Regular orbits and out-of-equilibrium phase transitions on systems with long-range interactions

Long-range interactions are encountered in a wide range of systems, from wave-particle interactions (plasma physics, Free Electron Lasers, etc) to astrophysics and Bose-Einstein condensates. There are characterized by an interaction potential in 1/r^a, with a>d the dimension of the system. Their dynamics exhibits a special feature: They get trapped in out-of-equilibrium regimes over very long times, diverging with the number of bodies in interaction. These dynamics are called “Quasi-Stationnary States” (QSS).

The QSS are here studied with the paradigmatic Hamiltonian Mean-Field model, composed of N particles globally coupled through the total magnetization of the system, which quantifies the spatial aggregation of the particles. The QSS can be decomposed in essentially two families: The ones with finite magnetization, and those with residual magnetization.

It can then be shown that when the number of degrees of freedom of the system increases (via the number of particles), regular orbits appear in the system, associated to invariant tori of the dynamics of a test-particle. These tori are another way to explain the presence of these long-lived regimes, while a statistical approach (based on the maximization of an out-of-quilibrium entropy) was formerly developed in this context. They also yield a dynamical version of the out-of-equilibrium phase transition present in the system, in terms of bifurcation of these invariant structures. We conclude by demonstrating the presence of a similar phenomenology in the dynamics of a Free Electron Laser.