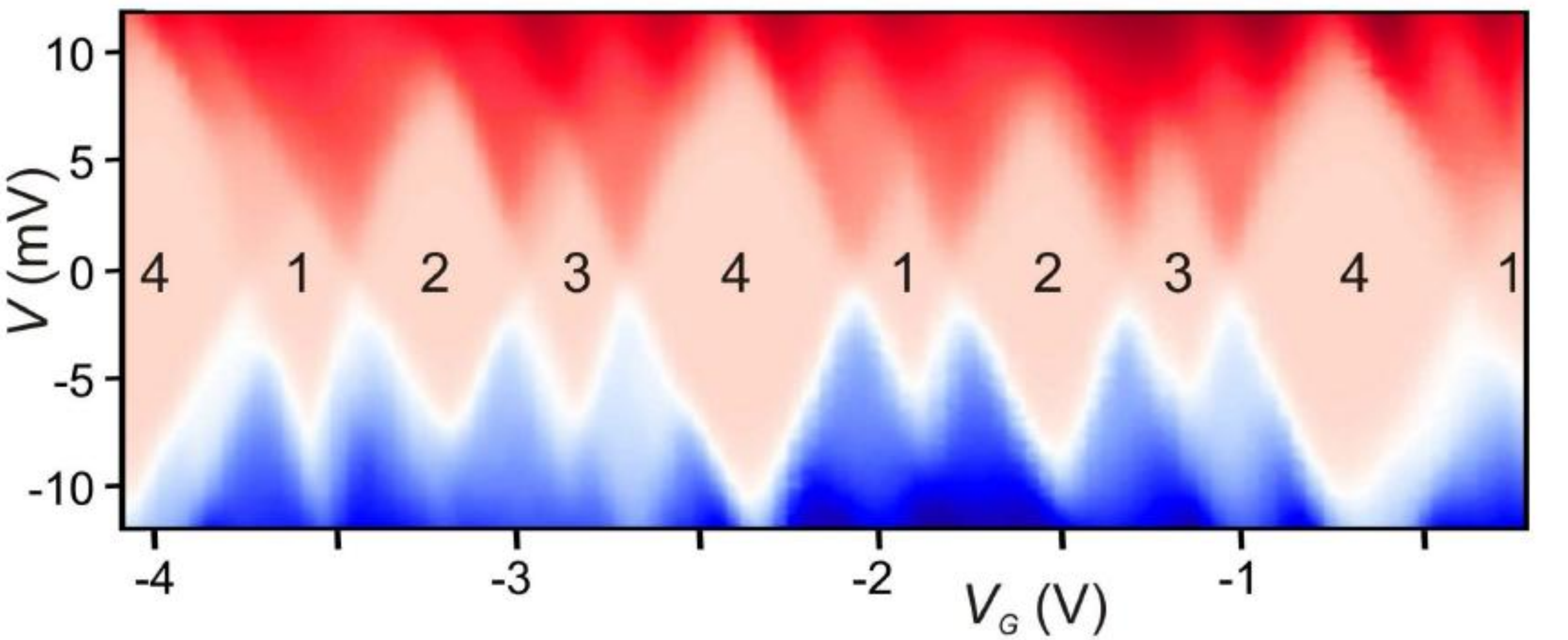
Kouwenhoven *et al.*, Z. Phys. B Condensed Matter **85**, 367 (1991)

Stability diagram

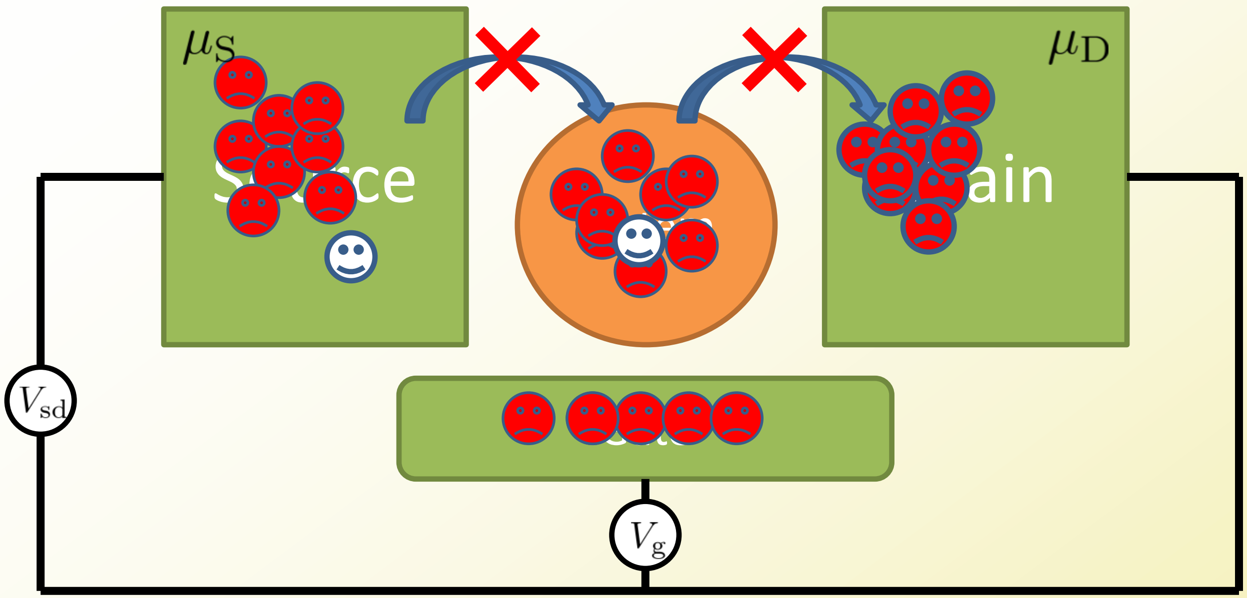




H. van der Zant, unpublished

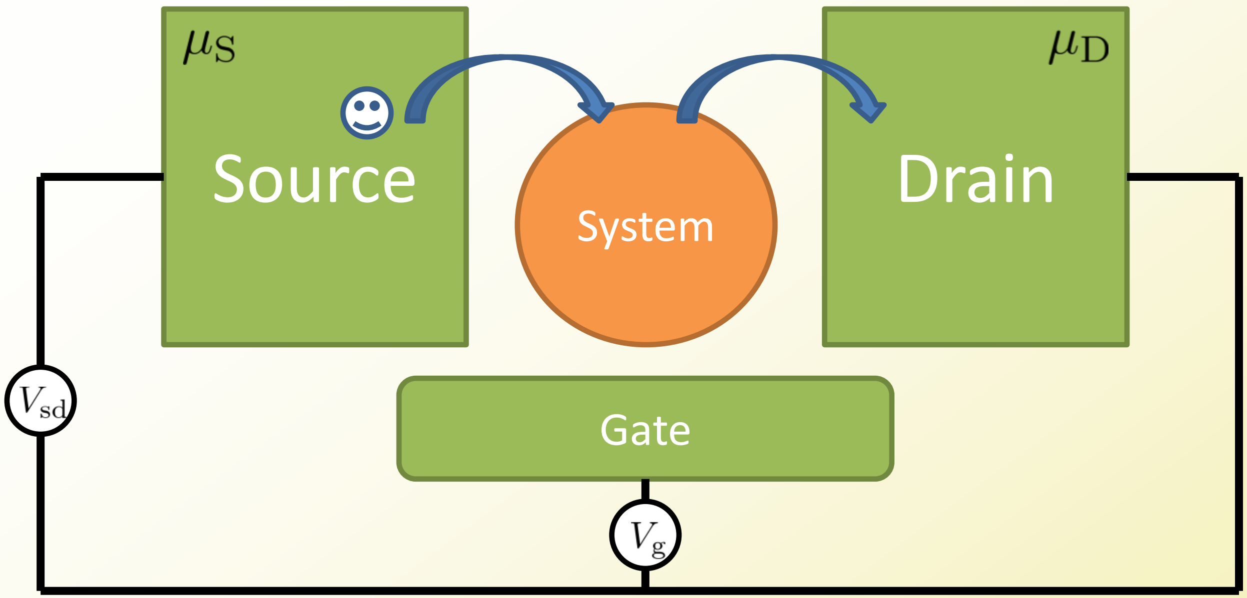
Coulomb blockade

$$\mu_S > \mu_D$$



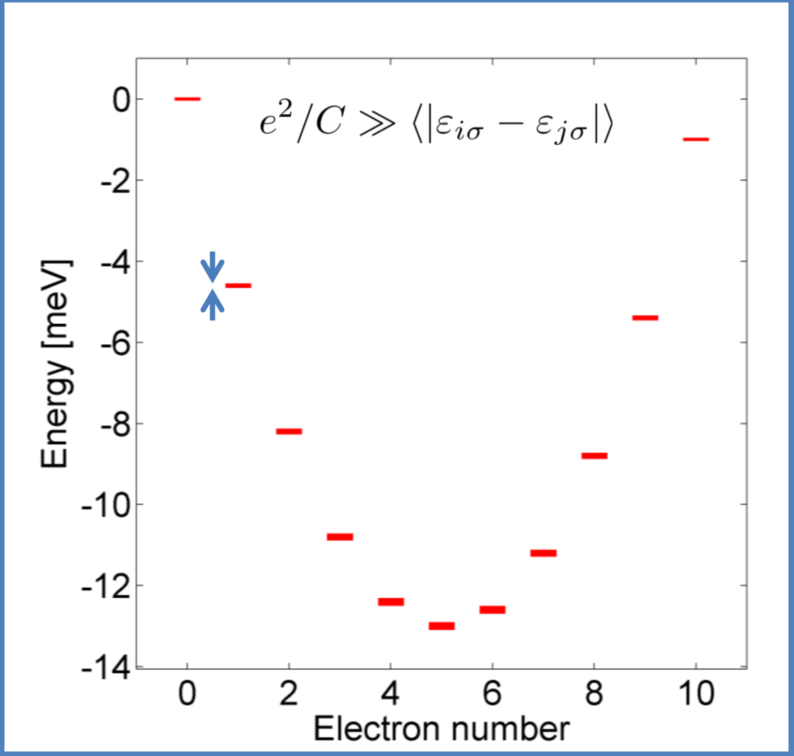
Single electron tunnelling

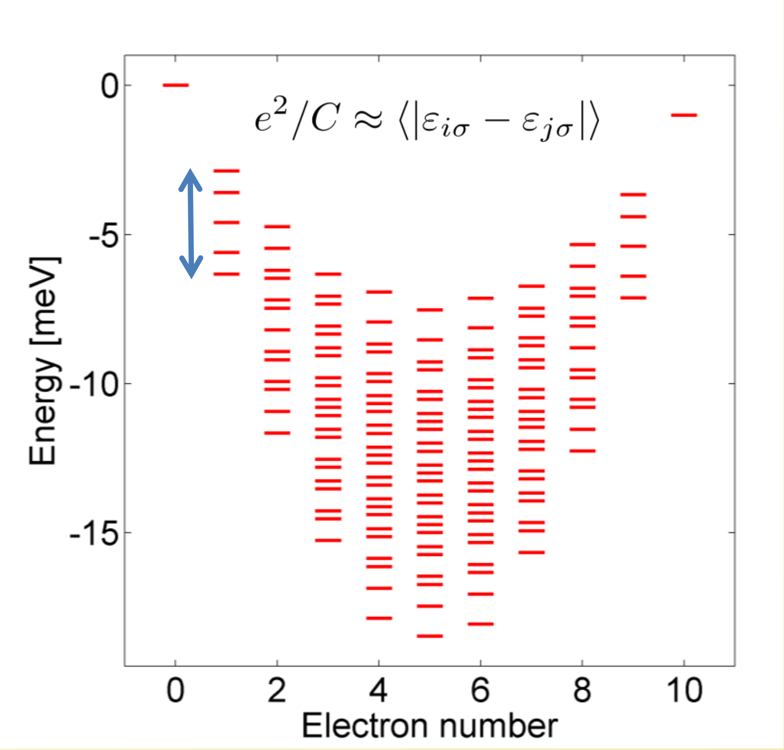
$$\mu_S > \mu_D$$



Discrete vs. Continuum

$$\hat{H}_{\rm sys} = \sum_{i\sigma} \varepsilon_{i\sigma} \hat{c}^\dagger_{i\sigma} \hat{c}_{i\sigma} + \frac{e^2}{C} \frac{\hat{N}(\hat{N}-1)}{2} - \hat{N} e V_{\rm g} \qquad \hat{N} = \sum_{i\sigma} \hat{c}^\dagger_{i\sigma} \hat{c}_{i\sigma}$$





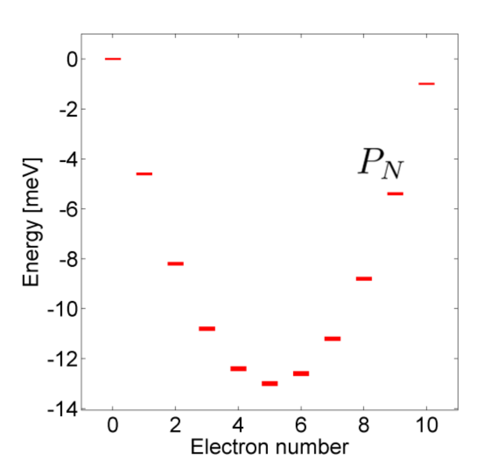
Averin & Likharev (1986)

