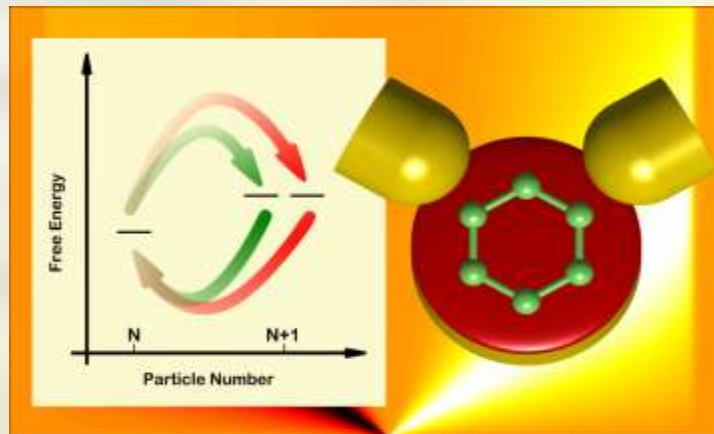
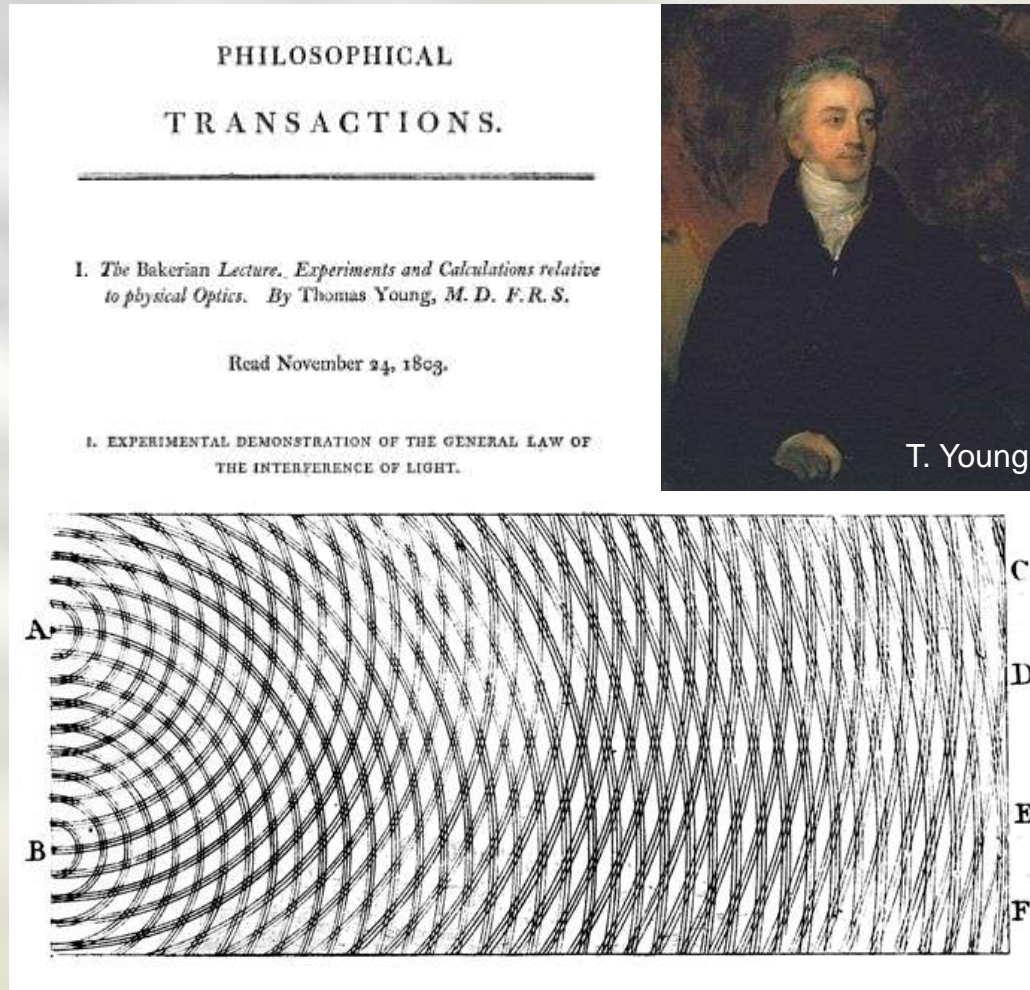


# Interference and interaction in symmetric nanojunctions

Andrea Donarini



# Double slit experiment: (London, 1801)



*Phil. Trans. R. Soc. Lon.*, **94**, 12 (1804)

# Double slit with electrons: (Tübingen, 1961)

Aus dem Institut für Angewandte Physik der Universität Tübingen

## **Elektroneninterferenzen an mehreren künstlich hergestellten Feinspalten**

Von

CLAUS JÖNSSON

Mit 14 Figuren im Text

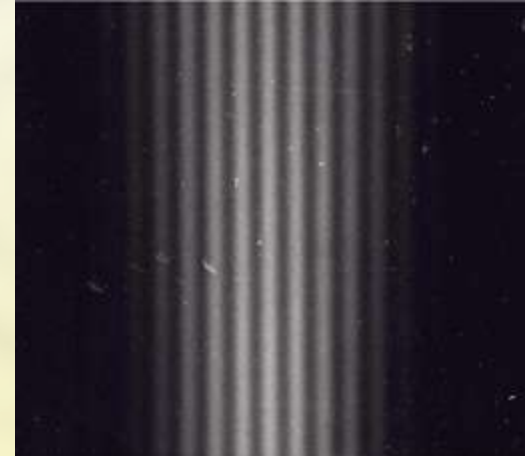
*(Eingegangen am 17. Oktober 1960)*

A glass plate covered with an evaporated silver film of about  $200 \text{ \AA}$  thickness is irradiated by a line-shaped electron-probe in a vacuum of  $10^{-4}$  Torr. A hydrocarbon polymerisation film of very low electrical conductivity is formed at places subjected to high electron current density. An electrolytically deposited copper film leaves these places free from copper. When the copper film is stripped a grating with slits free of any material is obtained.  $50 \mu$  long and  $0.3 \mu$  wide slits with a grating constant of  $1 \mu$  are obtained. The maximum number of slits is five. The electron diffraction pattern obtained using these slits in an arrangement analogous to Young's light optical interference experiment in the Fraunhofer plane and Fresnel region shows an effect corresponding to the well-known interference phenomena in light optics.

*Zeitschrift für Physik*, **161**, 454 (1961)



C.Jönsson



# Single electron interference (Bologna, 1974)

## On the statistical aspect of electron interference phenomena

P. G. Merli

*CNR-LAMEL, Bologna, Italy*

G. F. Missiroli and G. Pozzi

*CNR-GNSM, Istituto di Fisica, Laboratorio Microscopia Elettronica, Bologna, Italy*

(Received 29 May 1974; revised 17 October 1974)

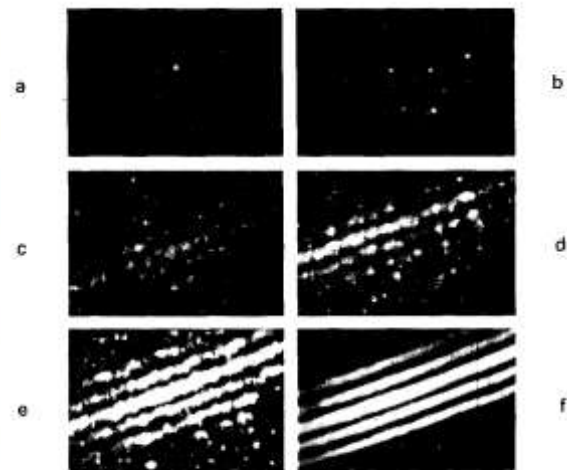


Fig. 1. (a-f) Electron interference fringe patterns filmed from a TV monitor at increasing current densities.

*Am. J. Phys.*, **44**, 306 (1976)

### Demonstration of single-electron buildup of an interference pattern

A. Tonomura, J. Endo, T. Matsuda, and T. Kawasaki  
*Advanced Research Laboratory, Hitachi, Ltd., Kokubunji, Tokyo 185, Japan*

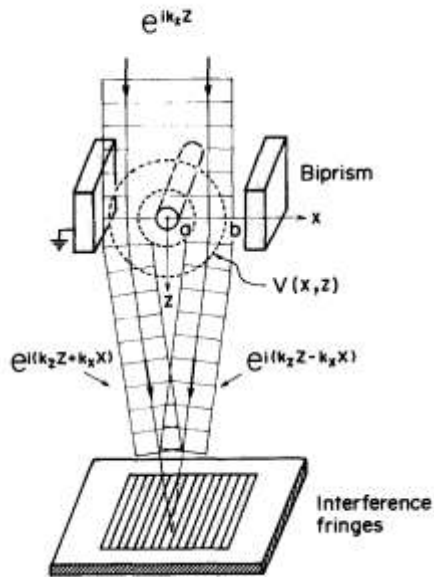
H. Ezawa  
*Department of Physics, Gakushuin University, Mejiro, Tokyo 171, Japan*

(Received 17 December 1987; accepted for publication 22 March 1988)

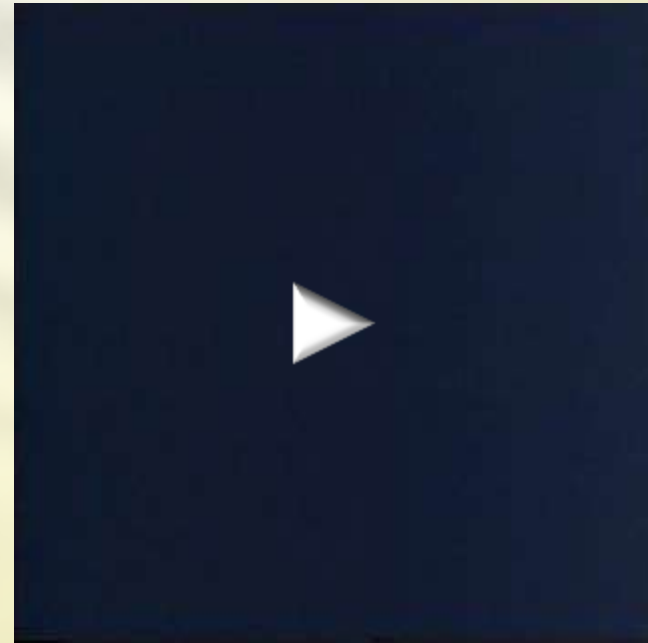
The wave-particle duality of electrons was demonstrated in a kind of two-slit interference experiment using an electron microscope equipped with an electron biprism and a position-sensitive electron-counting system. Such an experiment has been regarded as a pure thought experiment that can never be realized. This article reports an experiment that successfully recorded the actual buildup process of the interference pattern with a series of incoming single electrons in the form of a movie.



A. Tonomura



*Am. J. Phys.*, **57**, 117 (1989)



# In mesoscopic rings (Rehovot, 1995)

VOLUME 74, NUMBER 20

PHYSICAL REVIEW LETTERS

15 MAY 1995

## Coherence and Phase Sensitive Measurements in a Quantum Dot

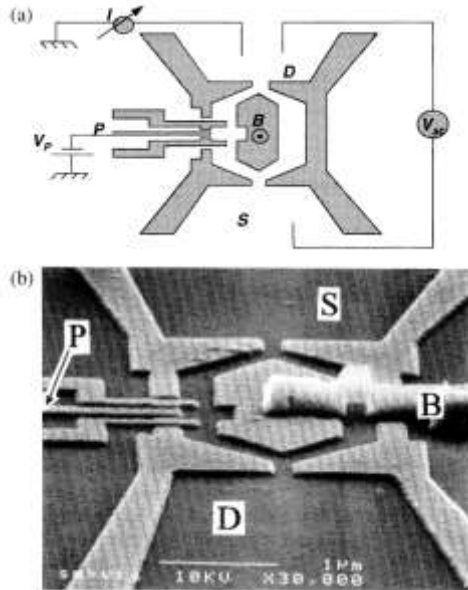
A. Yacoby, M. Heiblum, D. Mahalu, and Hadas Shtrikman

Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science,  
Rehovot 76100, Israel

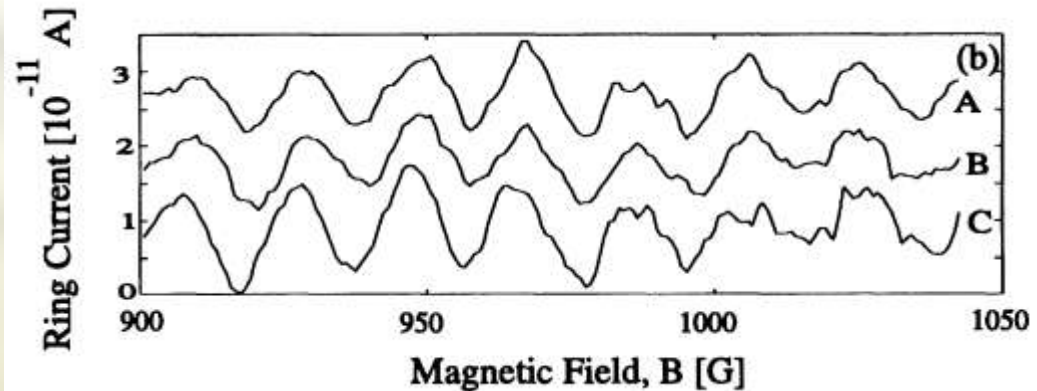
(Received 10 November 1994)

Via a novel interference experiment, which measures magnitude and phase of the transmission coefficient through a quantum dot in the Coulomb regime, we prove directly, for the first time, that transport through the dot has a coherent component. We find the same phase of the transmission coefficient at successive Coulomb peaks, each representing a different number of electrons in the dot; however, as we scan through a single Coulomb peak we find an abrupt phase change of  $\pi$ . The observed behavior of the phase cannot be understood in the single particle framework.

