**Problems to Introduction to Quantum Gravity I** WS 18/19

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**Problem 1: Metric determinant**

Show that in the ADM decomposition

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^a N_a & N_a \\ N_a & q_{ab} \end{pmatrix} \qquad g^{\mu\nu} = \begin{pmatrix} -1/N^2 & N^a/N^2 \\ N^a/N^2 & q^{ab} - N^a N^b/N^2 \end{pmatrix} \tag{1}$$

of the spacetime metric, we have $\sqrt{-\det g_{\mu\nu}} = |N|\sqrt{\det q_{ab}}$.

**Problem 2: Extrinsic curvature**

**a)** Show that $K_{\mu\nu}$ is symmetric.

Hint: $n_\mu \propto \partial_\mu f$, with $f(X_t(x^a)) = t$.

**b)** Show that $K_{\mu\nu} = \frac{1}{2}\mathcal{L}_n q_{\mu\nu}$.

Hint: Show that $2\nabla_{(\mu} n_{\nu)} = \mathcal{L}_n g_{\mu\nu}$.

**Problem 3: BONUS: Gauß and Codacci equations**

Derive the Gauß equation

$$R^{(3)}_{\mu\nu\rho\sigma} = -2K_{\rho[\mu}K_{\sigma]\nu} + q_\mu{}^{\mu'} q_\nu{}^{\nu'} q_\rho{}^{\rho'} q_\sigma{}^{\sigma'} R^{(4)}_{\mu'\nu'\rho'\sigma'} \tag{2}$$

and the Codacci equation

$$R^{(4)} = R^{(3)} + \left(K_{\mu\nu}K^{\mu\nu} - K^2\right) + 2\nabla_\mu \left(n^\nu \nabla_\nu n^\mu - n^\mu \nabla_\nu n^\nu\right). \tag{3}$$