Master equation

- The only relevant quantity is the probability that the system is occupied with N electrons.

$$\dot{P}_N = -\left(\Gamma^{N+1,N} + \Gamma^{N-1,N}\right)P_N + \Gamma^{N,N+1}P_{N+1} + \Gamma^{N,N-1}P_{N-1}$$

$$\Gamma^{N\pm1,N} = \Gamma_S^{N\pm1,N} + \Gamma_D^{N\pm1,N}$$



- The system relaxes to “local” thermal equilibrium after each tunneling event

$$\Gamma_\alpha^{N+1,N} = \gamma_\alpha \int d\varepsilon_\alpha d\varepsilon_{\rm sys} \underbrace{f^+(\varepsilon_\alpha - \mu_\alpha)}_{\text{lead-level occupation}} \underbrace{f^-(\varepsilon_{\rm sys} - \mu_{\rm sys})}_{\text{system-level occupation}} \underbrace{\delta(Ne^2/C - eV_{\rm g} + \varepsilon_{\rm sys} - \varepsilon_\alpha)}_{\text{energy conservation}}$$

$$\gamma_\alpha = \frac{2\pi}{\hbar}|t|^2 D_\alpha D_{\rm sys}$$

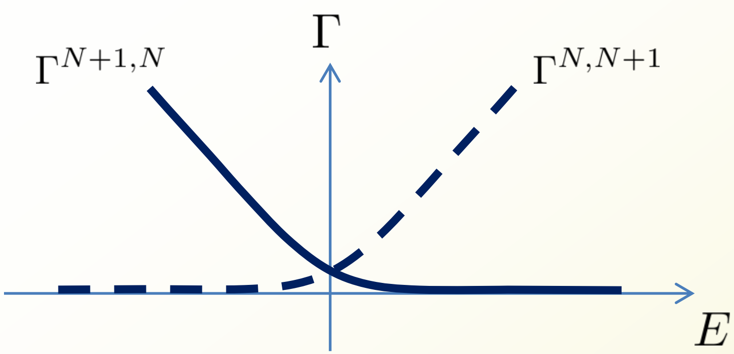
$$\Gamma_\alpha^{N+1,N} = \gamma_\alpha \frac{(Ne^2/C - eV_{\rm g} + \mu_{\rm sys} - \mu_\alpha)}{\exp[\beta(Ne^2/C - eV_{\rm g} + \mu_{\rm sys} - \mu_\alpha)] - 1}$$

Let us assume $\mu_{S/D} = \mu_{\rm sys} \mp \frac{eV_{\rm sd}}{2}$

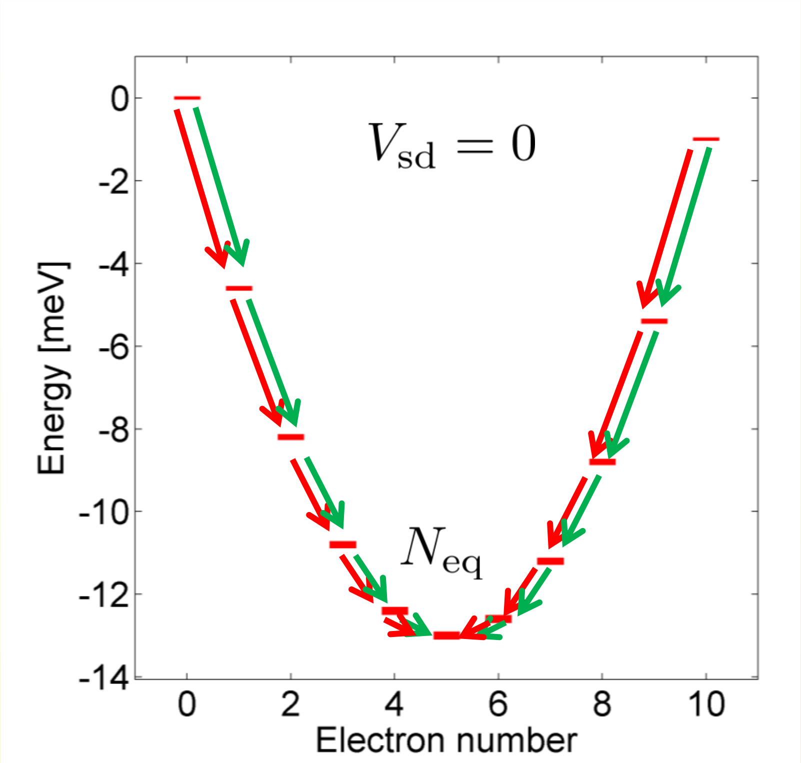
Orthodox theory: Averin & Likharev

Orthodox theory

$$\Gamma_{S/D}^{N+1,N} = \gamma_\alpha \frac{+(Ne^2/C - eV_g \pm eV_{sd}/2)}{\exp[+\beta(Ne^2/C - eV_g + \mu_{sys} - \mu_\alpha)] - 1}$$



$$\Gamma_{S/D}^{N,N+1} = \gamma_\alpha \frac{-(Ne^2/C - eV_g \pm eV_{sd}/2)}{\exp[-\beta(Ne^2/C - eV_g + \mu_{sys} - \mu_\alpha)] - 1}$$



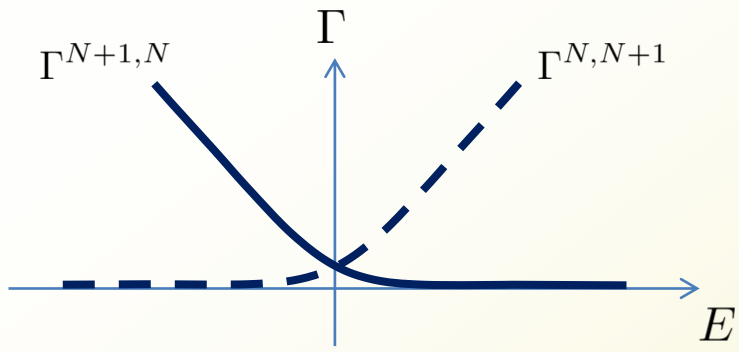
$$N_{eq} \approx V_g C/e$$

Particle number controlled by the gate

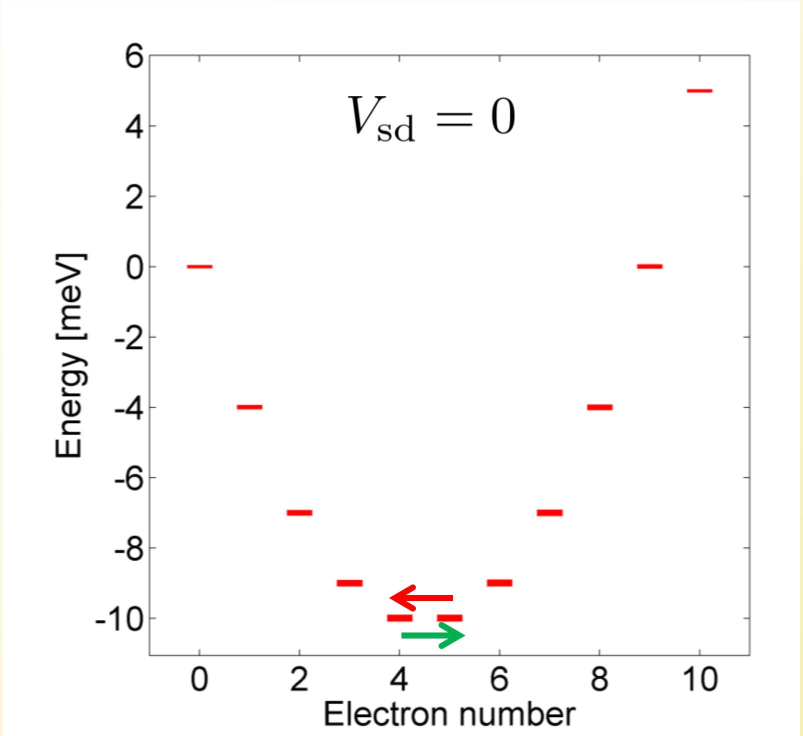
Orthodox theory: Averin & Likharev

Orthodox theory

$$\Gamma_{S/D}^{N+1,N} = \gamma_\alpha \frac{+(Ne^2/C - eV_{\rm g} \pm eV_{\rm sd}/2)}{\exp[+\beta(Ne^2/C - eV_{\rm g} + \mu_{\rm sys} - \mu_\alpha)] - 1}$$



$$\Gamma_{S/D}^{N,N+1} = \gamma_\alpha \frac{-(Ne^2/C - eV_{\rm g} \pm eV_{\rm sd}/2)}{\exp[-\beta(Ne^2/C - eV_{\rm g} + \mu_{\rm sys} - \mu_\alpha)] - 1}$$