PACS numbers: 73.20.Dx, 71.45.-d, 72.80.Ey, 73.40.Gk



M. Heiblum



Phys. Rev. Lett., **74**, 4047 (1995)

# ...counting single electrons (Zürich, 2008)

NANO  
LETTERS

2008  
Vol. 8, No. 8  
2547-2550

## Time-Resolved Detection of Single-Electron Interference

S. Gustavsson,<sup>\*</sup> R. Leturcq, M. Studer, T. Ihn, and K. Ensslin

*Solid State Physics Laboratory, ETH Zürich, CH-8093 Zürich, Switzerland*

D. C. Driscoll and A. C. Gossard

*Materials Department, University of California, Santa Barbara, California 93106*

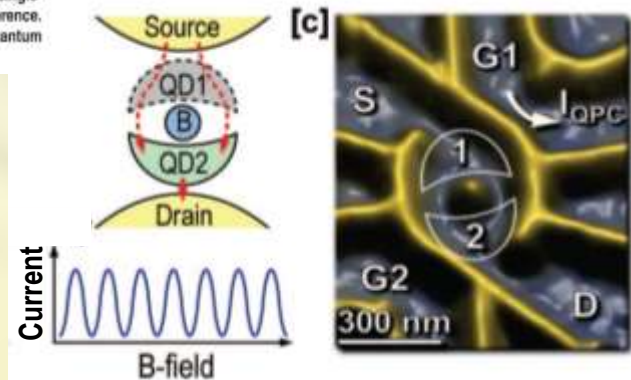
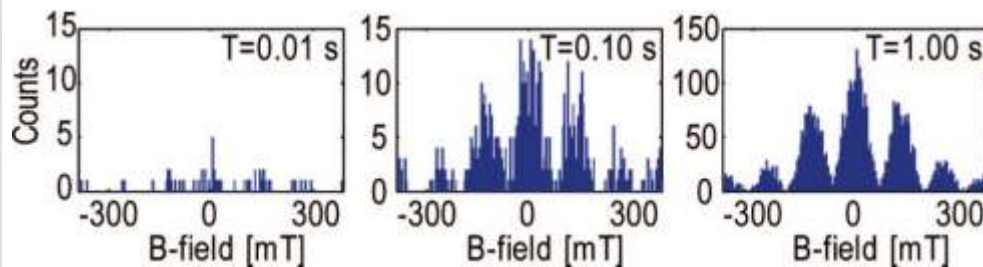
Received June 13, 2008

### ABSTRACT

We demonstrate real-time detection of self-interfering electrons in a double quantum dot embedded in an Aharonov–Bohm interferometer, with visibility approaching unity. We use a quantum point contact as a charge detector to perform time-resolved measurements of single-electron tunneling. With increased bias voltage, the quantum point contact exerts a back-action on the interferometer leading to decoherence. We attribute this to emission of radiation from the quantum point contact, which drives noncoherent electronic transitions in the quantum dots.

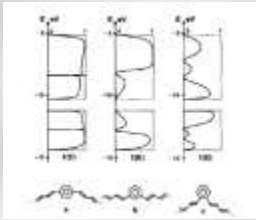


K. Ensslin

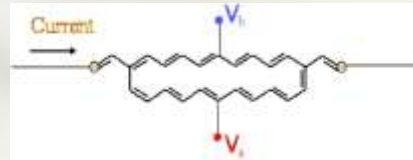


*Nano Lett.*, **8**, 2547 (2008)

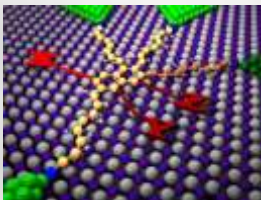
# Intramolecular interference



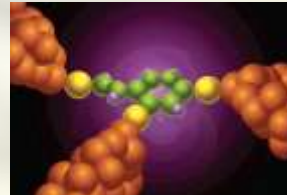
P. Sautet and C. Joachim  
*Chem. Phys. Lett.* **153**, 511 (1988)



R. Baer and D. Neuhauser  
*JACS*, **124**, 4200 (2002)



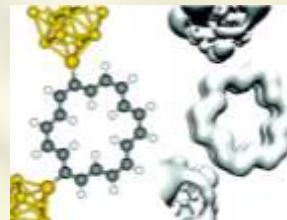
R. Stadler, et al.  
*Nanotechnology*, **14**, 138 (2003)



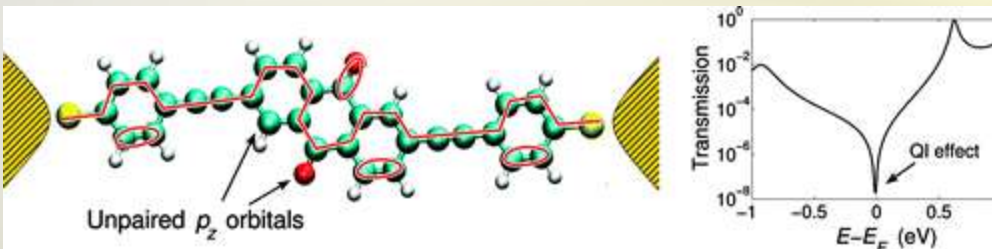
D. V. Cardamone, et al.  
*Nano Lett.*, **6**, 2422 (2006)



G. Solomon, et al.  
*JACS* **130**, 17307 (2008)



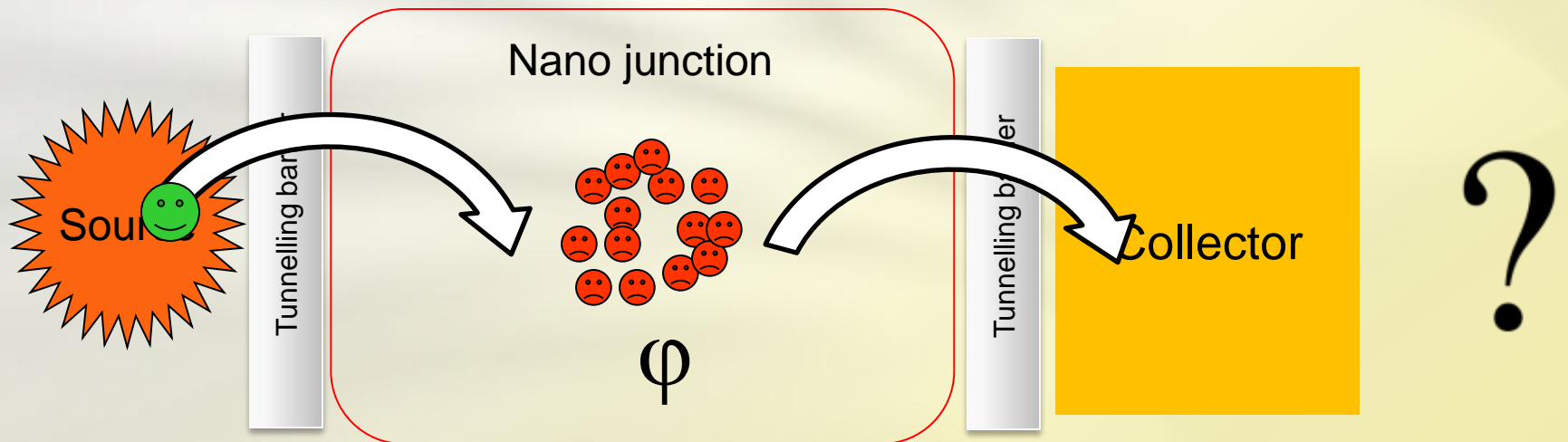
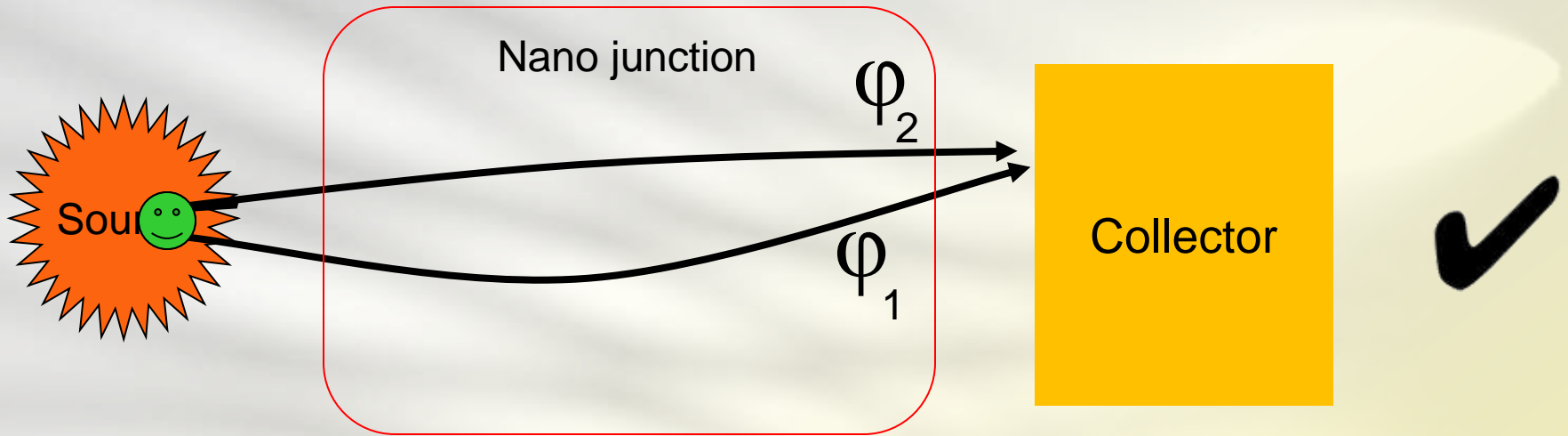
S.H. Ke, et al.  
*Nano Lett.*, **8**, 3257 (2008)



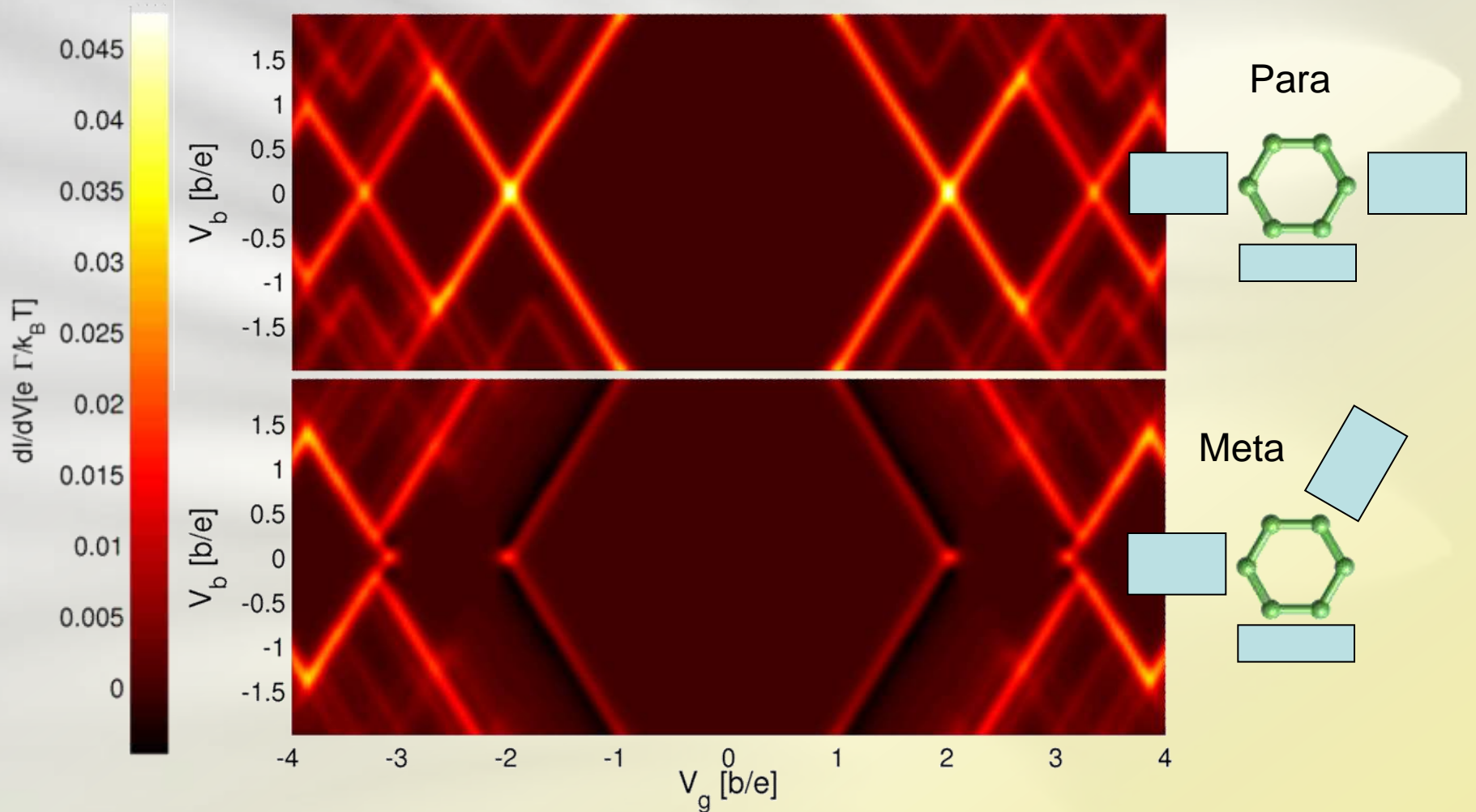
T. Markussen, et al.  
*Nano Lett.*, **10**, 4260 (2010)



