Problems to
Quantum Gravity I: Canonical General Relativity WS 20/21

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Sheet 5 — Handout: 30.11.2020 — To present: 07.12.2020

Problem 1: Dirac bracket and equations of motion

Consider the action

$$\int dt \, \big(f(\vec{x})_i \dot{x}^i - H(\vec{x})\big) \tag{1}$$

where $f(\vec{x})$ means that f depends on all of the x^i, and similarly for $H(\vec{x})$. f_i is chosen such that

$$\omega_{ij} := \frac{\partial f_j}{\partial x^i} - \frac{\partial f_i}{\partial x^j} \tag{2}$$

is non-degenerate everywhere in configuration space.

a) Compute the Lagrangian equations of motion.

b) Perform the Legendre transform and show that the Dirac stability analysis leads to a purely second class system.

c) Compute the Dirac bracket $\{x^i, x^j\}_*$ and use it to derive the unconstrained Hamiltonian equations of motion. Compare with the Lagrangian equations of motion.

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Problem 2: Henneaux & Teitelboim: problem 1.12

Consider a system with second-class constraints $\chi_\alpha \approx 0$. Let F be an arbitrary phase space function.

a) Show that one can define another function F^*, equal to F on the surface of the second-class constraints $\chi_\alpha \approx 0$,

$$F^* = F + \nu^\alpha \chi_\alpha \tag{3}$$

such that

$$\{F^*, \chi_\alpha\} \approx 0 \tag{4}$$

(Throughout this exercise, "weak equality" means "equality modulo the second class constraints only.") In particular, one finds that the function H^* is just the first-class Hamiltonian.

b) Show that $\{F^*, G\} \approx \{F^*, G^*\}$.

c) Verify that the Poisson bracket $\{F^*, G^*\}$ of F^* and G^* is weakly equal to the Dirac Bracket $\{F, G\}_*$ of F and G.

d) Show that $\{\{F, G\}_*, H\}_* \approx \{\{F,^* G^*\}, H^*\}$.

e) Infer from (c) and (d) that the Jacobi identity for the Dirac bracket holds weakly.

Problem 3: BONUS: Jacobi identity

Proof the Jacobi identity for the Dirac bracket by direct computation.