$$V_{\rm g} = (N + 1/2)\frac{e}{C}$$

Conductance peak => Coulomb oscillations

Orthodox theory: Averin & Likharev

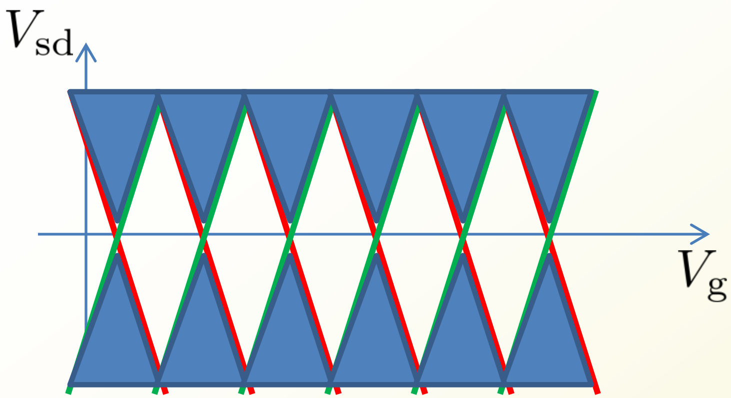
Orthodox theory

$$V_{\mathrm{sd}} > +2V_{\mathrm{g}} + 2Ne/C$$

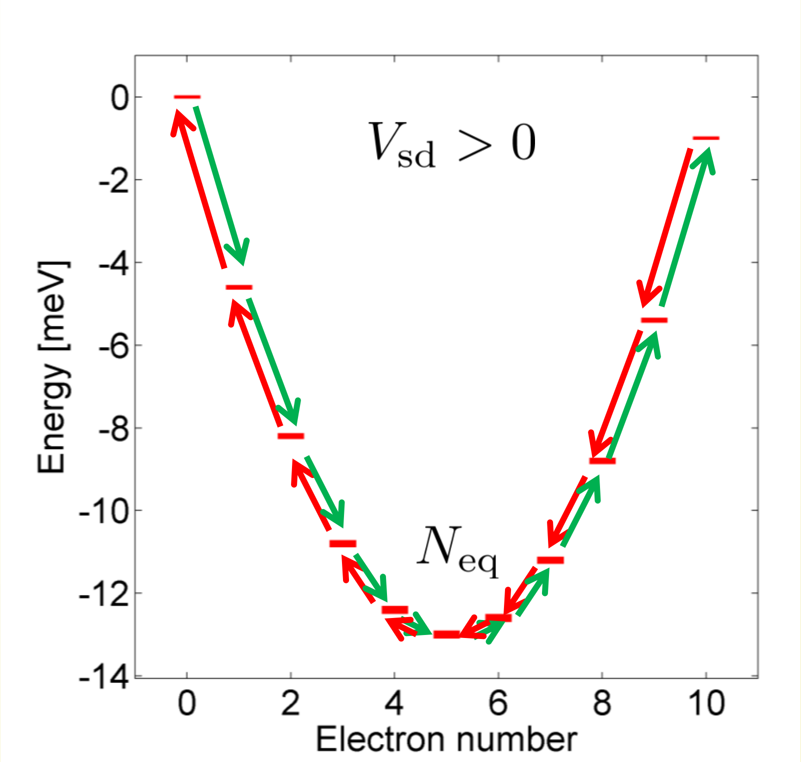
Source N -> N+1
Resonance condition

$$V_{\mathrm{sd}} > -2V_{\mathrm{g}} + 2Ne/C$$

Drain N -> N-1
Resonance condition



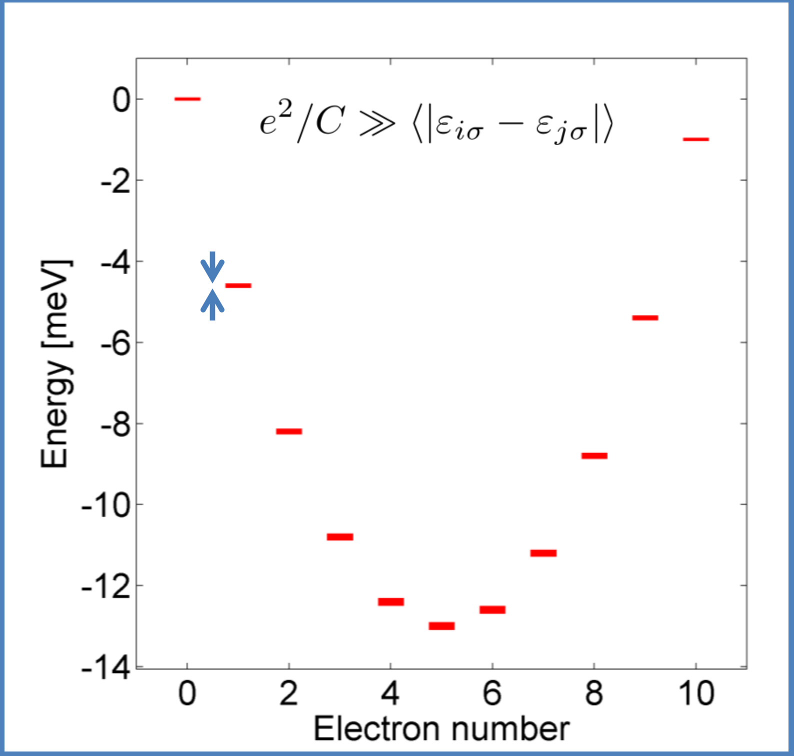
Coulomb diamonds

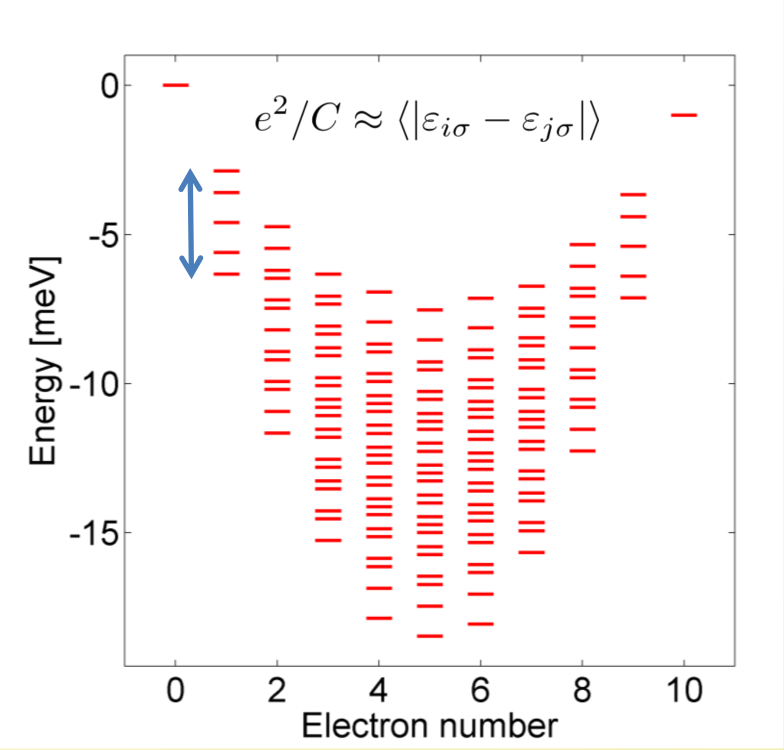


Orthodox theory: Averin & Likharev

Discrete vs. Continuum

$$\hat{H}_{\mathrm{sys}} = \sum_{i\sigma} \varepsilon_{i\sigma} \hat{c}^{\dagger}_{i\sigma} \hat{c}_{i\sigma} + \frac{e^2}{C} \frac{\hat{N}(\hat{N}-1)}{2} - \hat{N} e V_{\mathrm{g}} \qquad \hat{N} = \sum_{i\sigma} \hat{c}^{\dagger}_{i\sigma} \hat{c}_{i\sigma}$$





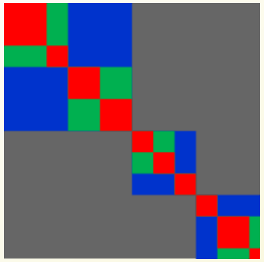
Beenakker (1991)

Generalized Master Equation

- We start with the **Liouville** equation: $$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H, \rho]$$

- We define the reduced density matrix $\sigma = \mathrm{Tr}_{S+T}\{\rho\}$ which is block-diagonal in

$\sigma =$



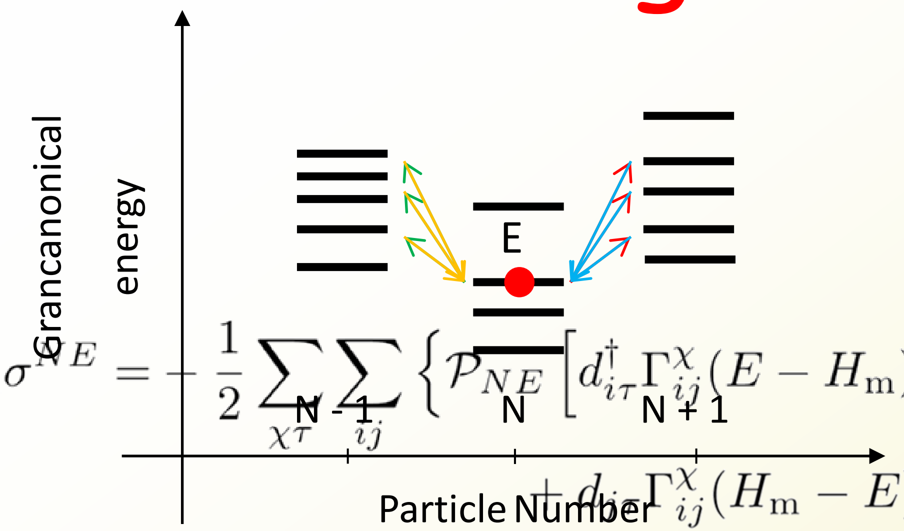
particle number
spin
energy

- We keep the coherences between orbitally degenerate states.

- The **Generalized Master Equation** is the equation of motion for σ :

$$\dot{\sigma} = \underbrace{-\frac{\mathrm{i}}{\hbar}[H_{\mathrm{m}}, \sigma]}_{\text{Coherent dynamics}} \underbrace{- \frac{\mathrm{i}}{\hbar}[H_{\mathrm{eff}}, \sigma]}_{\text{Effective internal dynamics}} + \underbrace{\mathcal{L}_{\mathrm{tun}}\sigma}_{\text{Tunnelling dynamics}} \quad := \mathcal{L}\sigma$$

Tunnelling Liouvillean



$$\mathcal{P}_{NE} = \sum_{\ell} |NE\ell\rangle\langle NE\ell|$$

Projector on the subspace of N particles and energy E.

$$\mathcal{L}_{\rm tun}\sigma^{NE} = -\frac{1}{2}\sum_{\chi\tau}\sum_{ij}\Big\{\mathcal{P}_{NE}\Big[d^\dagger_{i\tau}\Gamma^\chi_{ij}(E-H_{\rm m})f^-_\chi(E-H_{\rm m})d_{j\tau} + d_{j\tau}\Gamma^\chi_{ij}(H_{\rm m}-E)f^+_\chi(H_{\rm m}-E)d^\dagger_{i\tau}\Big]\sigma^{NE} + h.c.\Big\}$$

$$+\sum_{\chi\tau}\sum_{ijE'}\mathcal{P}_{NE}\Big[d^\dagger_{i\tau}\Gamma^\chi_{ij}(E-E')\sigma^{N-1E'}f^+_\chi(E-E')d_{j\tau} + d_{j\tau}\Gamma^\chi_{ij}(E'-E)\sigma^{N+1E'}f^-_\chi(E'-E)d^\dagger_{i\tau}\Big]\mathcal{P}_{NE}$$

Single particle rate matrix

$$\Gamma^{\chi}_{ij}(\Delta E) = \frac{2\pi}{\hbar} \sum_k (t^{\chi}_{ki})^* \, t^{\chi}_{kj} \, \delta(\varepsilon^{\chi}_k - \Delta E)$$

$$H_{\text{eff}} = \frac{1}{2\pi} \sum_{NE} \sum_{\chi\sigma} \sum_{ij} \mathcal{P}_{NE} \Big[d^{\dagger}_{i\sigma} \Gamma^{\chi}_{ij}(E - H_{\text{m}}) p_{\chi}(E - H_{\text{m}}) d_{j\sigma} + d_{j\sigma} \Gamma^{\chi}_{ij}(H_{\text{m}} - E) p_{\chi}(H_{\text{m}} - E) d^{\dagger}_{i\sigma} \Big] \mathcal{P}_{NE}$$

Effective Hamiltonian

$$I_{\chi} = \sum_{NE\sigma ij} \mathcal{P}_{NE} \Big[d_{j\sigma} \Gamma^{\chi}_{ij}(H_{\text{m}} - E) f^{+}_{\chi}(H_{\text{m}} - E) d^{\dagger}_{i\sigma} - d^{\dagger}_{i\sigma} \Gamma^{\chi}_{ij}(E - H_{\text{m}}) f^{-}_{\chi}(E - H_{\text{m}}) d_{j\sigma} \Big] \mathcal{P}_{NE}$$

Current operator

Many-body rate matrix

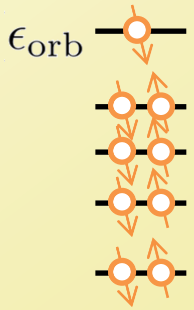
The **current** is proportional to the **transition rate** between **many-body states**

$$R^{\chi\tau}_{N\,E_0\to N+1\,E_1} = \sum_{ij} \langle N+1 E_1 | d^\dagger_{i\tau} | N E_0 \rangle \Gamma^{\chi}_{ij}(E_1 - E_0) \times \langle N E_0 | d_{j\tau} | N+1 E_1 \rangle f^+(E_1 - E_0 - \mu_\chi)$$

where

$$\Gamma^{\chi}_{ij}(E_1 - E_0) = \frac{2\pi}{\hbar} \sum_k (t^{\chi}_{ki})^* t^{\chi}_{kj} \delta(\epsilon^{\chi}_k - E_1 + E_0)$$

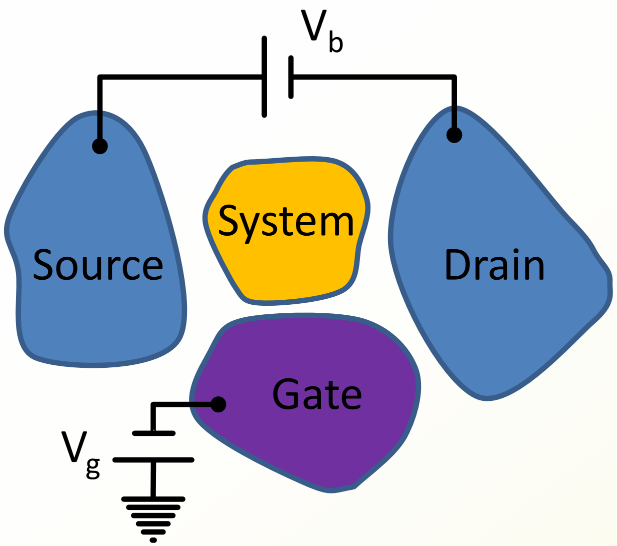
For **uncorrelated** and **non-degenerate systems** the many-body rate reduces to



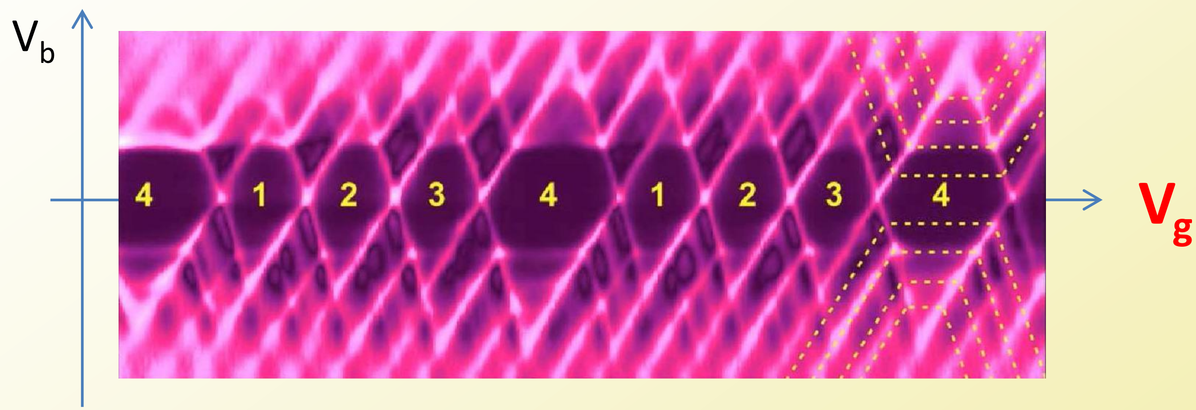
$$R^{\chi\tau}_{N\,E_0\to N+1\,E_1} = \Gamma^{\chi}_{\rm orb}(\epsilon_{\rm orb}) f^+(\epsilon_{\rm orb} - \mu_\chi)$$

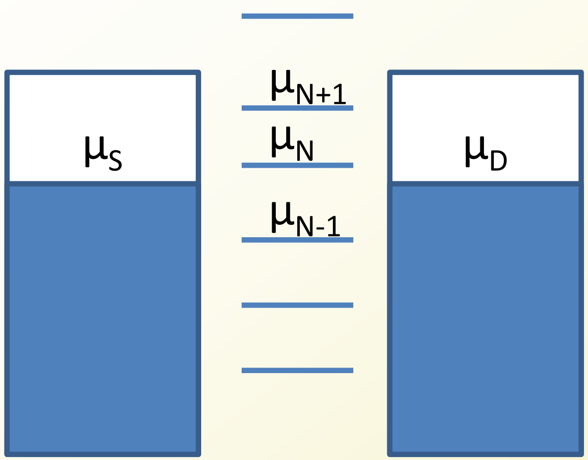
Negative differential conductance (NDC) with symmetric contacts