# Interference and dephasing



# Interference in weak coupling

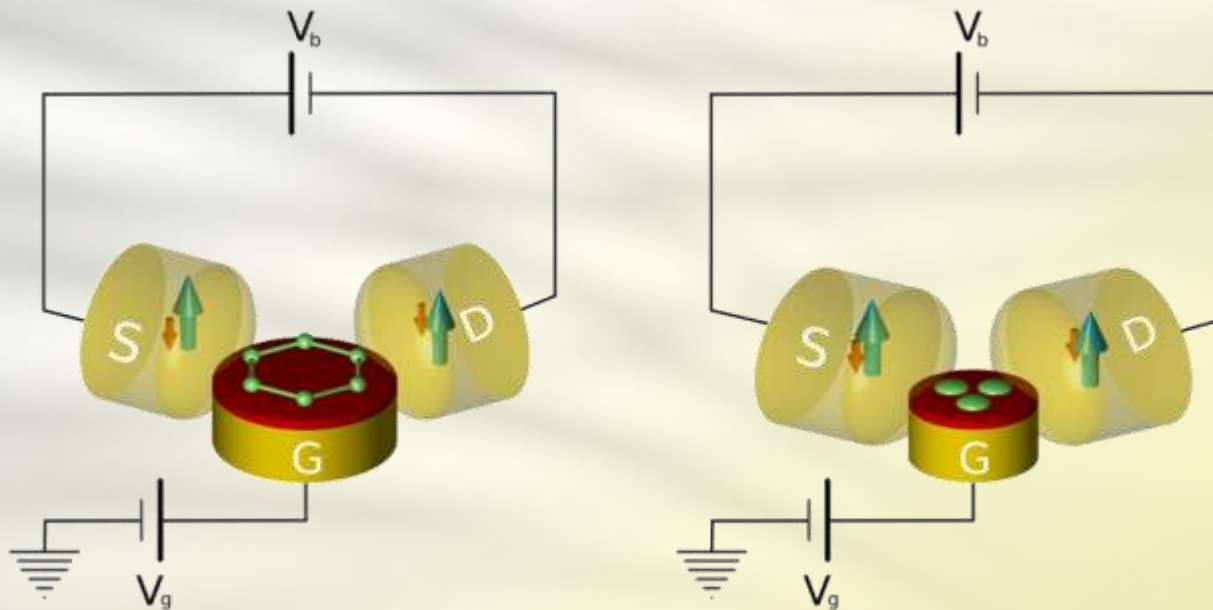


G. Begemann, D. Darau, **AD**, M. Grifoni, *Phys. Rev. B* **77**, 201406(R) (2008)

Regensburg - 31.01.2011

# Interference

## Single Electron Transistors

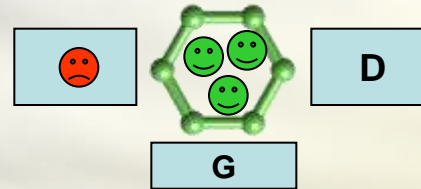


# (Benzene) ISET...

- **Weak coupling**
- **Coulomb** interaction
- Molecular **size**
- **Low** temperature



**Coulomb blockade**

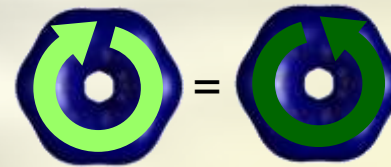


$$\hbar\Gamma \ll k_B T \ll \Delta E_{\text{ex}}$$

- **Rotational** symmetry



**Orbitally degenerate states**

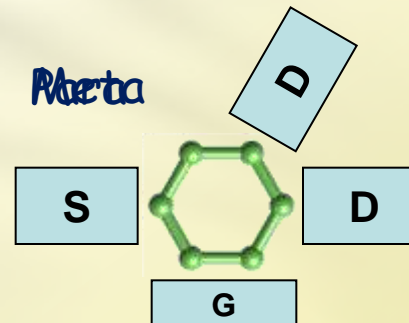


$$E_1 = E_2$$

- Contact **geometry**



**Contact symmetry breaking**



$$\frac{\gamma_{1L}}{\gamma_{2L}} \neq \frac{\gamma_{1R}}{\gamma_{2R}}$$

# ... with a magnetic flavour

- **Coulomb** interaction
- Molecular **size**



**Exchange splitting**

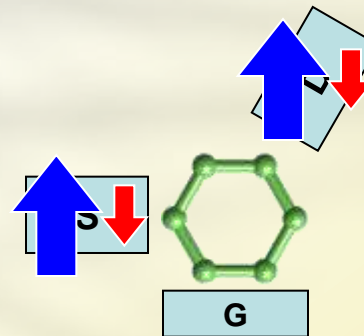


$$E_{\text{triplet}} \neq E_{\text{singlet}}$$

- Parallel **ferromagnetic** leads



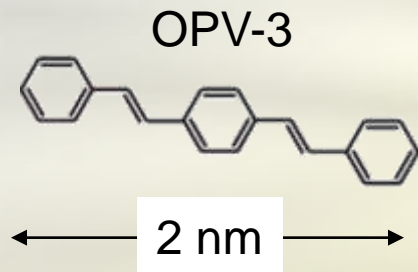
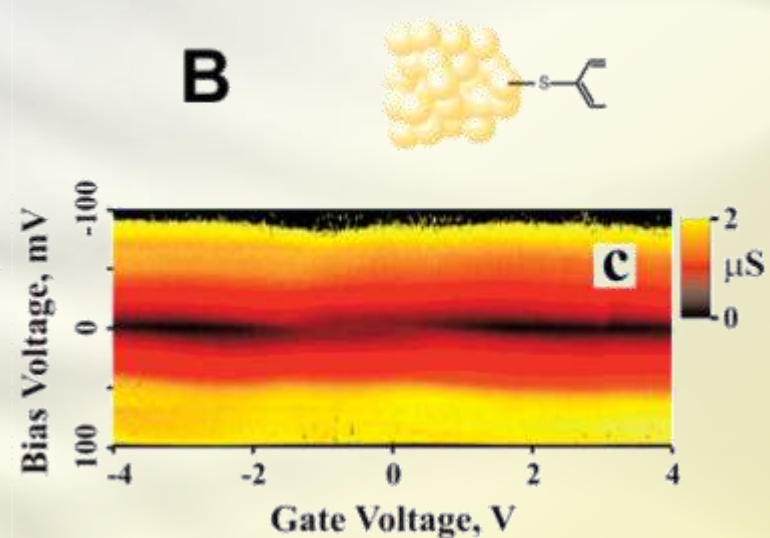
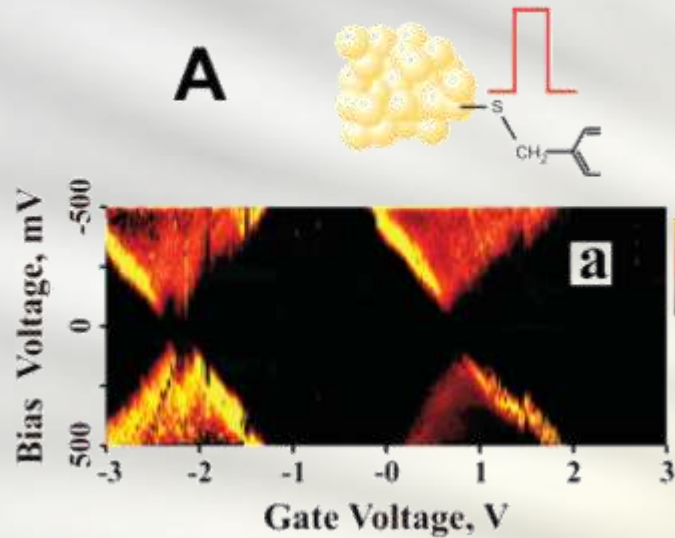
**Spin symmetry breaking**



$$\Gamma_{\alpha\uparrow} \neq \Gamma_{\alpha\downarrow}$$

Are these conditions **achievable** in today's experiments?

# Coulomb blockade

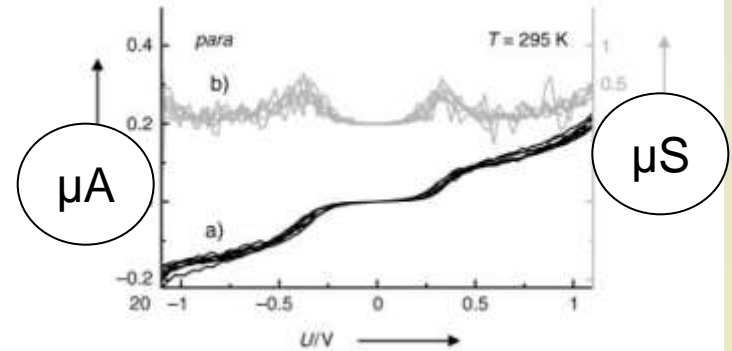
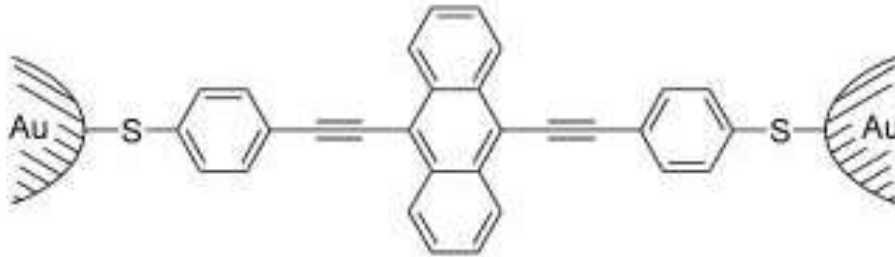


- **Gating** of 2 nm sized molecule
- **Weak coupling** realization with specific anchor groups

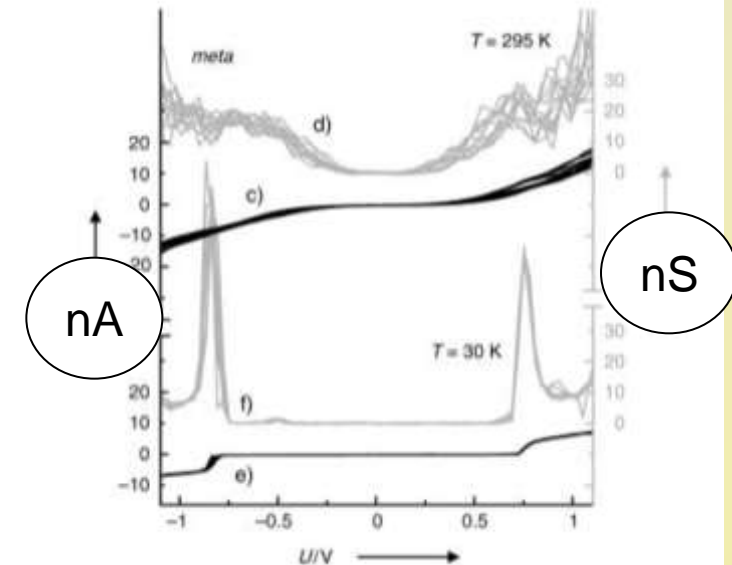
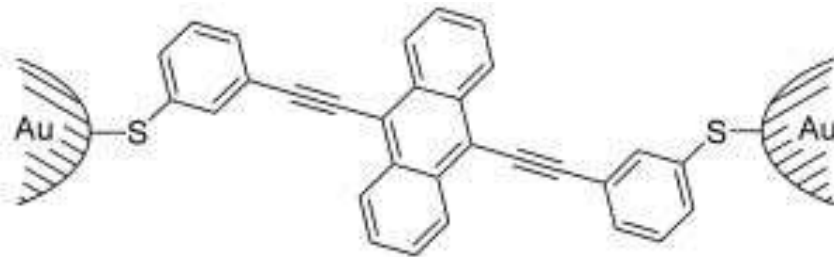
A. Danilov, S. Kubatkin, et al. *Nano Lett.* **8**, 1 (2008)

# Symmetry breaking contacts

**Para** configuration



**Meta** configuration

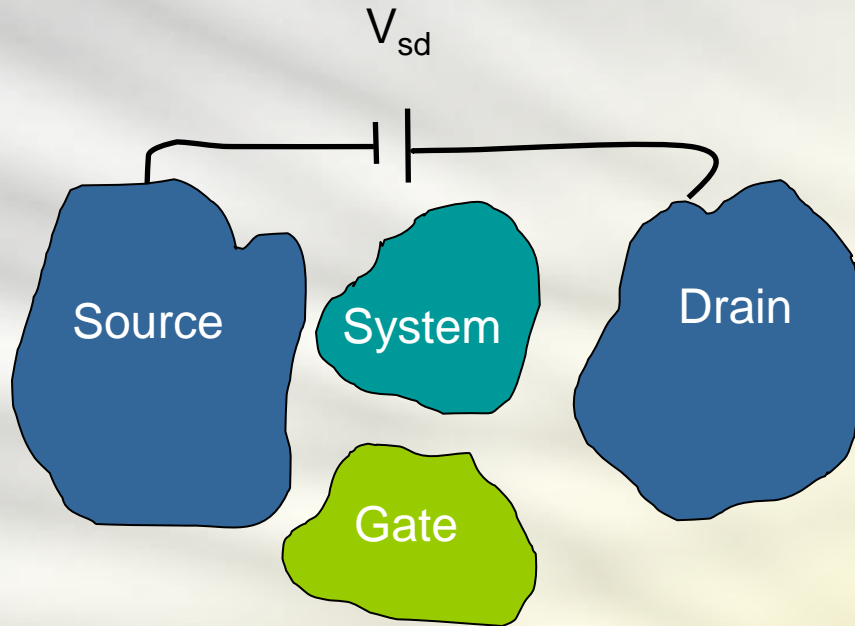


M. Mayor, H. Weber, et al. *Angew. Chem. Int. Ed.* **42** 5843 (2003)

Regensburg - 31.01.2011

physikalisches

# The Hamiltonian



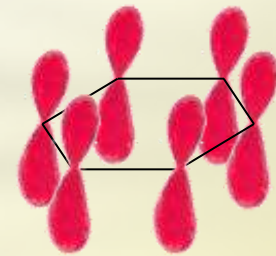
$$H = H_{\text{Sys}} + H_{\text{leads}} + H_{\text{tun}} \left\{ \begin{array}{l} H_{\text{Sys}} = H_{\text{ben}} / H_{\text{TD}} \\ H_{\text{leads}} = \sum_{\alpha k \sigma} \epsilon_k c_{\alpha k \sigma}^\dagger c_{\alpha k \sigma} \\ H_{\text{tun}} = t \sum_{\alpha k \sigma} \left( d_{\alpha \sigma}^\dagger c_{\alpha k \sigma} + c_{\alpha k \sigma}^\dagger d_{\alpha \sigma} \right) \end{array} \right.$$



# Interacting isolated benzene

- The **Pariser-Parr-Pople** Hamiltonian for isolated benzene reads:

$$\begin{aligned}
 H_{\text{ben}}^0 = & \xi_0 \sum_{i\sigma} d_{i\sigma}^\dagger d_{i\sigma} + b \sum_{i\sigma} \left( d_{i\sigma}^\dagger d_{i+1\sigma} + d_{i+1\sigma}^\dagger d_{i\sigma} \right) \\
 & + U \sum_i \left( n_{i\uparrow} - \frac{1}{2} \right) \left( n_{i\downarrow} - \frac{1}{2} \right) \\
 & + V \sum_i \left( n_{i\uparrow} + n_{i\downarrow} - 1 \right) \left( n_{i+1\uparrow} + n_{i+1\downarrow} - 1 \right)
 \end{aligned}$$



- The **size** of the Fock space for the many-body system  $4^6 = 4096$  since for each site there are 4 possibilities:  $|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle$
- Within this Fock space we diagonalize **exactly** the Hamiltonian.

# Symmetry of the ground states

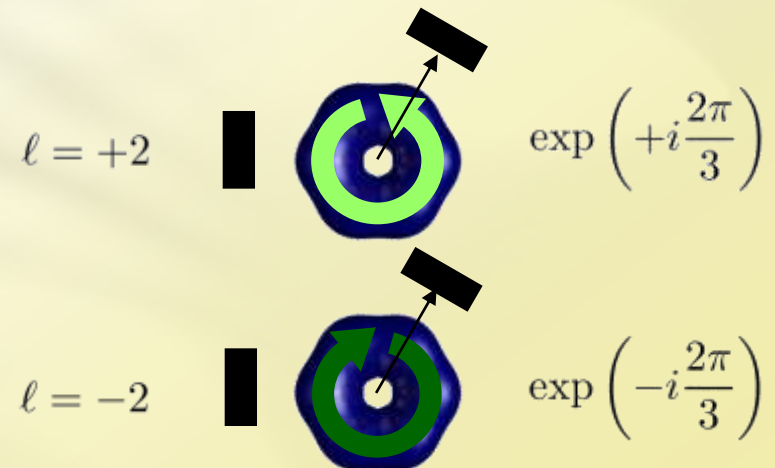
N	Degeneracy	GS energy[eV] (at $\xi = 0$ )	GS symmetry representation
0	1	0	$A_{1g}$
1	2	-22	$A_{2u}$
2	1	-42.25	$A_{1g}$
3	4	-57.42	$E_{1g}$
4	3	-68.875	$A_{2g}$
5	4	-76.675	$E_{1g}$
6	1	-81.725	$A_{1g}$
7	4	-76.675	$E_{2u}$
8	3	-68.875	$A_{2g}$
9	4	-57.42	$E_{2u}$
10	1	-42.25	$A_{1g}$
11	2	-22	$B_{2g}$
12	1	0	$A_{1g}$

## Rotation phase factors

Under rotation of an angle  $\phi = \frac{n\pi}{3}$

- $\mathcal{R}_\phi |6_g\rangle = |6_g\rangle$       No phase acquired

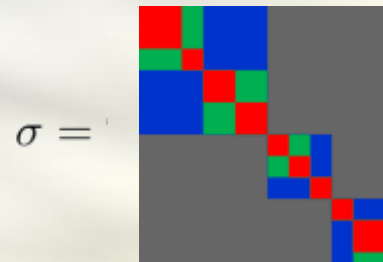
- $\mathcal{R}_\phi |7_g \ell\rangle = e^{-i\ell\phi} |7_g \ell\rangle$        $\ell = \pm 2$



# Generalized Master Equation

- We start with the **Liouville** equation:  $\dot{\rho} = -\frac{i}{\hbar}[\mathcal{H}, \rho]$

- We define the reduced density matrix  $\sigma = \text{Tr}_{\text{Leads}}\{\rho\}$  which is **block-diagonal** in



particle number  
spin  
energy

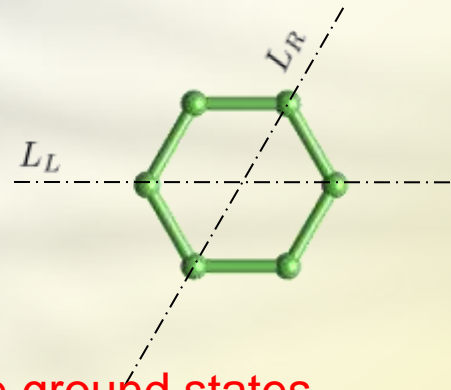
- We keep the coherences between **orbitally** degenerate states.
- The **Generalized Master Equation** is the equation of motion for  $\sigma$  :

$$\dot{\sigma} = \underbrace{-\frac{i}{\hbar}[H_{\text{sys}}, \sigma]}_{\text{Coherent dynamics}} - \underbrace{\frac{i}{\hbar}[H_{\text{eff}}, \sigma]}_{\text{Effective internal dynamics}} + \underbrace{\mathcal{L}_{\text{tun}}\sigma}_{\text{Tunnelling dynamics}}$$

# The effective Hamiltonian

The effective Hamiltonian is expressed in terms of **angular momentum** operators and **renormalization frequencies**:

$$H_{\text{eff}} = \sum_{\alpha\sigma} \omega_{\alpha\sigma} L_{\alpha}$$



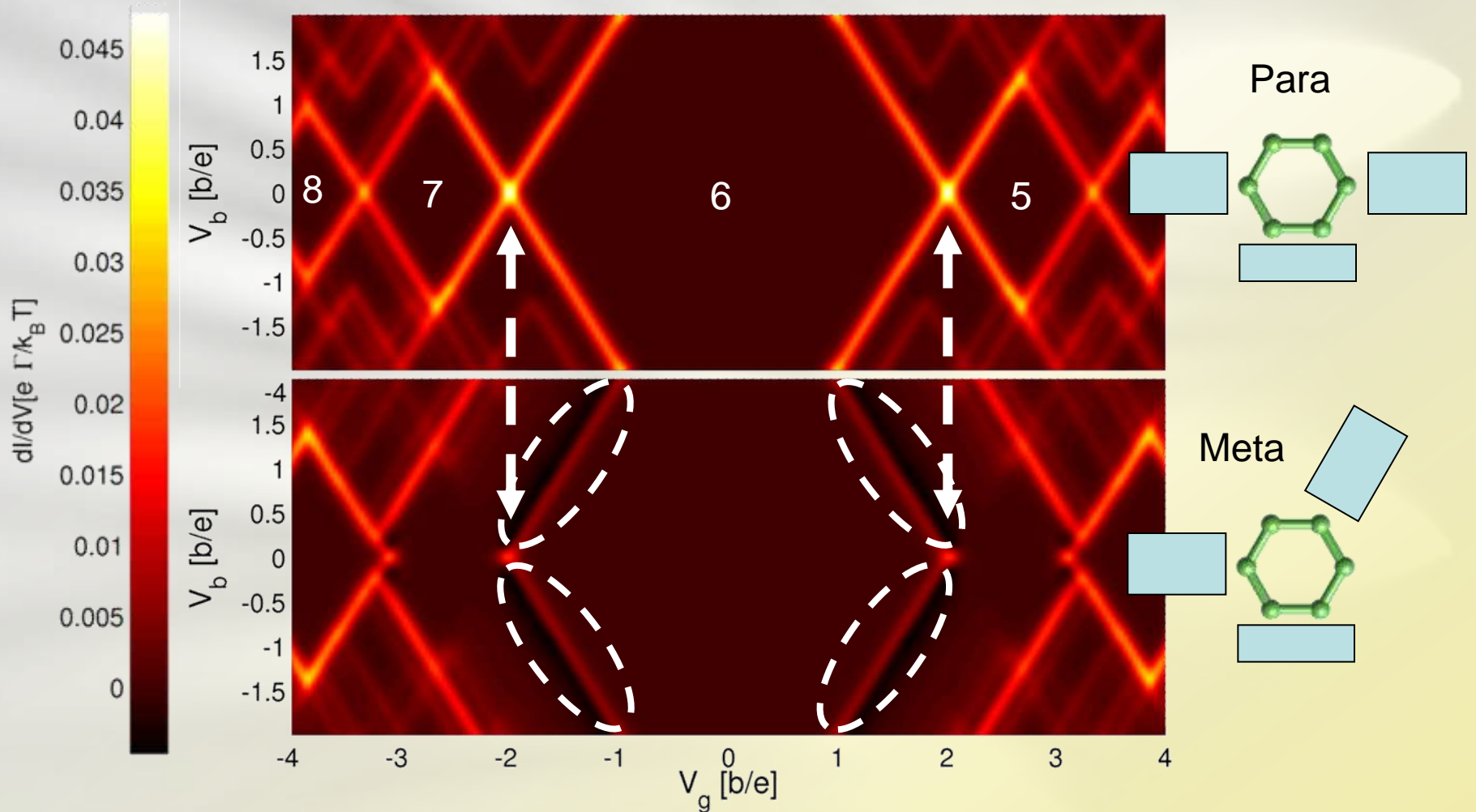
In particular in the Hilbert space of the **7 particle ground states**

$$L_{\alpha} = \frac{\hbar}{2} \begin{pmatrix} 1 & e^{i2|\ell|\phi_{\alpha}} \\ e^{-i2|\ell|\phi_{\alpha}} & 1 \end{pmatrix}$$

$$\omega_{\alpha\sigma} = \frac{1}{\pi} \sum_{\sigma' \{E\}} \Gamma_{\alpha\sigma'}^0 \left[ \langle 7_g \ell \sigma | d_{M\sigma'} | 8\{E\} \rangle \langle 8\{E\} | d_{M\sigma'}^{\dagger} | 7_g m \sigma \rangle p_{\alpha}(E - E_{7_g}) + \langle 7_g \ell \sigma | d_{M\sigma'}^{\dagger} | 6\{E\} \rangle \langle 6\{E\} | d_{M\sigma'} | 7_g m \sigma \rangle p_{\alpha}(E_{7_g} - E) \right]$$

← Bias and gate dependent

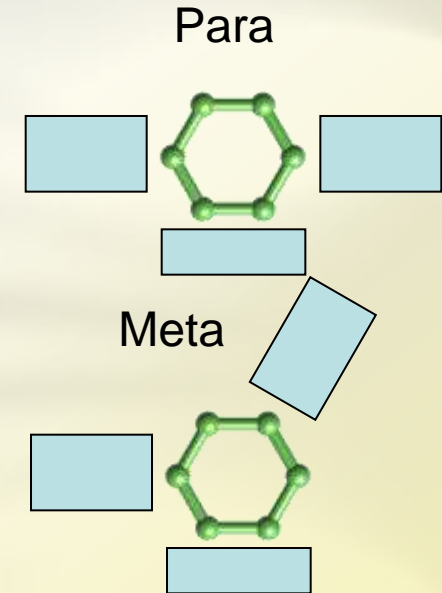
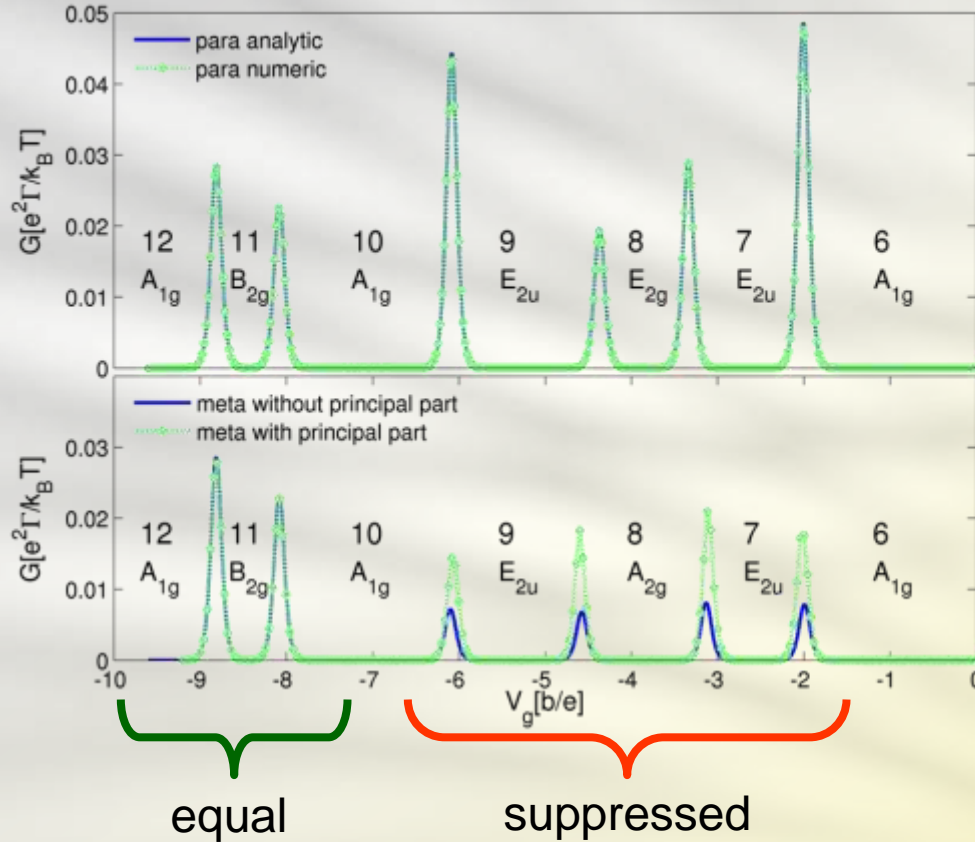
# Para vs. Meta



G. Begemann, D. Darau, **AD**, M. Grifoni, *Phys. Rev. B* **77**, 201406(R) (2008)

Regensburg - 31.01.2011

# Conductance suppression



**A:** non-degenerate  $\longleftrightarrow$  **B:** non-degenerate  $\Rightarrow$  Equal

**A:** non-degenerate  $\longleftrightarrow$  **E:** degenerate  $\Rightarrow$  Suppressed

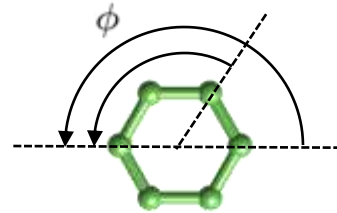
# Destructive interference

Interference  
factor

$$\Lambda = \left| \sum_{nm\tau} \langle N, n | d_{L\tau} | N+1, m \rangle \langle N+1, m | d_{R\tau}^\dagger | N, n \rangle \right|^2$$

$$\Lambda = \left| \sum_{nm\tau} |\langle N, n | d_{L\tau} | N+1, m \rangle|^2 e^{i\phi_{nm}} \right|^2$$

$$d_{R\tau}^\dagger = \mathcal{R}_\phi^\dagger d_{L\tau}^\dagger \mathcal{R}_\phi$$