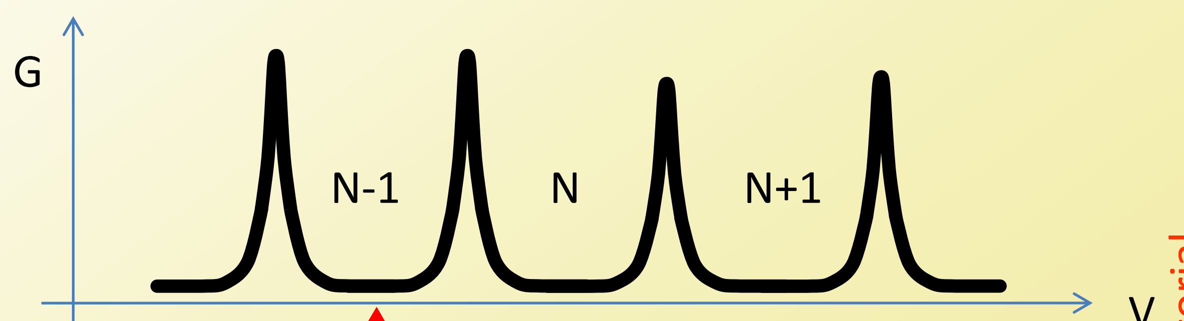
Single electron transistor



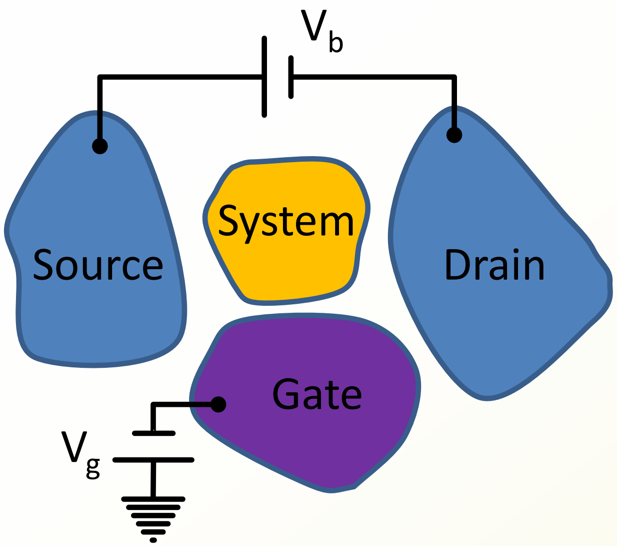
Small System size
+ weak System-Lead
Tunnelling coupling
→
Strong e-e
interaction
on the System
→
Single electron
control



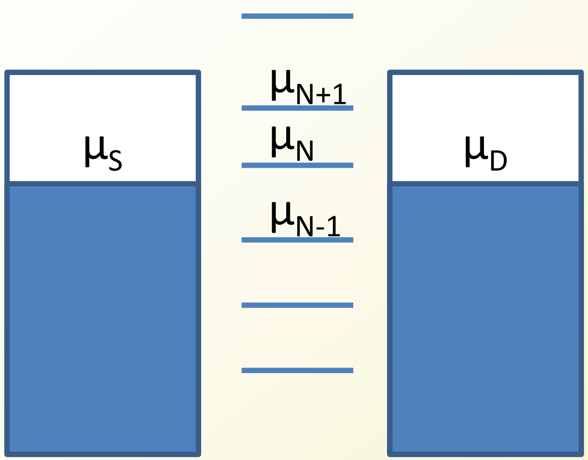


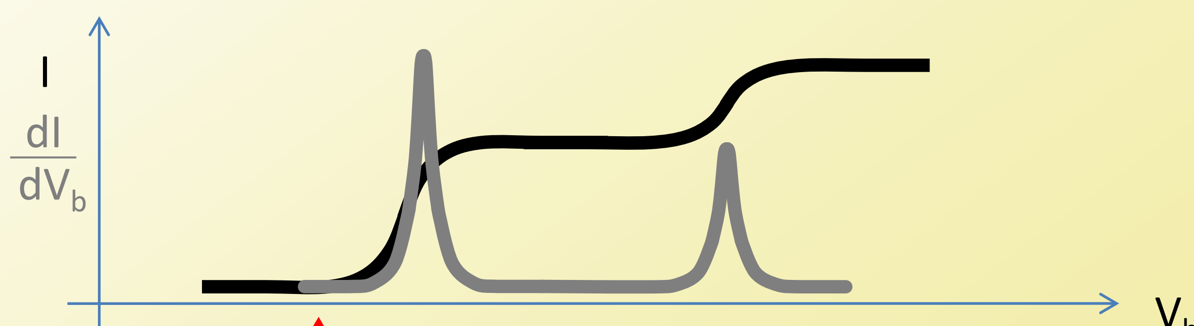


Single electron transistor

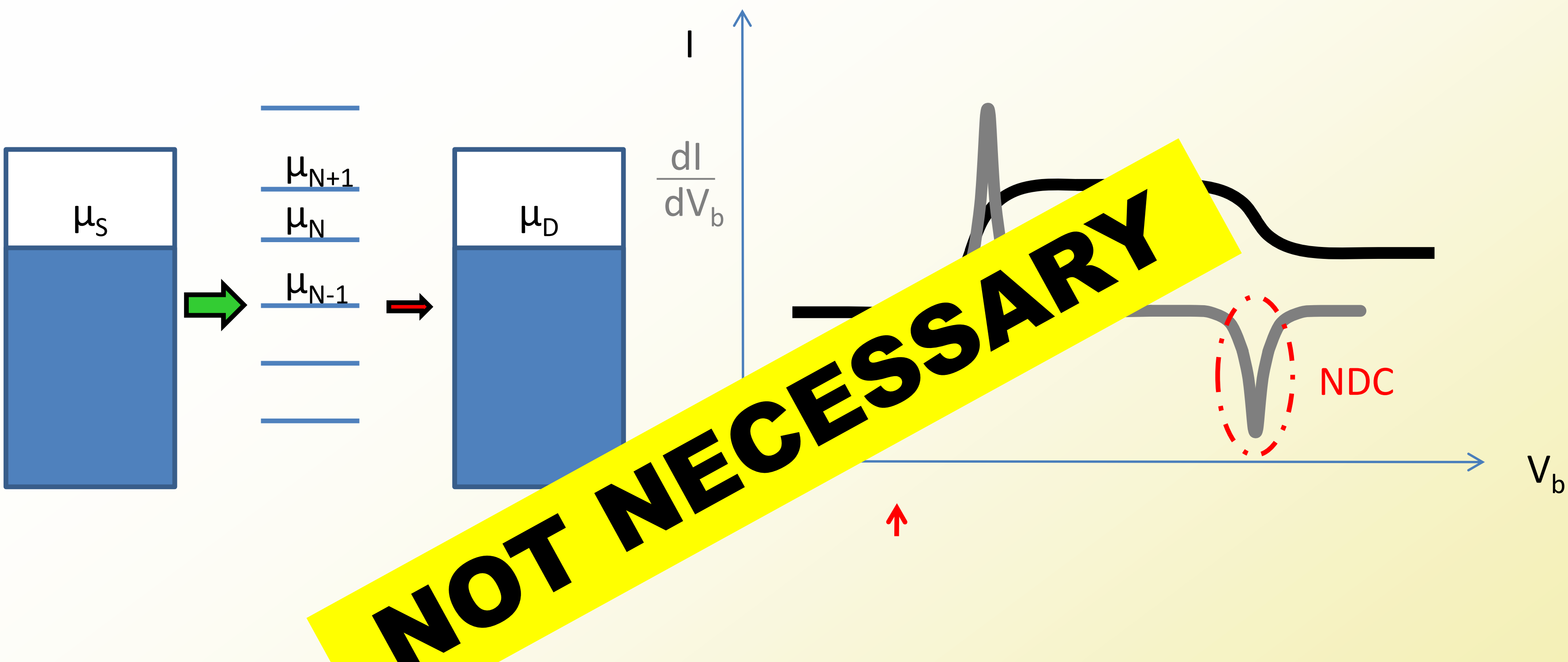


Small System size + weak System-Lead Tunnelling coupling → Strong e-e interaction on the System → Single electron control



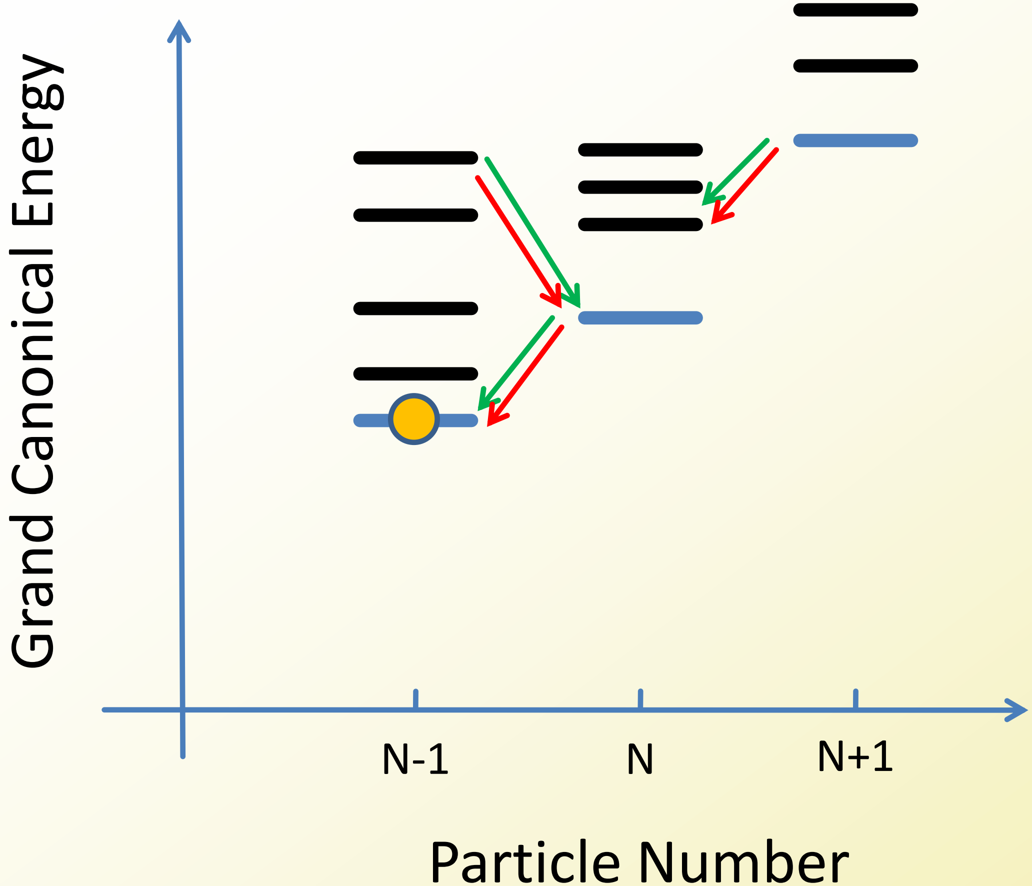


Negative Differential Conductance



Negat[illegible] differential conductance (NDC) is usually associated with a **strong asymmetry** in the coupling to the leads

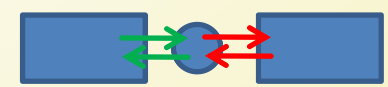
The "free energy" formulation

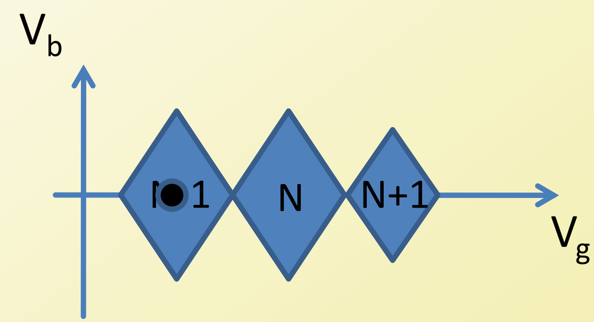


$F = H - \mu_0 N$ GC energy

Source transition

Drain transition

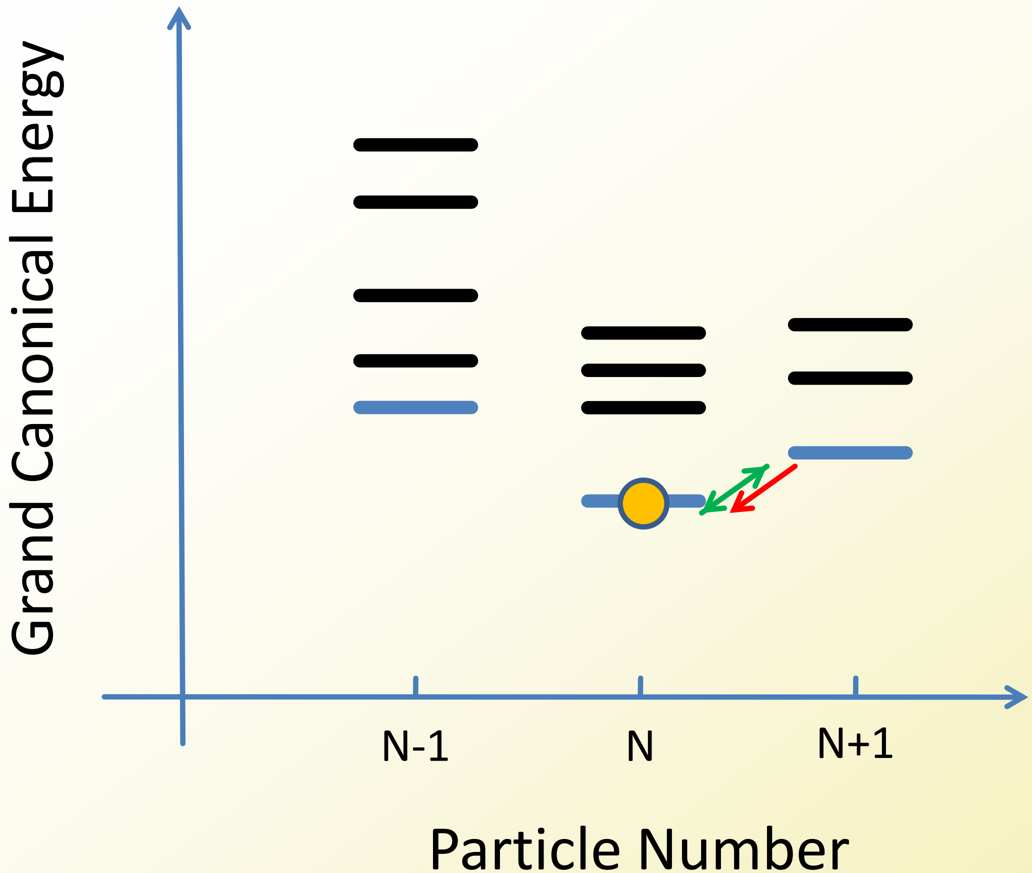


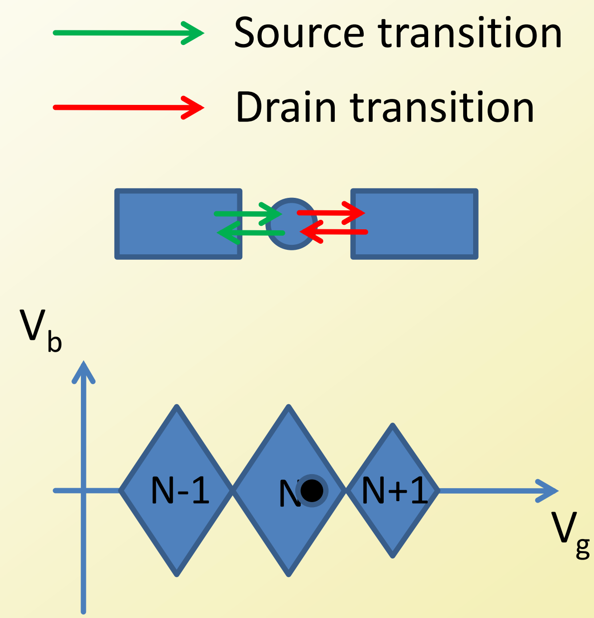


AD, G Begemann, Milena Grifoni
PRB, **82**, 125451 (2010)

The "free energy" formulation

$F = H - \mu_0 N$ GC energy





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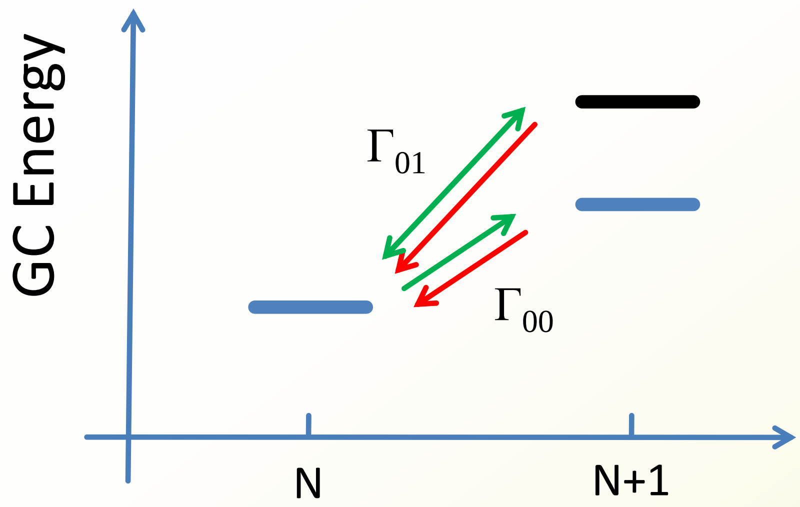
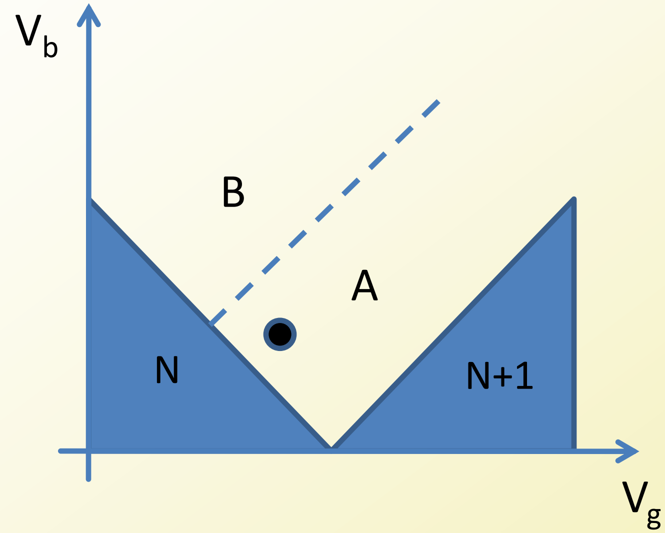
PRB, **82**, 125451 (2010)

NDC with symmetric set-up





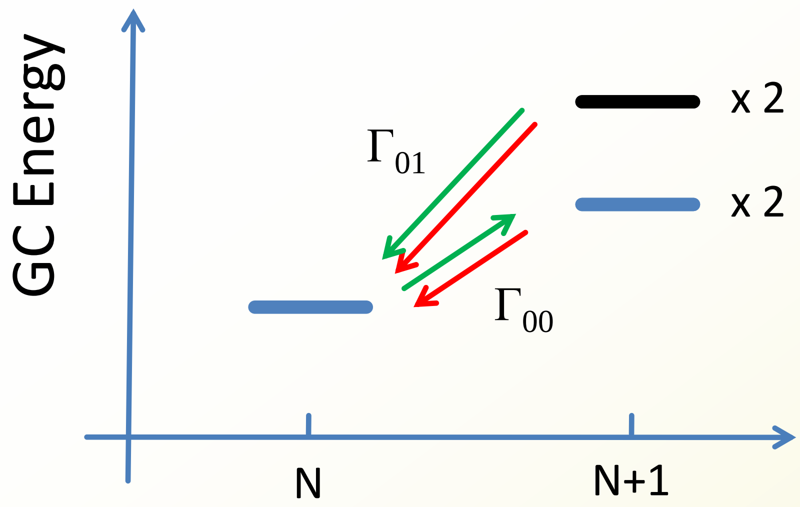
The dynamics is evaluated with the **master equation**: in the **stationary limit**

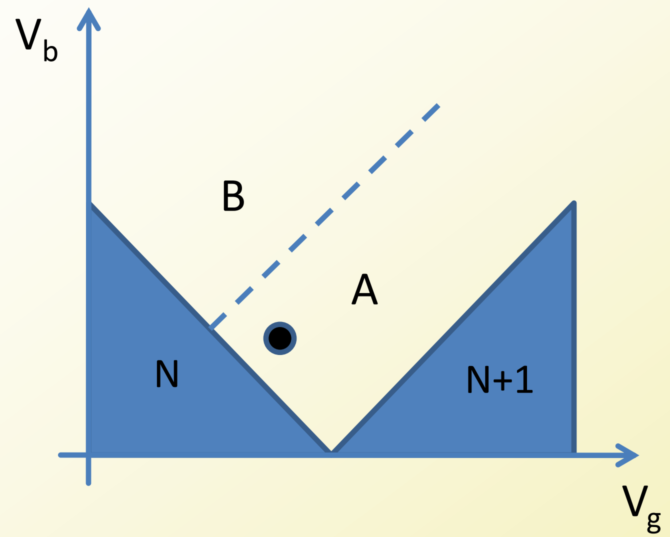
	$P_{N,0}$	$P_{N+1,0}$	$P_{N+1,1}$	Current
A	1/2	1/2	0	$\Gamma_{00}/2$
B	1/3	1/3	1/3	$1/3(\Gamma_{00}+\Gamma_{01})$

NDC if $I_B < I_A$

$\Gamma_{01} < \Gamma_{00}/2$

Role of the degeneracies





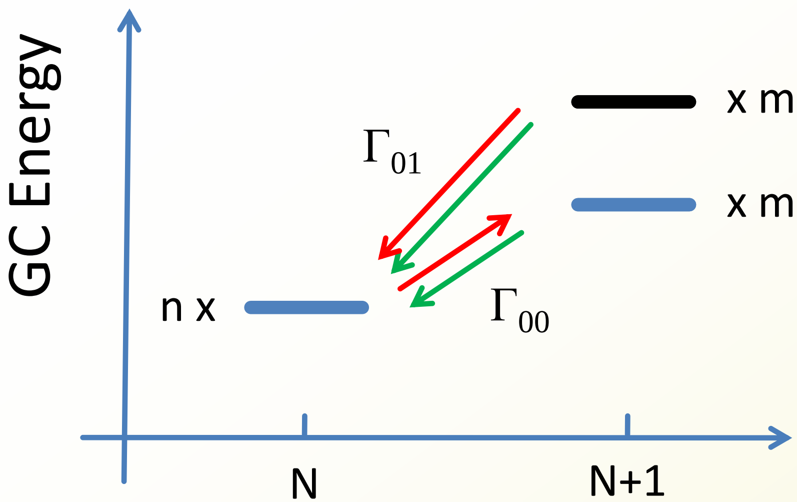
The dynamics is evaluated with the **master equation**: in the **stationary limit**

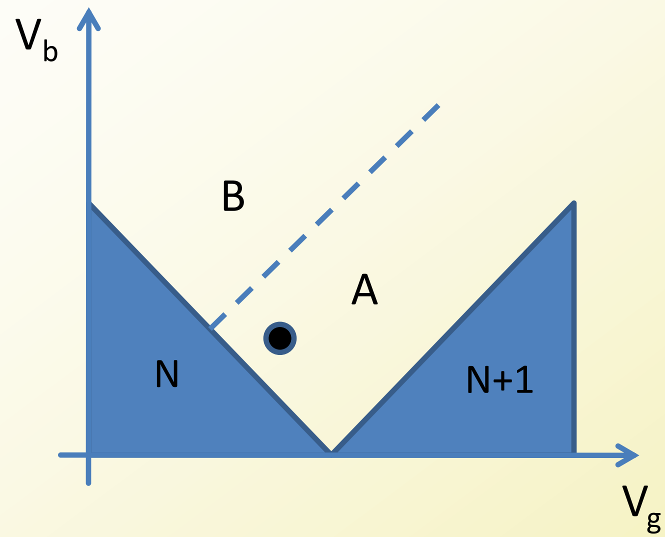
	$P_{N,0}$	$P_{N+1,0}$	$P_{N+1,1}$	Current
A	1/3	1/3	0	$2\Gamma_{00}/3$
B	1/5	1/5	1/5	$2/5(\Gamma_{00} + \Gamma_{01})$

NDC if $I_B < I_A$

$\Gamma_{01} < 2\Gamma_{00}/3$

Role of the degeneracies





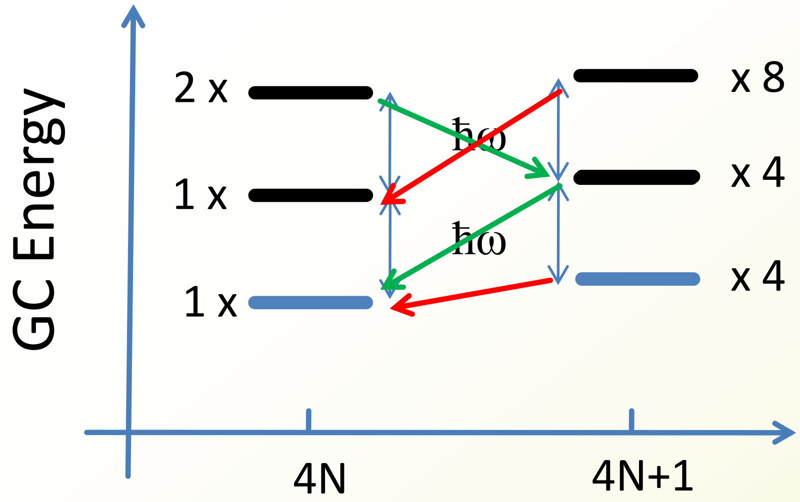
The dynamics is evaluated with the **master equation**: in the **stationary limit**