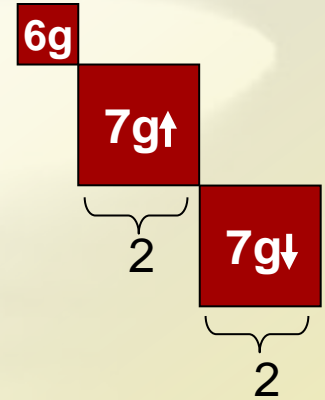
In particular for the transition **6 -7** in the **meta** configuration:

$$\Lambda = \left| |\langle 6_g | d_{L\tau} | 7_g, +2, \tau \rangle|^2 e^{+i\frac{2\pi}{3}} + |\langle 6_g | d_{L\tau} | 7_g, -2, \tau \rangle|^2 e^{-i\frac{2\pi}{3}} \right|^2$$

$$= \left| \left( \text{green path} e^{+i\frac{2\pi}{3}} + \text{red path} e^{-i\frac{2\pi}{3}} \right)^2 \right.$$

# Negative Differential Conductance

- The 7 particle ground state has spin and orbital **degeneracies**;
- **Physical basis**: the basis that diagonalizes the stationary density matrix;
- The physical basis **depends on the bias**: in whatever reference basis, **coherences** are essential for a correct description of the system;
- The **visualization tool**: **position resolved** transition probability to the physical basis:



$$P(x, y; \ell\tau) = \lim_{L \rightarrow \infty} \sum_{\sigma} \frac{1}{2L} \int_{-L/2}^{L/2} dz |\langle 7_g \ell\tau | \psi_{\sigma}^{\dagger}(\vec{r}) | 6_g \rangle|^2$$



# Interference blockade

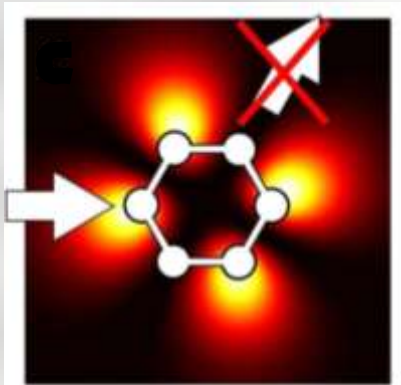


## Geometry

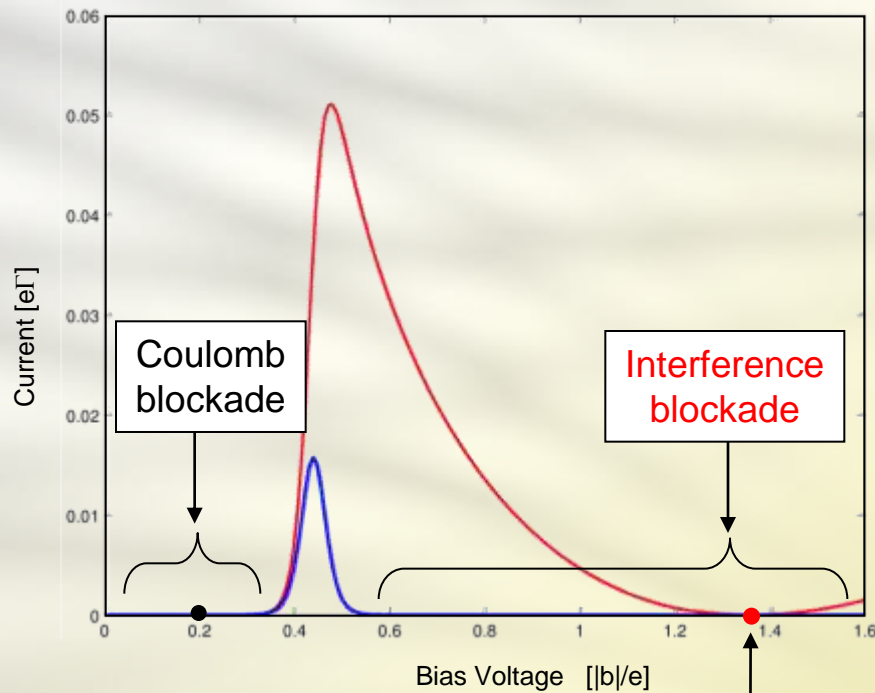
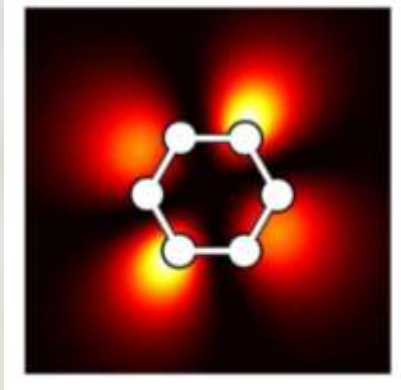
## I-V for transition 6 -7

## Energetics

### Blocking state

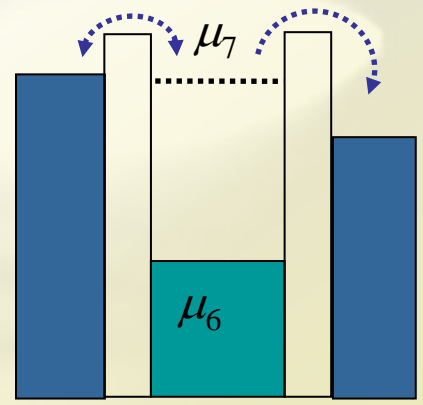


### Non-blocking state

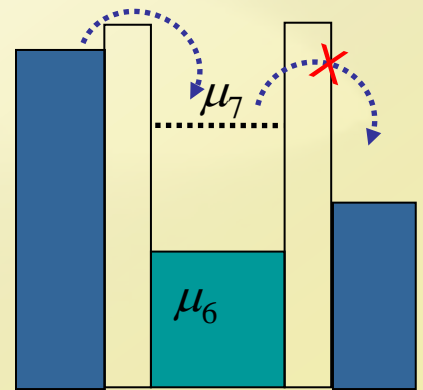


The **blocking** state is an eigenstate of the effective Hamiltonian

$$\omega_{L\sigma} = 0$$

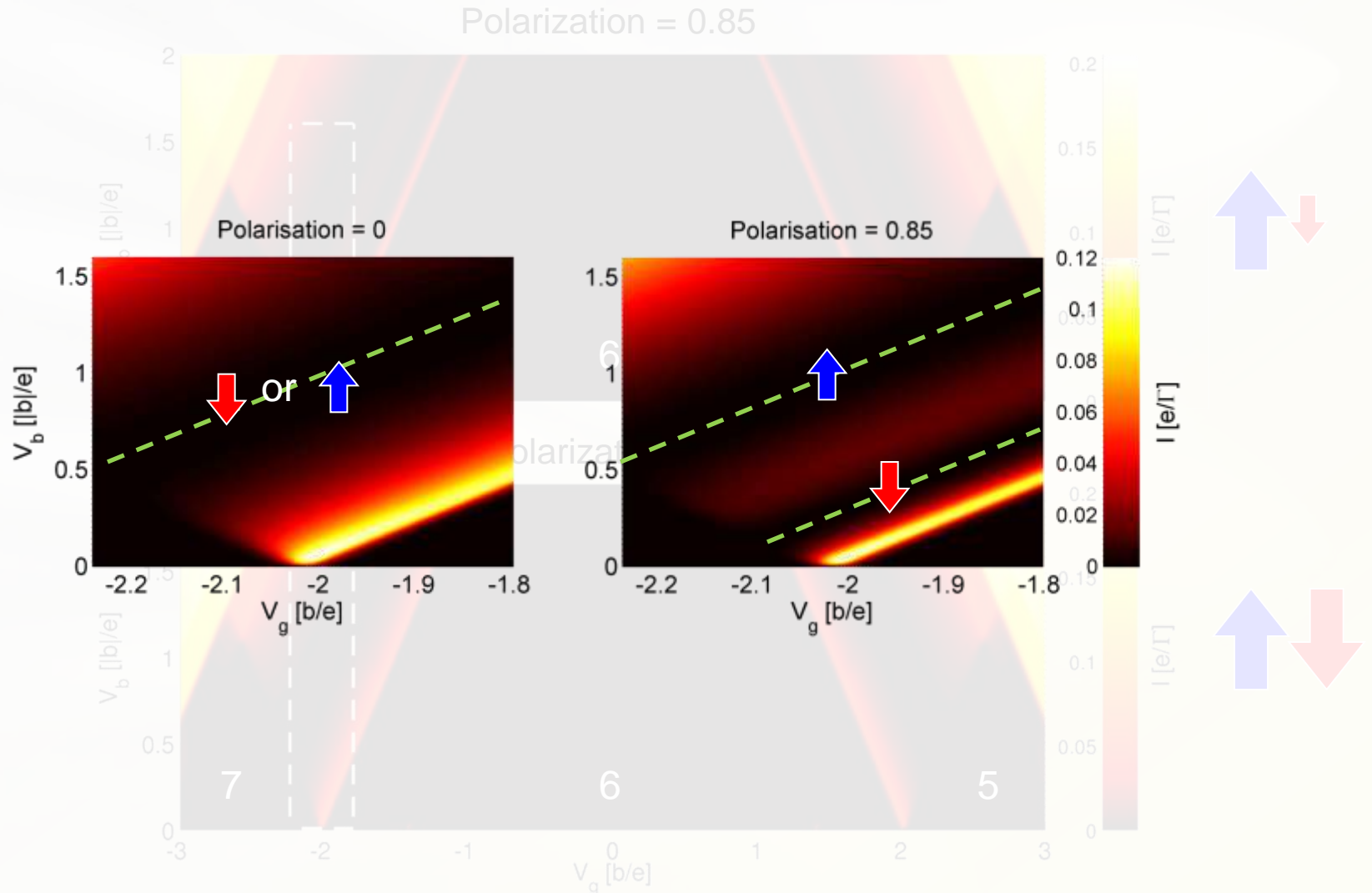


current onset

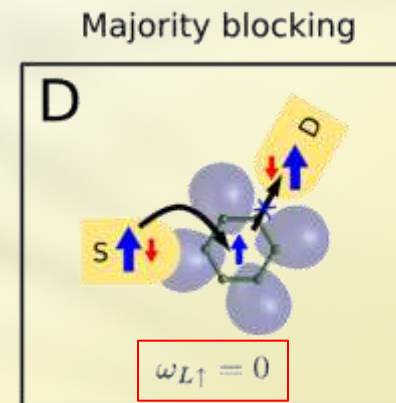
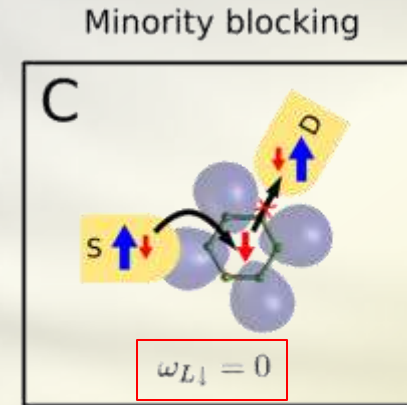
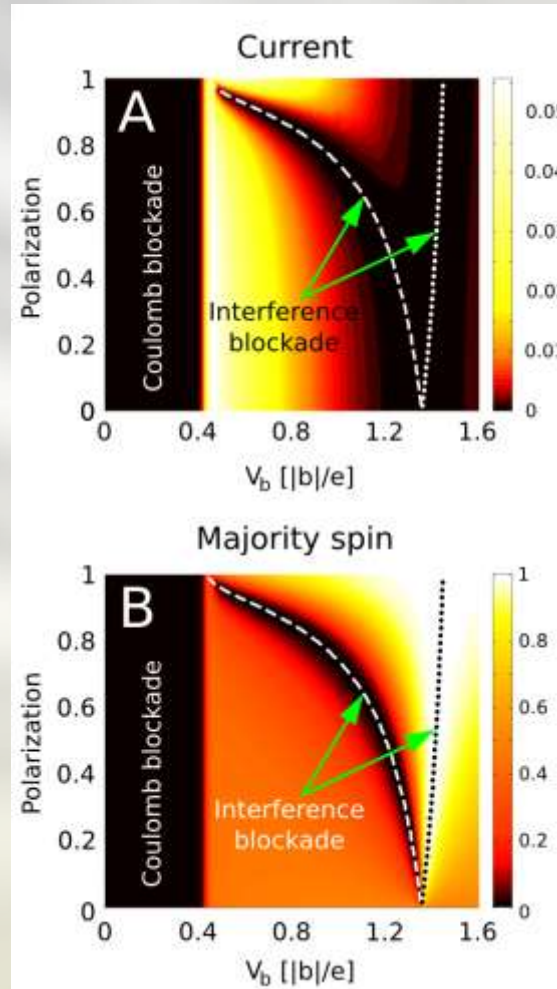


blockade

# Normal vs. ferromagnetic leads



# Selective Interference Blocking



AD, G. Begemann, and M. Grifoni *Nano Lett.* **9**, 2897 (2009)