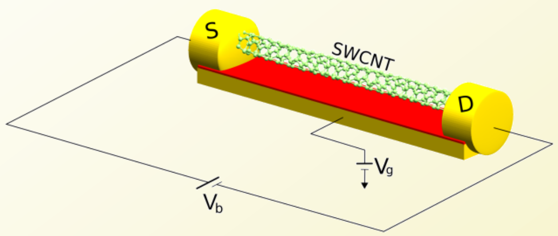
	$P_{N,0}$	$P_{N+1,0}$	$P_{N+1,1}$	Current
A	$\frac{1}{n+m}$	$\frac{1}{n+m}$	0	$\frac{nm}{n+m}\Gamma_{00}$
B	$\frac{1}{n+2m}$	$\frac{1}{n+2m}$	$\frac{1}{n+2m}$	$\frac{nm}{n+2m}(\Gamma_{00}+\Gamma_{01})$

NDC if $I_B < I_A$

$$\Gamma_{01} < \frac{m}{n+m}\Gamma_{00}$$

An example: a suspended CNT



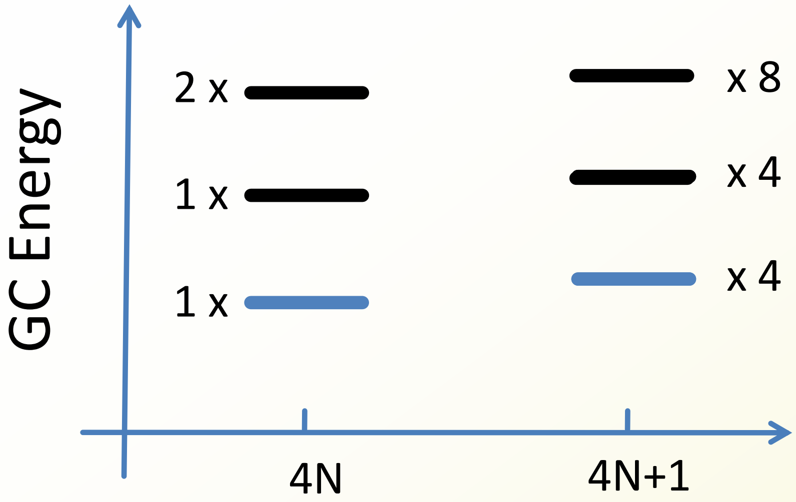


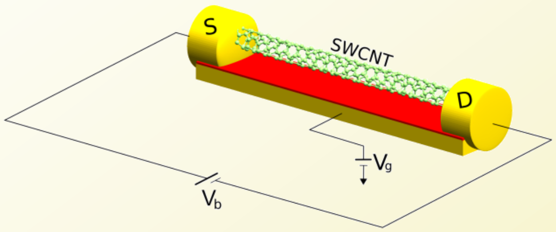
The transition rates are proportional to product of Frack-Condon coefficients

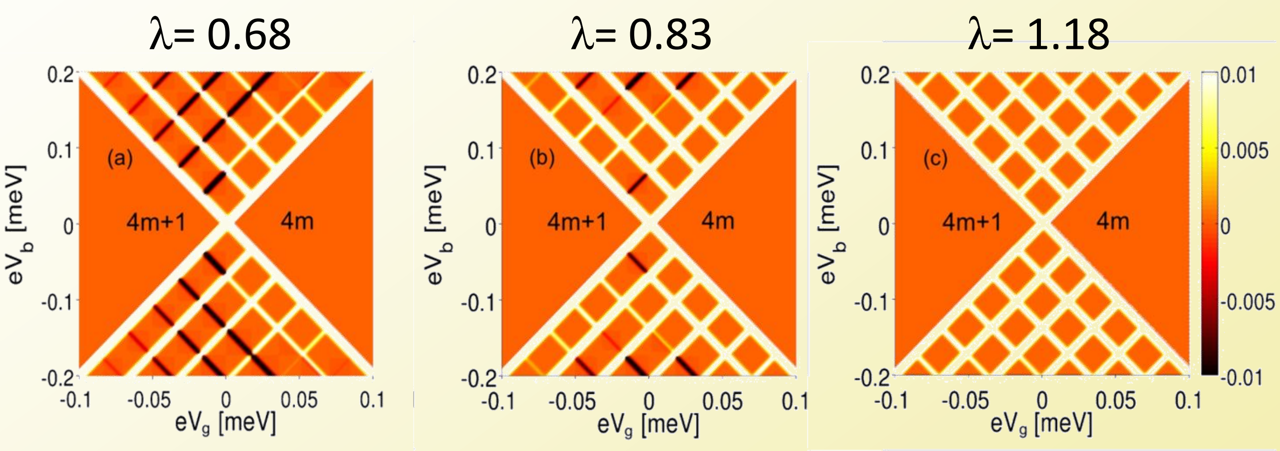
$$\Gamma_{ij} = \prod_{n} FC^{(n)}(i, j; \lambda)$$

A. Yar, **AD**, S. Koller, and M. Grifoni, arXiv:1101.3892 (2011)

An example: a suspended CNT

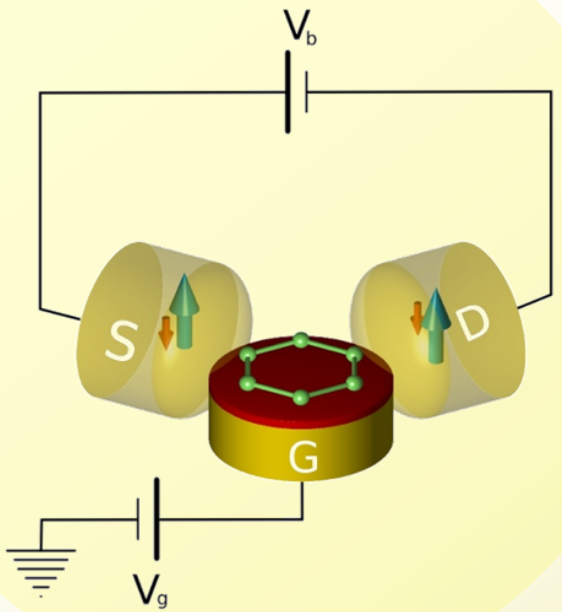


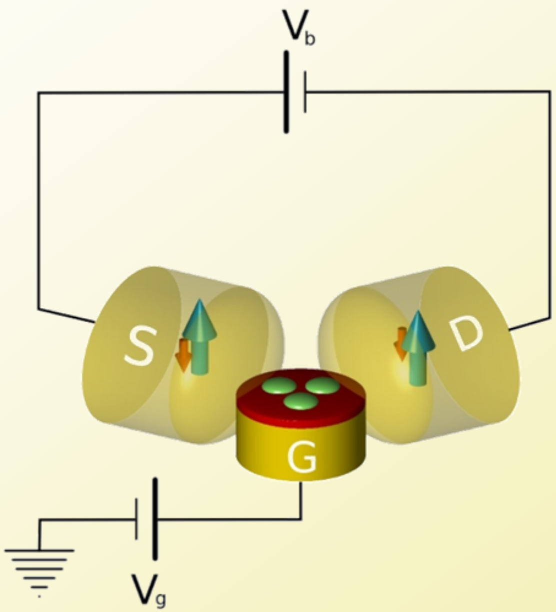




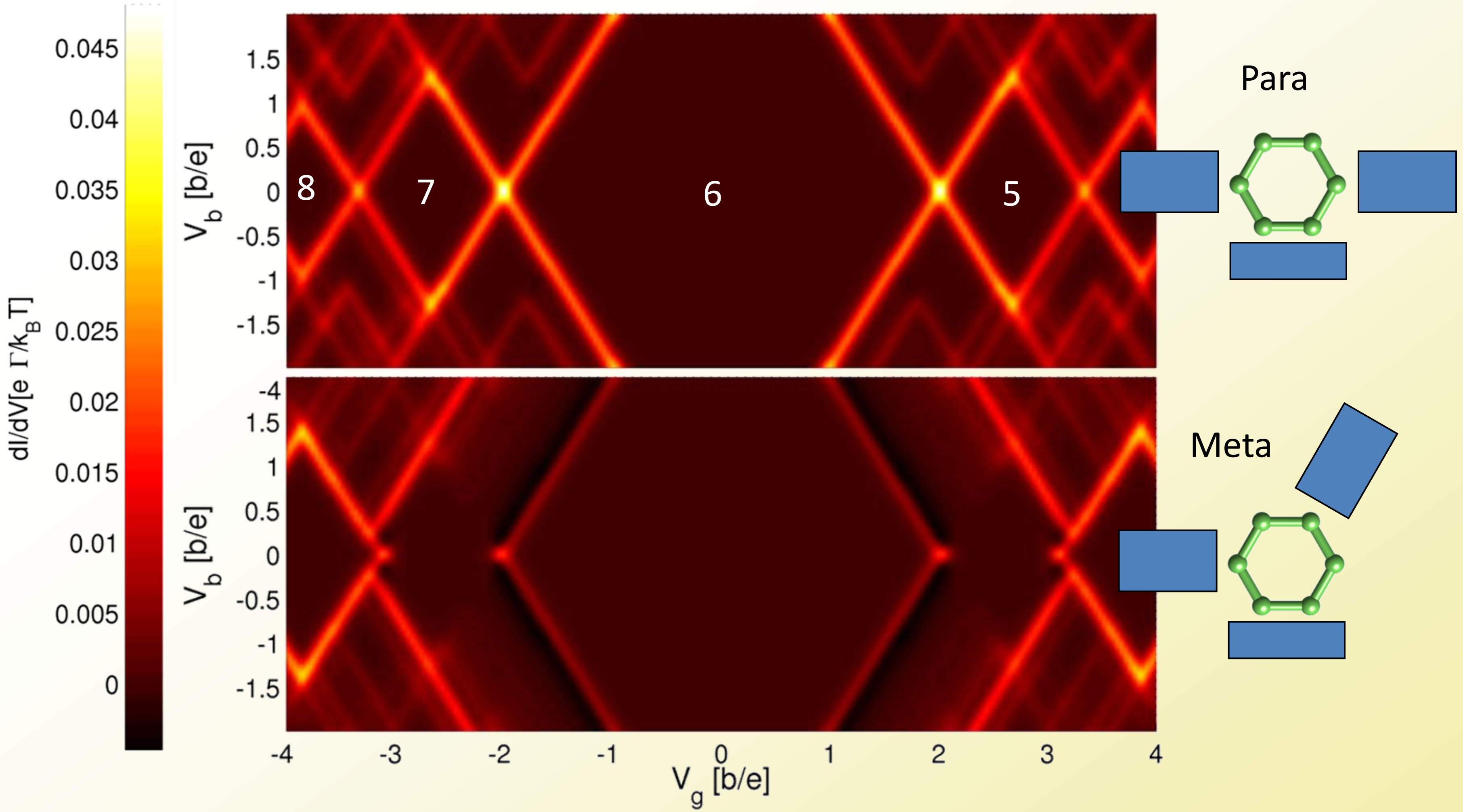
A. Yar, **AD**, S. Koller, and M. Grifoni, arXiv:1101.3892 (2011)

Interference Single Electron Transistors





Para vs. Meta

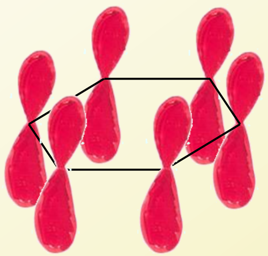


G. Begemann, D. Darau, **AD**, M. Grifoni, *Phys. Rev. B* **77**, 201406(R) (2008)

Interacting isolated benzene

- The Pariser-Parr-Pople Hamiltonian for isolated benzene reads:

$$H^0_{\rm ben} = \xi_0 \sum_{i\sigma} d^\dagger_{i\sigma} d_{i\sigma} + b \sum_{i\sigma} \left(d^\dagger_{i\sigma} d_{i+1\sigma} + d^\dagger_{i+1\sigma} d_{i\sigma} \right)$$
$$+U \sum_i \left(n_{i\uparrow} - \tfrac{1}{2}\right)\left(n_{i\downarrow} - \tfrac{1}{2}\right)$$
$$+V \sum_i \left(n_{i\uparrow} + n_{i\downarrow} - 1\right)\left(n_{i+1\uparrow} + n_{i+1\downarrow} - 1\right)$$



- The **size** of the Fock space for the many-body system **4^6 = 4096**

since for each site there are 4 possibilities: $|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle$

- Within this Fock space we diagonalize **exactly** the Hamiltonian.

Symmetry of the ground states

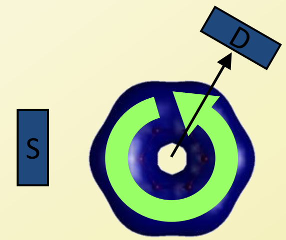
N	Degeneracy	GS energy[eV] (at $\xi = 0$)	GS symmetry representation
0	1	0	A_{1g}
1	2	-22	A_{2u}
2	1	-42.25	A_{1g}
3	4	-57.42	E_{1g}
4	3	-68.875	A_{2g}
5	4	-76.675	E_{1g}
6	1	-81.725	A_{1g}
7	4	-76.675	E_{2u}
8	3	-68.875	A_{2g}
9	4	-57.42	E_{2u}
10	1	-42.25	A_{1g}
11	2	-22	B_{2g}
12	1	0	A_{1g}

Rotation phase factors

Under rotation of an angle $\phi = \frac{n\pi}{3}$

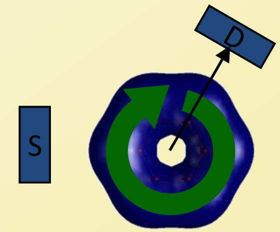
- $\mathcal{R}_\phi |6_g\rangle = |6_g\rangle$ No phase acquired
- $\mathcal{R}_\phi |7_g\,\ell\rangle = e^{-i\ell\phi} |7_g\,\ell\rangle$ $\ell = \pm 2$

$\ell = +2$



$\exp\left(+i\frac{2\pi}{3}\right)$

$\ell = -2$



$\exp\left(-i\frac{2\pi}{3}\right)$

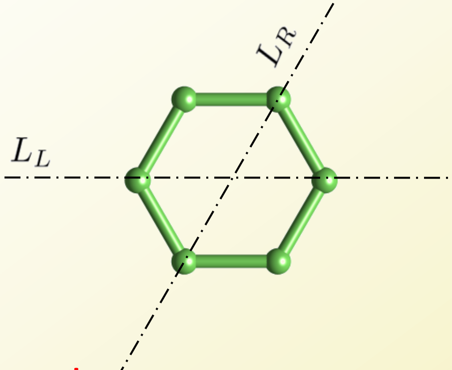
Generalized Master Equation

$$\dot{\sigma} = -\frac{\mathrm{i}}{\hbar}[H_{\mathrm{m}}, \sigma] - \frac{\mathrm{i}}{\hbar}[H_{\mathrm{eff}}, \sigma] + \mathcal{L}_{\mathrm{tun}}\sigma \quad := \mathcal{L}\sigma$$

The effective Hamiltonian

The effective Hamiltonian is expressed in terms of **angular momentum** operators and **renormalization frequencies**:

$$H_{\text{eff}} = \sum_{\alpha\sigma} \omega_{\alpha\sigma} L_{\alpha}$$



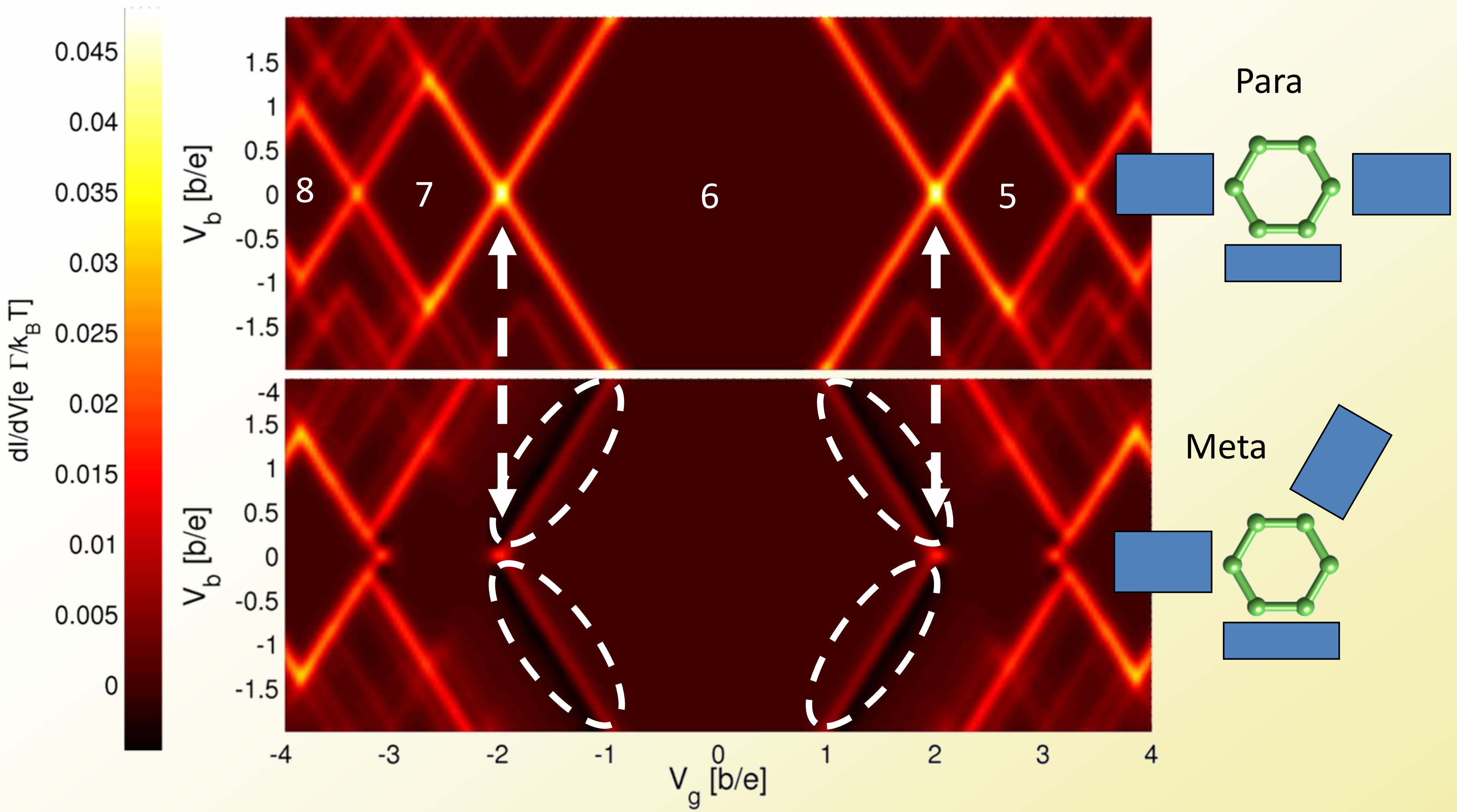
In particular in the Hilbert space of the 7 particle ground states

$$L_{\alpha} = \frac{\hbar}{2} \begin{pmatrix} 1 & e^{i2|\ell|\phi_{\alpha}} \\ e^{-i2|\ell|\phi_{\alpha}} & 1 \end{pmatrix}$$

$$\omega_{\alpha\sigma} = \frac{1}{\pi} \sum_{\sigma'\{E\}} \Gamma^0_{\alpha\sigma'} \Big[\langle 7_g \ell\sigma | d_{M\sigma'} | 8\{E\}\rangle\langle 8\{E\} | d^{\dagger}_{M\sigma'} | 7_g m\sigma\rangle p_{\alpha}(E - E_{7_g}) + \langle 7_g \ell\sigma | d^{\dagger}_{M\sigma'} | 6\{E\}\rangle\langle 6\{E\} | d_{M\sigma'} | 7_g m\sigma\rangle p_{\alpha}(E_{7_g} - E) \Big]$$

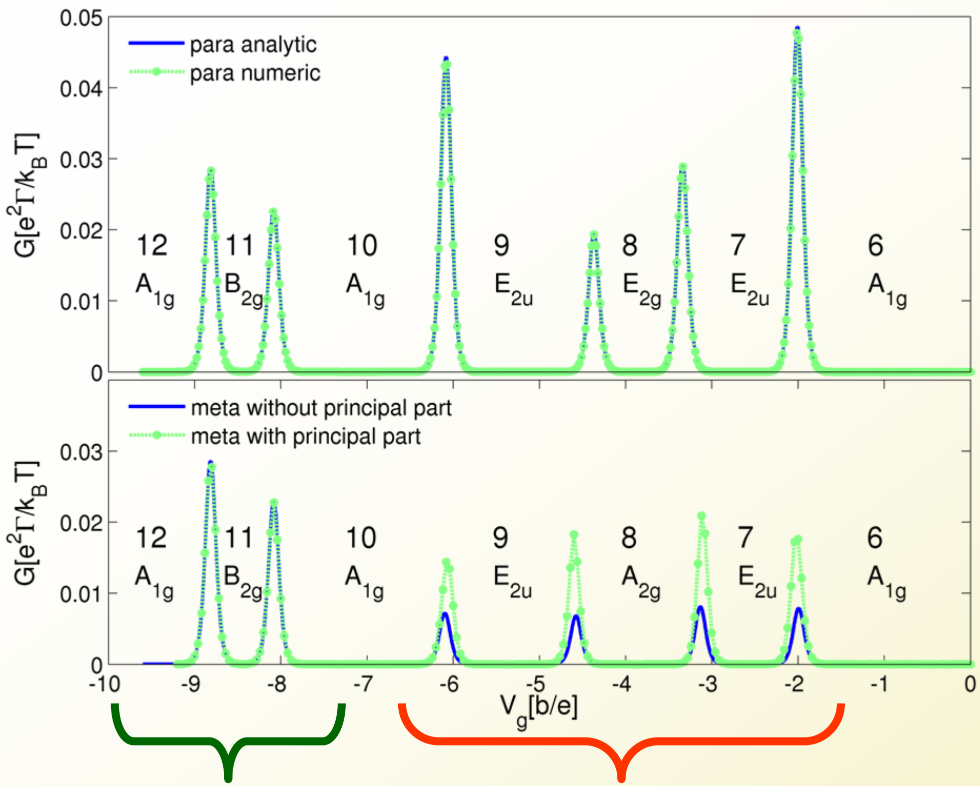
Bias and gate dependent

Para vs. Meta

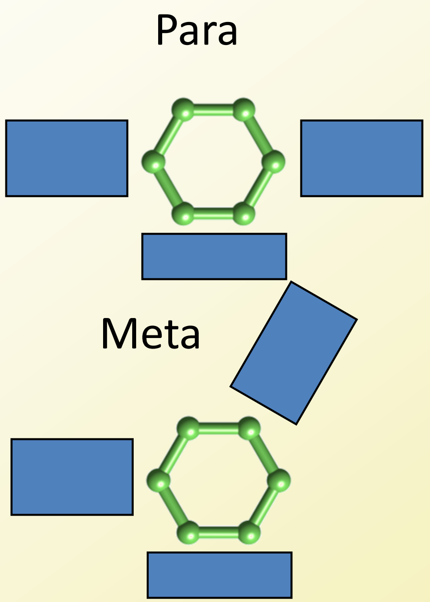


G. Begemann, D. Darau, **AD**, M. Grifoni, *Phys. Rev. B* **77**, 201406(R) (2008)

Conductance suppression



equal suppressed



A: non- degenerate ⟷ **B: non- degenerate** ⇒ Equal

A: non- degenerate ⟷ **E: degenerate** ⇒ Suppressed

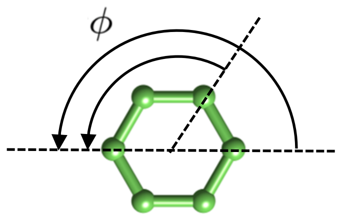
Destructive interference

$$\Lambda = \left|\sum_{nm\tau} \langle N, n|d_{\mathrm{L}\tau}|N+1, m\rangle\langle N+1, m|d^{\dagger}_{\mathrm{R}\tau}|N, n\rangle\right|^2$$

Interference factor

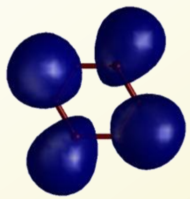
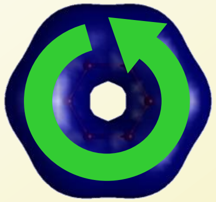
$$\Lambda = \left|\sum_{nm\tau} |\langle N, n|d_{\mathrm{L}\tau}|N+1, m\rangle|^2 e^{i\phi_{nm}}\right|^2$$

$$d^{\dagger}_{\mathrm{R}\tau} = \mathcal{R}^{\dagger}_{\phi} d^{\dagger}_{\mathrm{L}\tau} \mathcal{R}_{\phi}$$



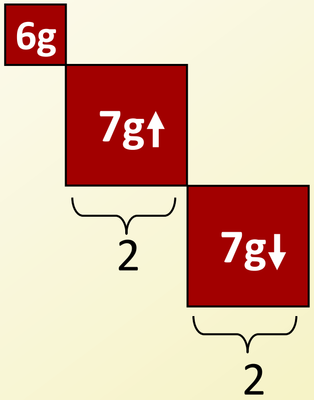
In particular for the transition **6 -7** in the **meta** configuration:

$$\Lambda = \left| |\langle 6_g|d_{\mathrm{L}\tau}|7_g, +2, \tau\rangle|^2 e^{+i\frac{2\pi}{3}} + |\langle 6_g|d_{\mathrm{L}\tau}|7_g, -2, \tau\rangle|^2 e^{-i\frac{2\pi}{3}} \right|^2$$

 = |  $e^{+i\frac{2\pi}{3}}$ +  $e^{-i\frac{2\pi}{3}}$ | ^{2}

Negative Differential Conductance

- The 7 particle ground state has spin and orbital **degeneracies**;
- **Physical basis**: the basis that diagonalizes the stationary density matrix;
- The physical basis **depends on the bias:** in whatever reference basis, **coherences** are essential for a correct description of the system;