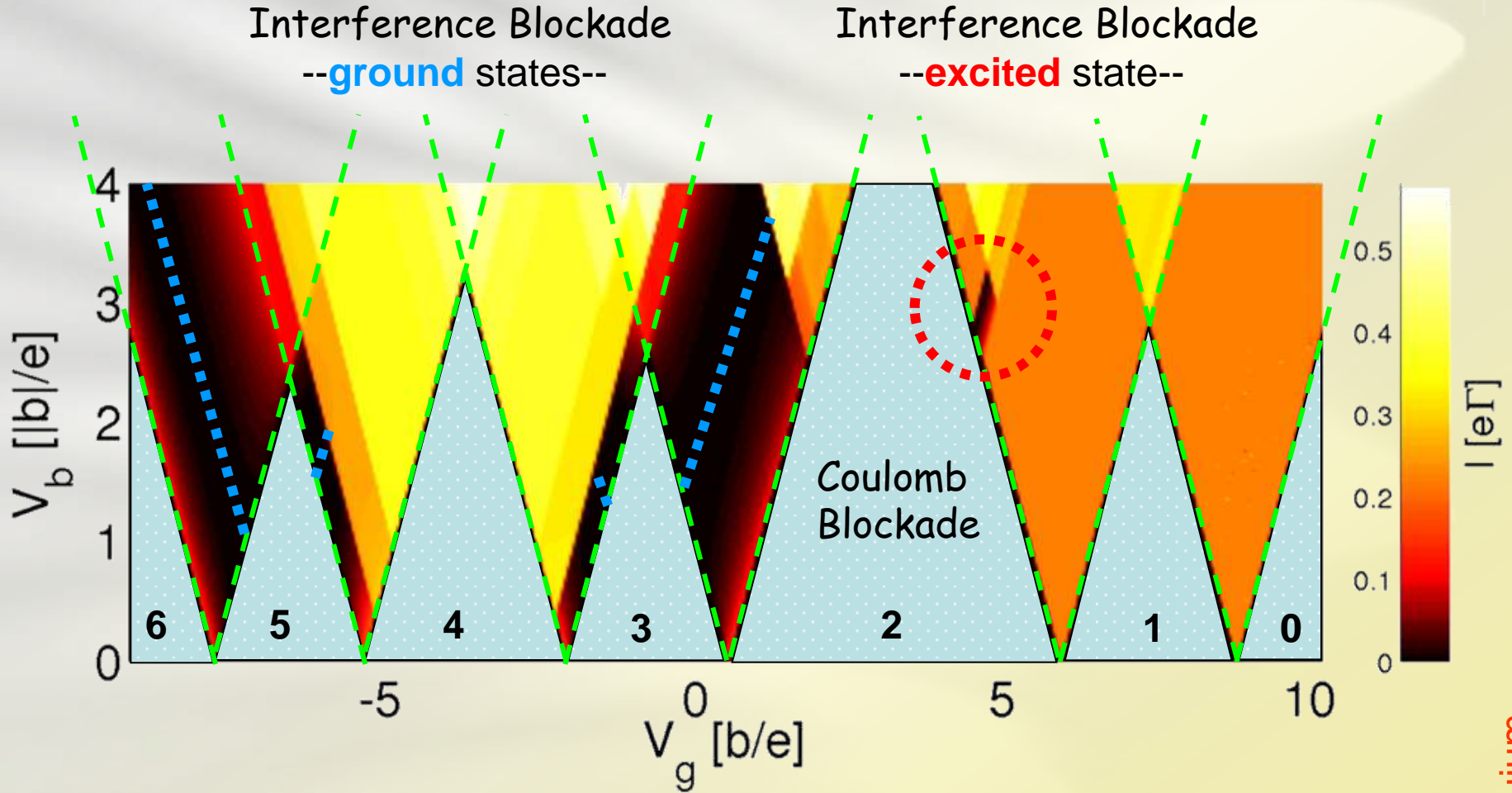
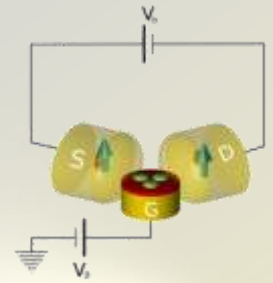
# Level renormalization in presence of polarized leads

We obtain a difference in the renormalization frequencies for the 2 spin directions linear in the **polarization of the leads**:

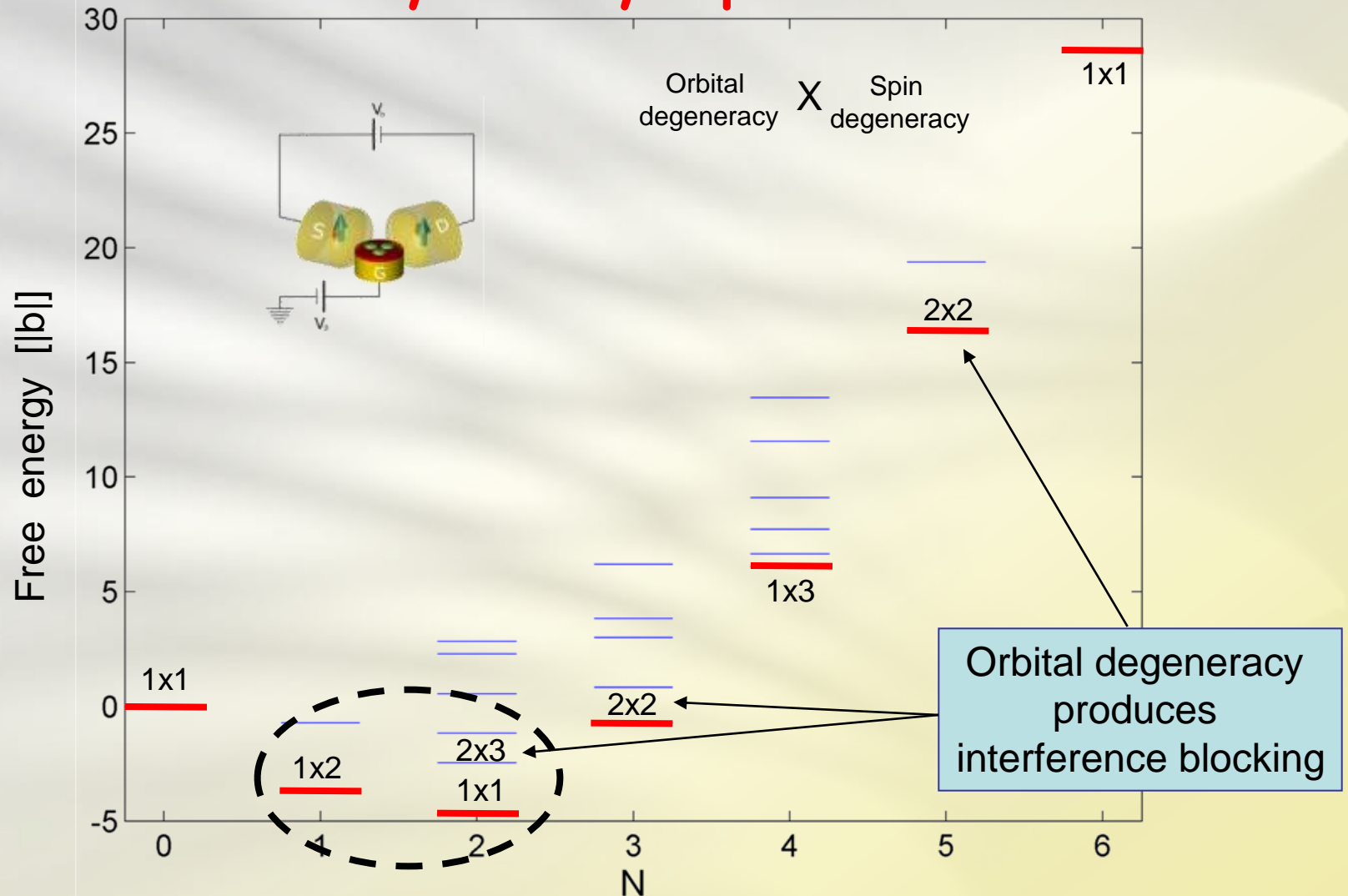
$$\omega_{\alpha\uparrow} - \omega_{\alpha\downarrow} = 2\bar{\Gamma}_{\alpha}^{-0} P_{\alpha} \frac{1}{\pi} \sum_{\{E\}} \left[ \begin{aligned} &\langle 7_g \ell \uparrow | d_{M\uparrow} | 8\{E\} \rangle \langle 8\{E\} | d_{M\uparrow}^{\dagger} | 7_g m \uparrow \rangle p_{\alpha}(E - E_{7_g}) \\ &+ \langle 7_g \ell \uparrow | d_{M\uparrow}^{\dagger} | 6\{E\} \rangle \langle 6\{E\} | d_{M\uparrow} | 7_g m \uparrow \rangle p_{\alpha}(E_{7_g} - E) \\ &- \langle 7_g \ell \uparrow | d_{M\downarrow} | 8\{E\} \rangle \langle 8\{E\} | d_{M\downarrow}^{\dagger} | 7_g m \uparrow \rangle p_{\alpha}(E - E_{7_g}) \\ &- \langle 7_g \ell \uparrow | d_{M\downarrow}^{\dagger} | 6\{E\} \rangle \langle 6\{E\} | d_{M\downarrow} | 7_g m \uparrow \rangle p_{\alpha}(E_{7_g} - E) \end{aligned} \right]$$

The splitting of the level renormalization depends crucially on the Coulomb interaction on the molecule and **vanishes in absence of exchange**.

# Triple dot ISET



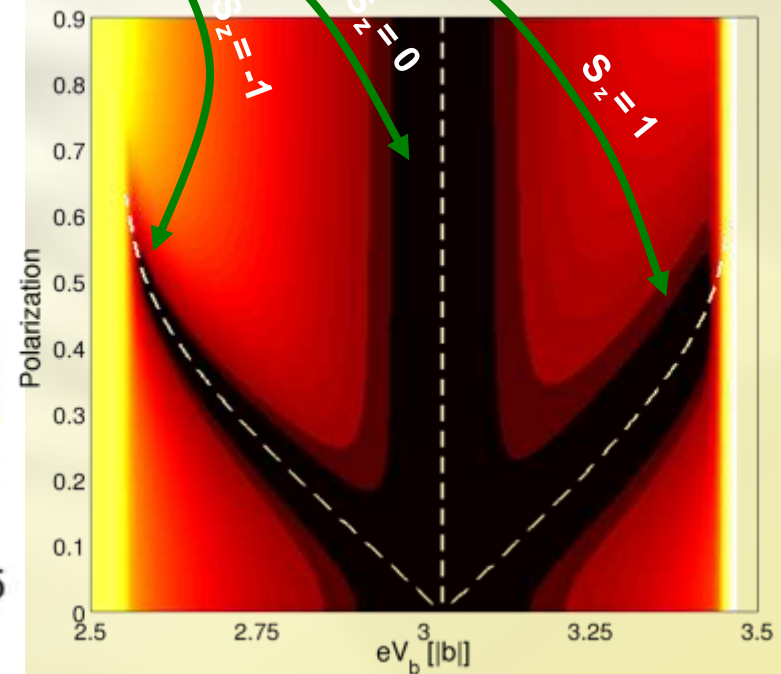
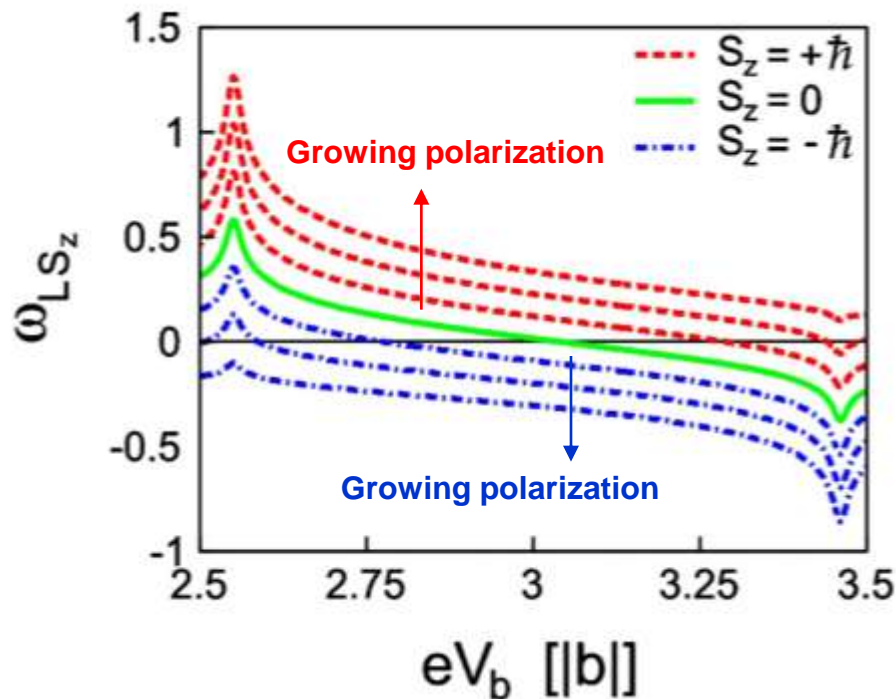
# Many-body spectrum



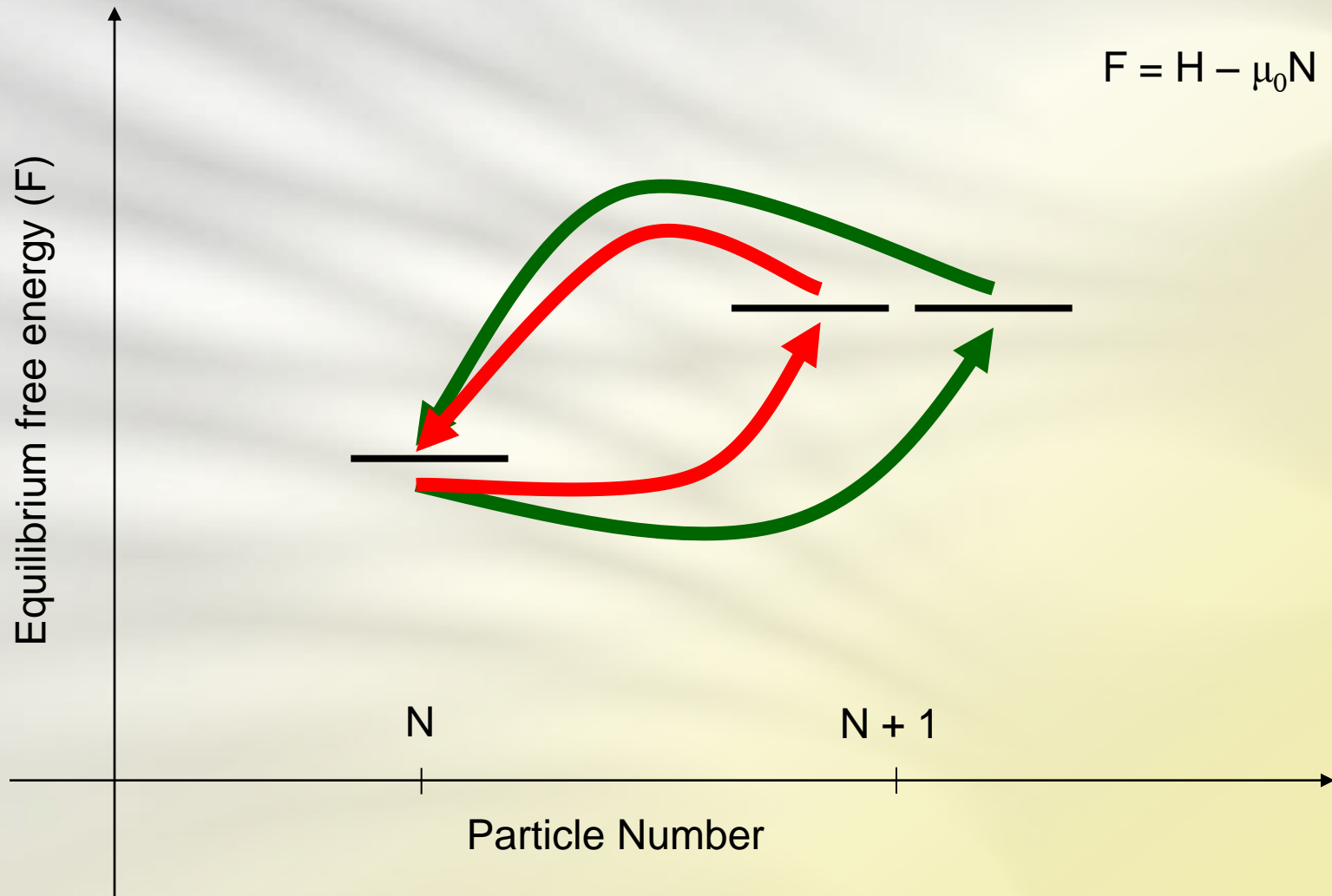
# Triplet splitting

The states decoupled from the right lead are eigenstates of  $L_R$ . They are eigenstates of  $H_{\text{eff}}$  only if