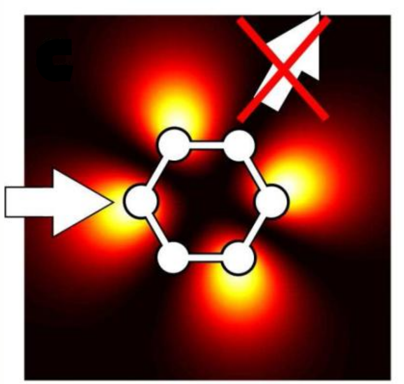
- The **visualization tool**: **position resolved** transition probability to the physical basis:

$$P(x, y; \ell\tau) = \lim_{L\to\infty} \sum_{\sigma} \frac{1}{2L} \int_{-L/2}^{L/2} \mathrm{d}z |\langle 7_g\, \ell\tau | \psi_\sigma^\dagger(\vec{r}) | 6_g \rangle|^2$$

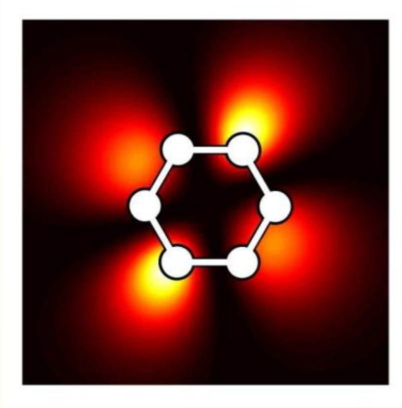
Interference blockade

Geometry

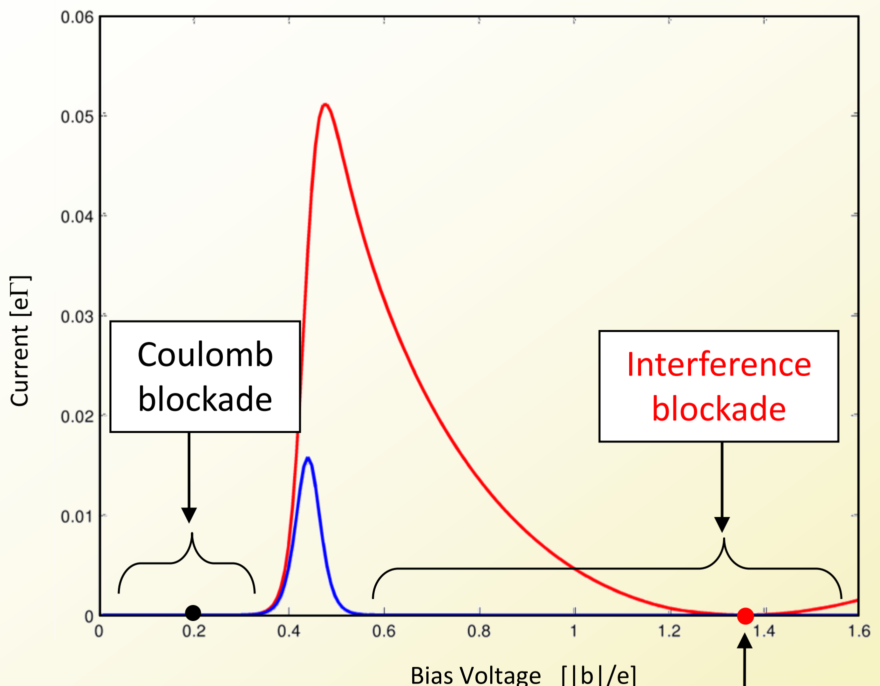
Blocking state



Non-blocking state



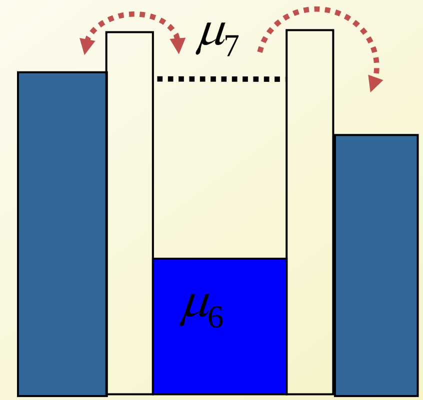
I-V for transition 6 -7



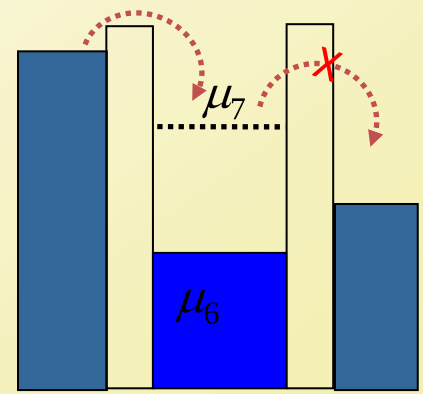
The blocking state is an eigenstate of the effective Hamiltonian

$$\omega_{L\sigma} = 0$$

Energetics

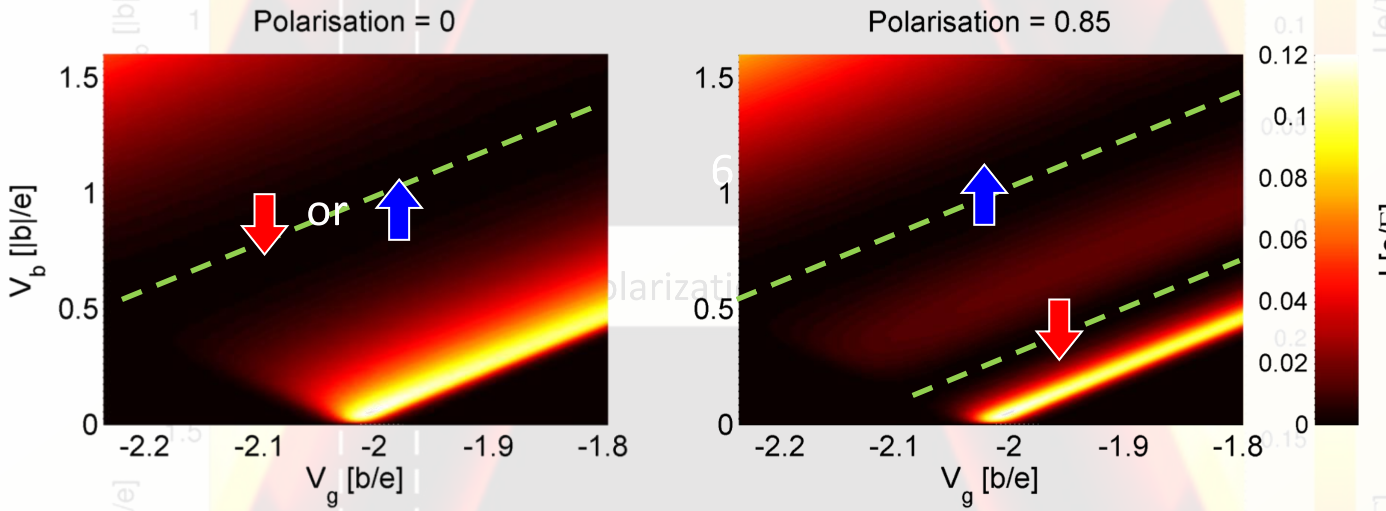


current onset

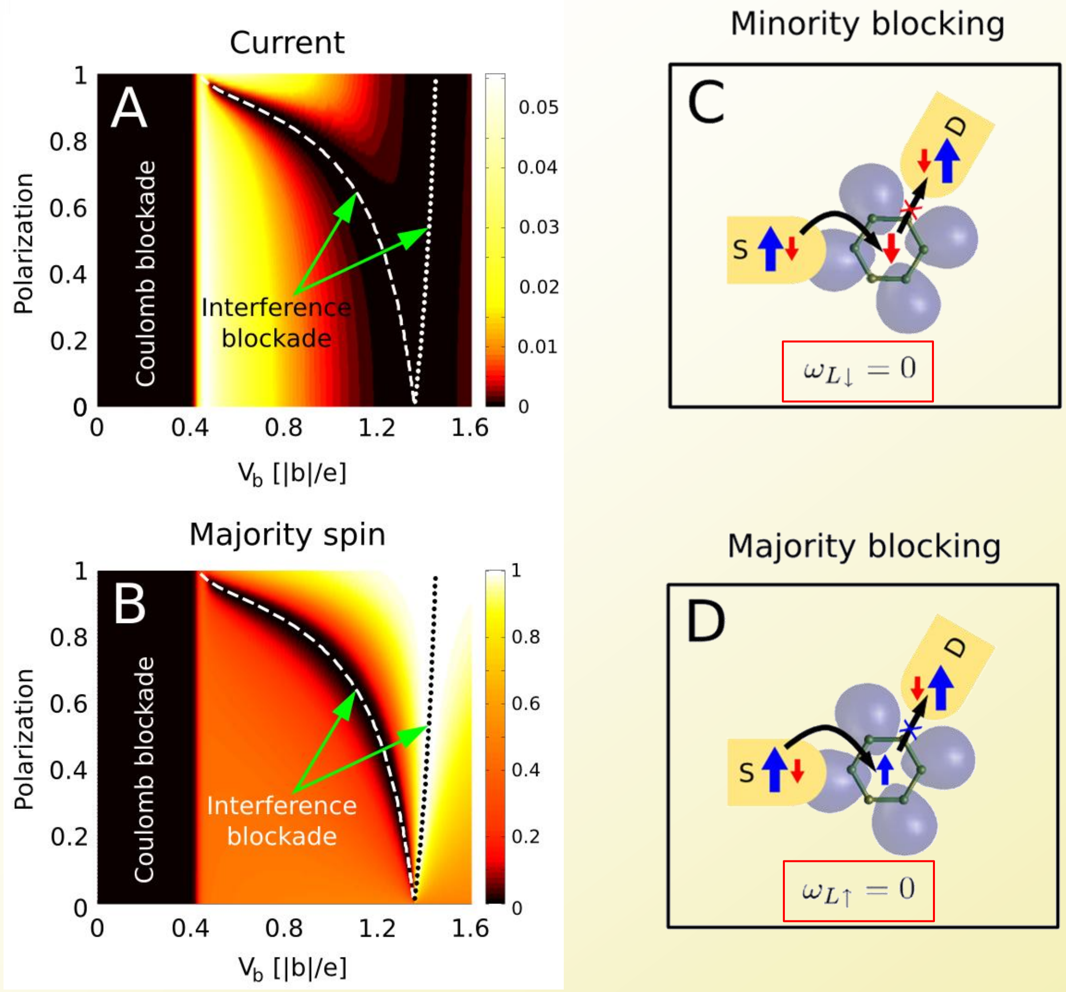


blockade

Normal vs. ferromagnetic leads



Selective Interference Blocking



AD, G. Begemann, and M. Grifoni *Nano Lett.* **9**, 2897 (2009)

Level renormalization in presence of polarized leads

We obtain a difference in the renormalization frequencies for the 2 spin directions linear in the **polarization of the leads**:

$$\omega_{\alpha\uparrow} - \omega_{\alpha\downarrow} = 2\bar{\Gamma}^0_\alpha P_\alpha \frac{1}{\pi} \sum_{\{E\}}$$

$$\Big[\langle 7_g \ell \uparrow | d_{M\uparrow} | 8\{E\} \rangle \langle 8\{E\} | d^\dagger_{M\uparrow} | 7_g m \uparrow \rangle p_\alpha(E - E_{7_g})$$

$$+ \langle 7_g \ell \uparrow | d^\dagger_{M\uparrow} | 6\{E\} \rangle \langle 6\{E\} | d_{M\uparrow} | 7_g m \uparrow \rangle p_\alpha(E_{7_g} - E)$$

$$- \langle 7_g \ell \uparrow | d_{M\downarrow} | 8\{E\} \rangle \langle 8\{E\} | d^\dagger_{M\downarrow} | 7_g m \uparrow \rangle p_\alpha(E - E_{7_g})$$

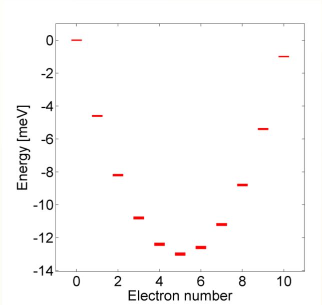
$$- \langle 7_g \ell \uparrow | d^\dagger_{M\downarrow} | 6\{E\} \rangle \langle 6\{E\} | d_{M\downarrow} | 7_g m \uparrow \rangle p_\alpha(E_{7_g} - E) \Big]$$

The splitting of the level renormalization depends crucially on the Coulomb interaction on the molecule and **vanishes in absence of exchange**.

Summary

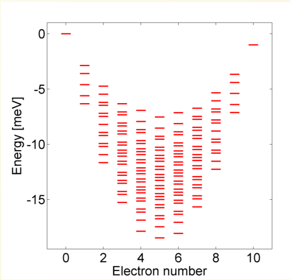
- Pedagogical introduction to the physics of the **sequential tunnelling regime** in transport through micro- and nano-junctions.
- Basic **theory** of Coulomb blockade and single electron tunneling in two flavours

Metallic island



Master equation

Quandum dot



Generalized Master equation

- Application of the **GME** to different types of **quantum dot molecules**

Acknowledgements

Georg Begemann

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Dana Darau

A. Yar

S. Kolmeder

B. Siegert

S.Pfaller

Thank you for your attention!