$$\omega_L S_z = 0$$



# The "two paths" in the ISET





# Robustness

- We have tested the **robustness** of the effects against:
  - Residual **potential drop** on the (artificial) molecule (in weak coupling to the leads the potential drop is concentrated at the contacts)
  - On-site **energy renormalization** of the contact atom due to different anchor groups
  - Lifting of the electronic degeneracy due to deformation (**static Jahn-Teller effect**)
- The minimal necessary condition is **quasi-degeneracy**:

$$\delta E \ll \hbar\Gamma$$

D. Darau, G. Begemann, **AD**, and M. Grifoni, *PRB*, **79**, 235404 (2009)

# Blocking conditions

The interference blocking state:

- is a **linear combination** of (quasi-)degenerate system eigenstates
- is **achievable from the global minimum** via a finite number of allowed transitions
- has **vanishing tunnelling amplitudes** for all energetically allowed outgoing transitions

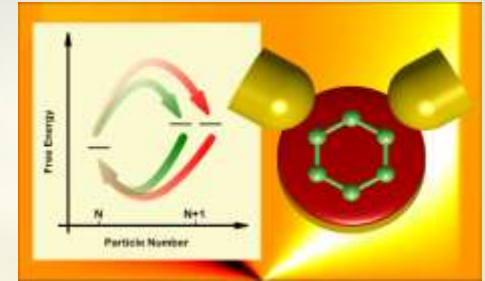
$$\mathcal{L}_{\text{tun}}\sigma_B = 0$$

- is an eigenstate of the **effective Hamiltonian**

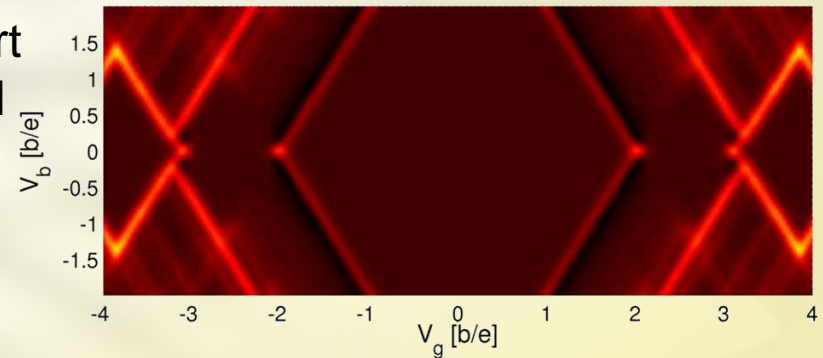
$$[H_{\text{eff}}, \sigma_B] = 0$$

# Conclusions

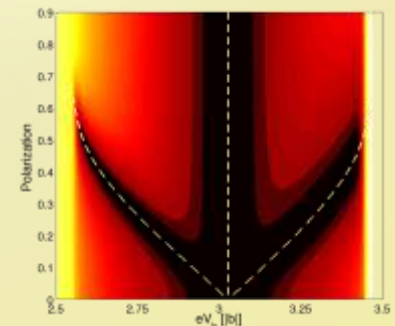
- Interference does occur even in the single-electron tunnelling regime when energetically equivalent paths involving **degenerate states** contribute to the dynamics.



- Interference effects dominates the transport characteristics of ISET both in the linear and non linear regime producing selective **suppression of the conductance** and interference **current blocking**.



- In the presence of ferromagnetic leads, the interplay between interference and exchange on the ISET allows to achieve **all-electrical spin control** of the junction.



# Thanks



Georg Begemann



Milena Grifoni



Dana Darau

Deutsche  
Forschungsgemeinschaft

DFG

in the research programs



SPP 1243 Quantum Transport  
at the Molecular Scale

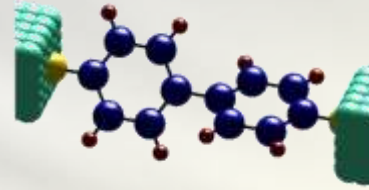


SFB 689 Spinphänomene  
in reduzierten Dimensionen

# Beyond... ISET

Vibronic effects in transport through conjugated molecules

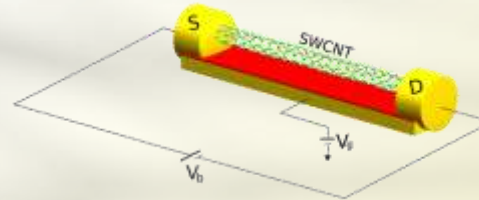
*Phys. Rev. Lett.*, **97**, 166801 (2006)



K. Richter

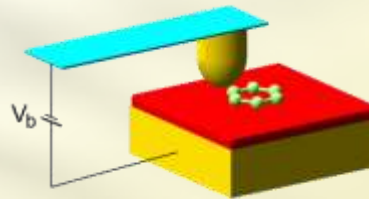
Transport through suspended single wall carbon nanotube quantum dots

*arXiv:1101.3892 submitted to PRB*



A. Yar

Interference effects in transport through single molecules in the STM set-up

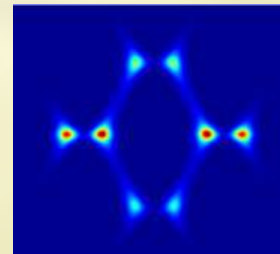


S. Kolmeder



J. Repp

Transport through double dot structures with multiple gates



D. Preusche

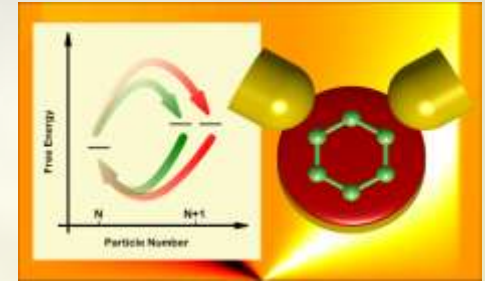


A. Hüttel

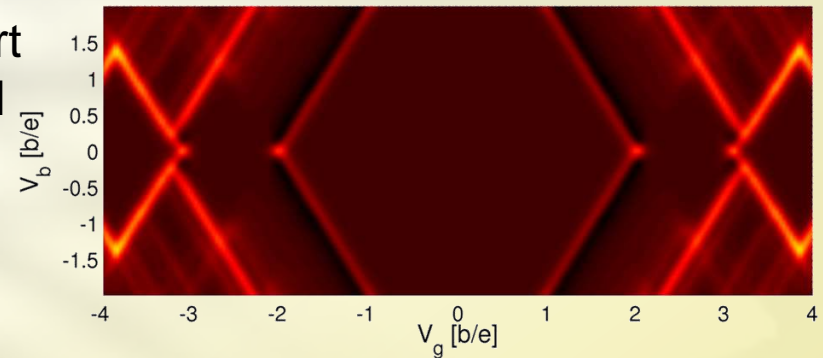
Thank you for your attention!

# Conclusions

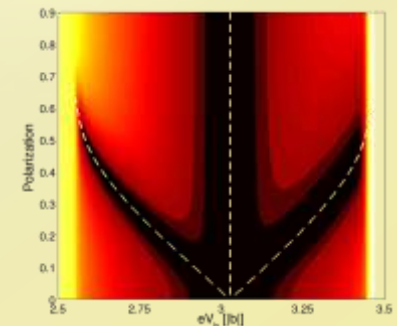
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- Interference effects dominates the transport characteristics of ISET both in the linear and non linear regime producing selective **suppression of the conductance** and interference **current blocking**.



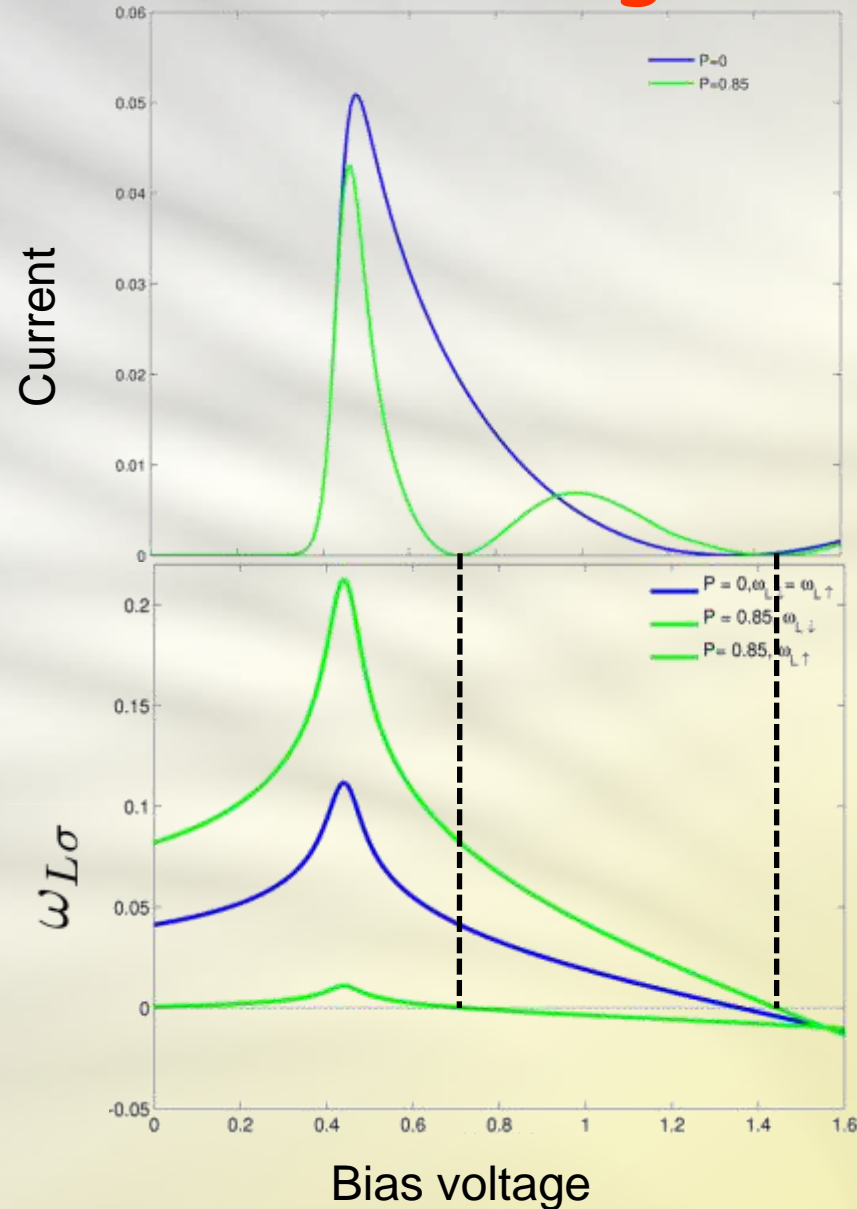
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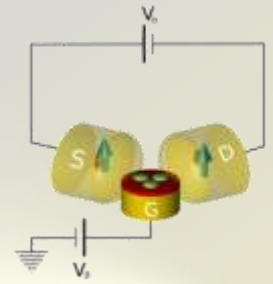
# Supplementary material



# Normal vs ferromagnetic leads



# The triple dot ISET



$$H = H_{\text{sys}} + H_{\text{leads}} + H_{\text{tun}}$$

$$\begin{aligned}
 H_{\text{sys}} = & \xi_0 \sum_{i\sigma} d_{i\sigma}^\dagger d_{i\sigma} + b \sum_{i\sigma} \left( d_{i\sigma}^\dagger d_{i+1\sigma} + d_{i+1\sigma}^\dagger d_{i\sigma} \right) \\
 & + U \sum_i \left( n_{i\uparrow} - \frac{1}{2} \right) \left( n_{i\downarrow} - \frac{1}{2} \right) \\
 & + V \sum_i \left( n_{i\uparrow} + n_{i\downarrow} - 1 \right) \left( n_{i+1\uparrow} + n_{i+1\downarrow} - 1 \right)
 \end{aligned}$$

**Extended Hubbard**  
Hamiltonian with on-site  
and nearest neighbors  
**Coulomb interaction**

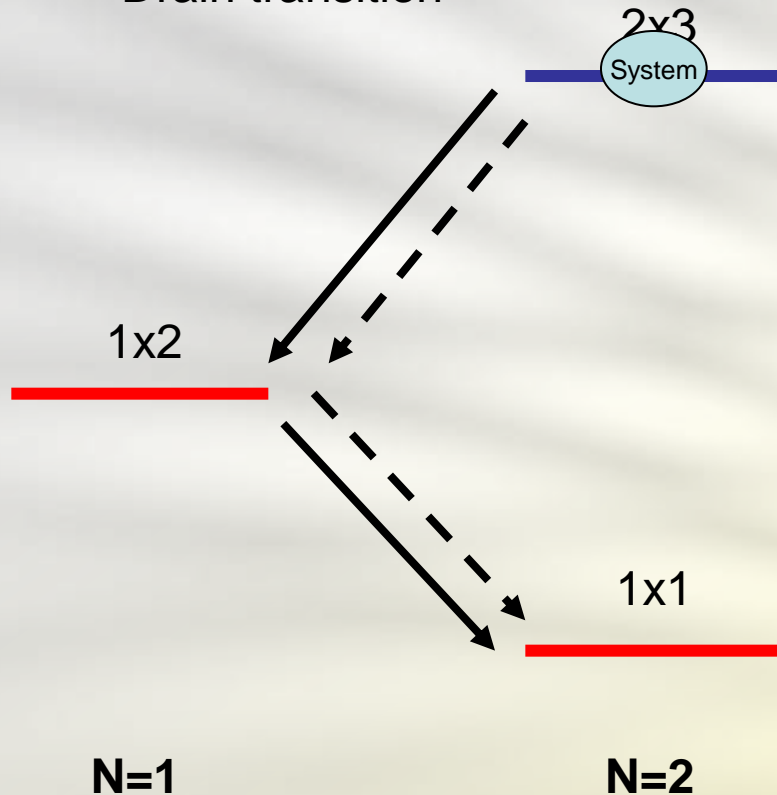
$$H_{\text{tun}} = t \sum_{\alpha k \sigma} \left( c_{\alpha k \sigma}^\dagger d_{\alpha \sigma} + d_{\alpha \sigma}^\dagger c_{\alpha k \sigma} \right)$$

Tunnelling restricted to the dot  
closest to the corresponding lead

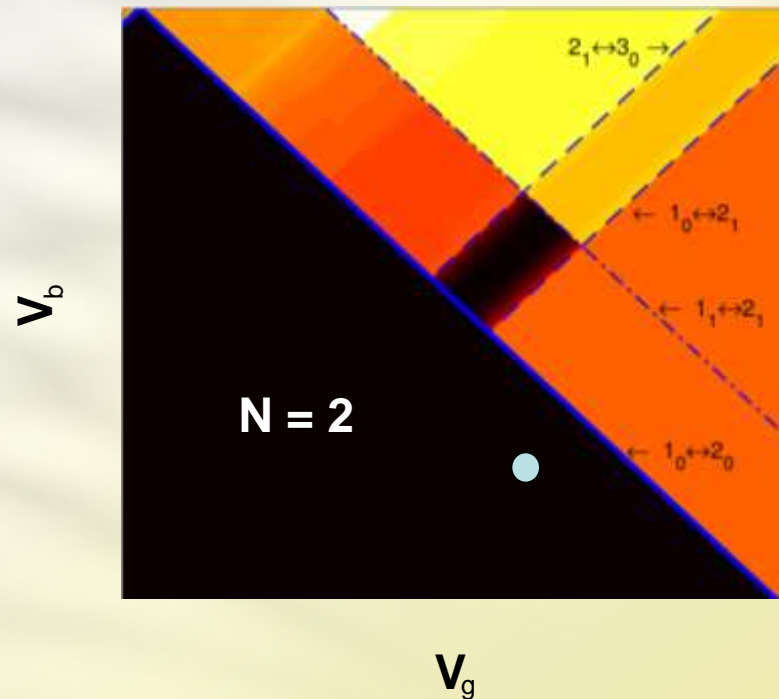
$H_{\text{leads}}$  Ferromagnetic leads with equal parallel polarization

# Excited state blocking

—▶ Source transition  
 - -▶ Drain transition

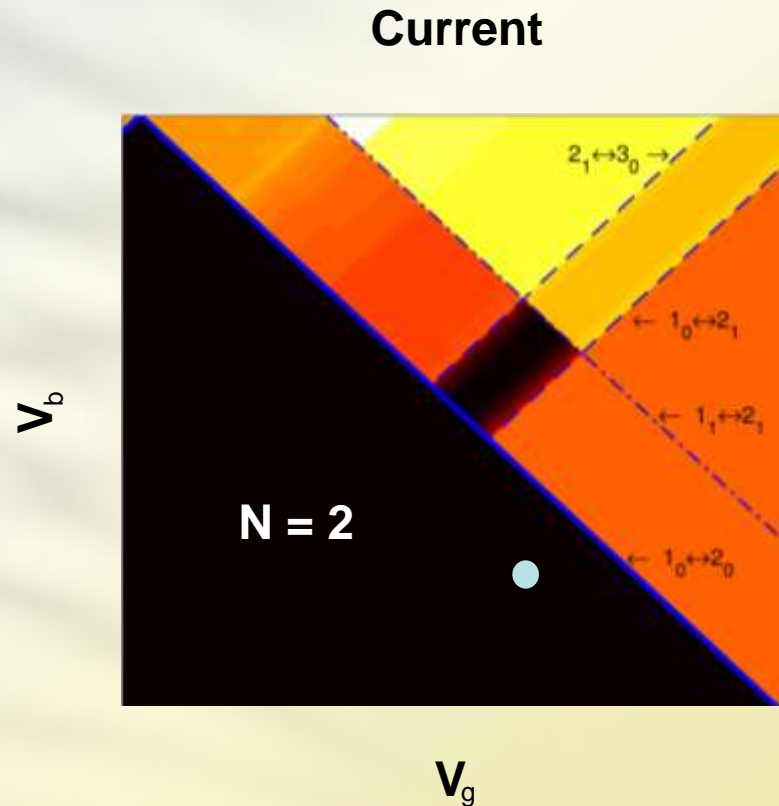
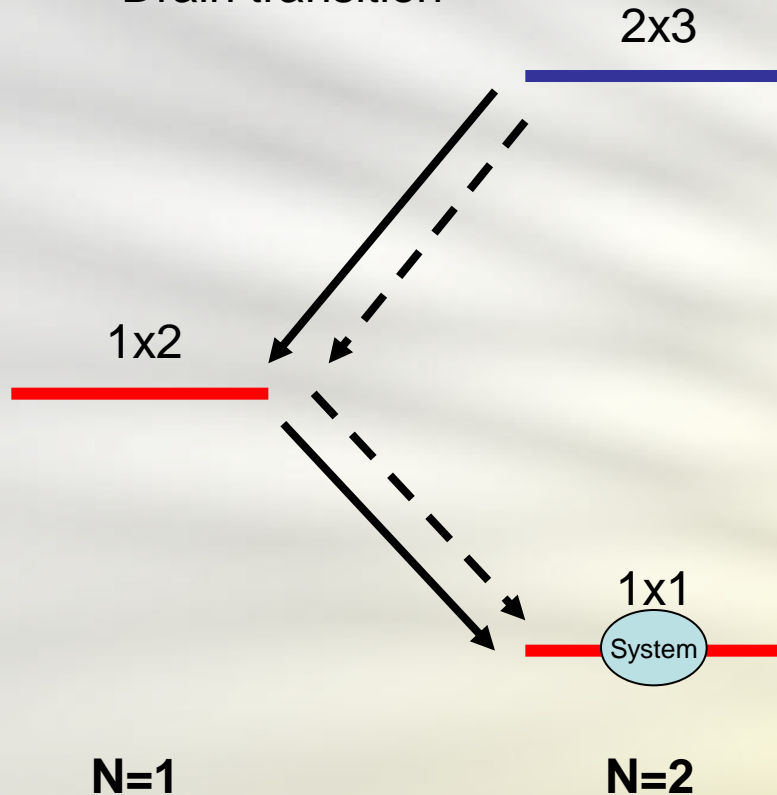


## Coulomb Blockade



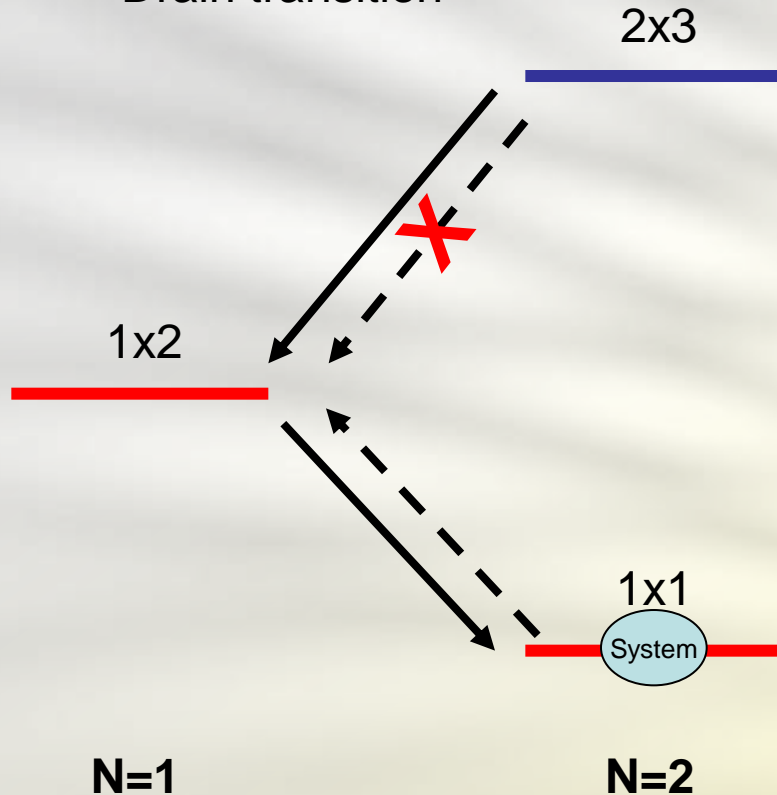
# Excited state blocking

—▶ Source transition  
 - -▶ Drain transition

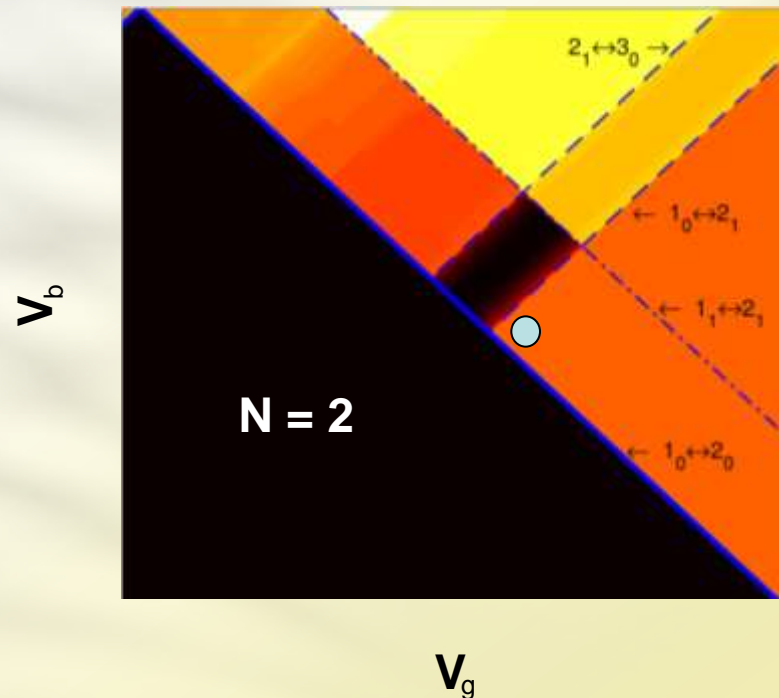


# Excited state blocking

—▶ Source transition  
 - -▶ Drain transition

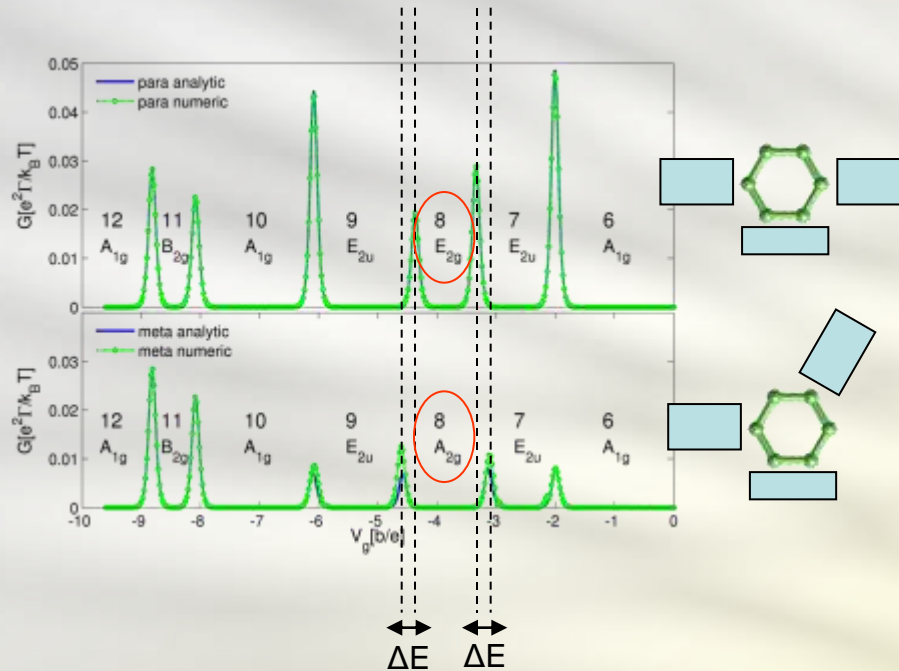


## Interference Blockade

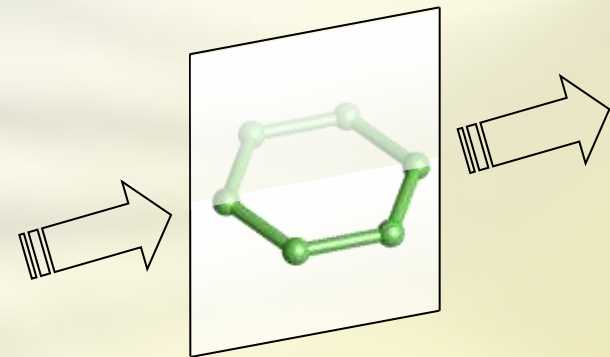


**Three** linear combinations of 2-particle excited states are coupled **ONLY to the source**.

# The 8 electrons "anomaly"



Mirror symmetry of the para-configuration



The tunnelling preserves this **mirror symmetry**: the lowest 8 electron state involved in transport is the mirror-symmetric (first excited) state with  $E_{2g}$  symmetry.